Presupposition Repairs: a Static, Trivalent Approach to Predicting Projection*

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Abstract

This paper presents a new theory of presupposition projection similar to the strong Kleene (1952) trivalent logic. The primary merits of the present theory are its unique and fine-grained predictions regarding the presuppositions associated with quantified sentences, and its predictiveness; the issue of overgeneration discussed by Soames (1989) and Heim (1990) is avoided within a static, trivalent theory of pesupposition. A number of alternative theories based on the strong Kleene approach are also explored, and their strengths and weaknesses are compared.

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1 Approaching the Projection Problem

The essence of the projection problem for presuppositions can be found in the fact that (1-a) presupposes that Smith is incompetent, but (1-b) does not:

(1) a. Smith knows that he is incompetent.
b. Smith is incompetent and knows it.

Seeing that a complex sentence may have weaker presuppositions than its simpler constituents, we find ourselves in need of a nontrivial theory of presupposition projection to provide us with rules for computing the presuppositions of a whole from those of its parts. This paper presents the case for one such theory, framed in a trivalent semantics, without recourse to the resources of dynamic semantics or to an account of presuppositions as implicatures.

There is, of course, no shortage of theories of presupposition projection, but recent work (such as that of Chemla (2007) and of Schlenker (2006, 2008a)[1]) has raised a number of concerns about the data taken for granted by most established approaches[2] and about the conceptual and methodological limitations of older theories. The present theory is an attempt to address these issues, and an exercise in showing that these issues can be addressed within a static semantics.

I begin with a review of the concept of presupposition and of the projection problem, followed by an overview of the major new and old data for quantifiers and connectives. Next, I review a number of related theories, so that the empirical differences between the new theory and those that came before will be clear. Finally, the theory itself is presented in formal detail. An interesting possible refinement is discussed in appendix A.

1.1 Presupposition

The presuppositions of an utterance are generally characterized as what that utterance takes for granted. Of course, in normal conversation, we take all

[1]Although this recent work has called new attention to the issues involved, uncertainty about the presuppositions of quantified sentences can be seen as a component of longstanding disputes about universal and existential presuppositions, and the overgeneration problem recently discussed by Schlenker is also mentioned by Soames (1989) and Heim (1990).

[2]Most work has attributed the same projection behavior to all quantifiers, while, on closer inspection the strength of judgments varies considerably between quantifiers.

kinds of things for granted, but some such things are tied specifically to particular expressions. If I say *the king of France is bald*, I am presupposing that France has a king, and asserting of that king that he is bald; if I say *Smith has stopped smoking*, I am presupposing that Smith once smoked, and asserting that this state of affairs has changed; and if I say *Jones knows that sugar is often refined with bone char*, I am presupposing that sugar is often refined with bone char, and asserting that Jones knows this.

There are two main families of empirical and intuitive phenomena associated presupposition. The first thing to note is that, since a presupposition is taken for granted, it is very poor conversational form to presuppose something that might be contentious - the presuppositions of any utterance should either be among the commonly accepted facts of the conversation or else be claims concerning which the speaker has some kind of authority.[3] Presuppositions made without meeting these criteria will result in discourses that sound funny or result in confusion or resentment. It is for this reason that questions beginning with *have you stopped ...* are famously loaded. When its presuppositions are not well-supported, an utterance is not merely wrong - it is broken.[4]

Second, the idea that presupposition failure goes beyond an utterance's normal correctness or incorrectness gives rise to one of the most common empirical tests of presupposition: the negation test of Keenan (1971). Under this test, a sentence φ presupposes another sentence ψ (in a particular utterance context) if ψ follows from both φ and its natural negation. For the presupposition triggers *the*, *know*, and *stop*, we can see this in action in the following entailments, which remain valid with or without negation of the premise:

(2) a. The king of France (is/isn't) bald
$\vDash$ France has a king.
b. Smith (has/hasn't) stopped smoking

[3]Within many discourse models, this notion can be understood as meaning that presupposition is the part of semantic meaning that gets evaluated against the common ground, or against some contextually salient set of generally accepted facts, as distinct from the primary, asserted content, which is evaluated against the private beliefs of the speaker.

[4]Of course, this brokenness need not correspond to any particular theoretical status. It might be a property associated with any of a wide variety of semantic, pragmatic, or perhaps even syntactic modules. This intuition of brokenness does not by itself force us to abandon a bivalent semantics: it might well be that every sentence suffering presupposition failure is either true or false, and that failure is encoded in some other way. This last approach is in fact taken by many very promising pragmatic accounts, including those recently offered in Schlenker (2006, 2008a), Chemla (2008), and Simons (2006).

$\vDash$ Smith once smoked.

c. Jones (knows/doesn't know) that sugar is often refined with bone char
$\vDash$ Sugar is often refined with bone char.

It is, of course, not *a priori* clear that the negation test and the discourse intuitions are picking out the same phenomena, and each criterion is subject to numerous confounding factors. Nonetheless, throughout this paper, I will adopt the standard assumption that these two criteria pick out the same phenomenon of presupposition.

1.2 Projection

Under standard accounts of presupposition, the presuppositions of a sentence must be met for us to even reach the point of considering its other semantic content. Depending on our theory, a sentence that suffers presupposition failure might still be true or false, but even if the sentence has a truth value, the presupposition failure makes it irrelevant in ordinary conversational circumstances. In light of this, we might simply expect a trigger with unsatisfied presuppositions to poison any constituent in which it appears - after all, once we have an item that cannot be evaluated, how would we hope to evaluate its combination? This is, in fact, what is explicitly done for presuppositions in early accounts like that of Keenan (1970), and it is at least implicit in the simple accounts of presupposition based on partial functions that are sometimes used in works primarily concerned with other topics.

This might be the obvious thing to expect, but it's been known for quite a while that it's not at all what we see. It is quite easy to devise examples of sentences that contain presupposition triggers with unsatisfied presuppositions, that are nevertheless quite true, as (3) attests:

(3) Either France has no king, or the king of France is a recluse.

This is not simply a matter of certain contexts suppressing presupposition effects - in many cases, the presuppositions of a complex sentence containing a trigger are different from the presuppositions we'd associate with that trigger in a simpler sentence.

In general terms, the projection problem is just the problem of how to compute these effects. From very early in its history as a subject of research, however, the projection problem has served as a key battlefield for disputes about the semantic and pragmatic machinery associated with presuppositions, and for methodological disputes between different semantic theories. Perceived methodological excesses of pre-dynamic approaches to projection were given by Heim (1983) as a motivation for the dynamic approach to projection and the dynamic semantics in general,[5] and more recently, Schlenker (2006) complained of methodological weaknesses of classic dynamic accounts in advocating for a pragmatic theory of presupposition. This paper is in large part an exercise in showing that the theoretical concerns discussed by Heim (1983) and Schlenker (2006) are not inherently liabilities of static or semantic approaches, by producing an example of a static, semantic theory of projection that responds to their concerns. It is also an attempt to show that a revised theory in the tradition of earlier work on trivalence like that of Kleene (1952), Peters (1977), and van Fraassen (1969) can make interesting and novel predictions about some cases where new empirical concerns have recently been raised.

2 Background: Data

The projection behavior of the truth functions *and*, *or*, and *not*, of common nominal quantifiers, and of the conditional, has been discussed in the literature extensively and from a number of angles. I will be ignoring the conditional here (because, in the absence of an agreed-upon theory of conditionals, I am unable to make any predictions for it, especially since this paper develops only the extensional version of my theory), but the (lamentably incomplete and tentative) presupposition data for the major truth functions and for a number of quantifiers will be described in this section. Where the data are uncertain or ambiguous, I have tried to note these difficulties, and I have called attention to a few of the confounding factors that I find relevant, although the confounds discussed here are not meant to be an exhaustive list.[6]

[5] Here I mean the complaint of Gazdar (1979) that the theory of Karttunen and Peters (1979) treated presupposition inheritance and truth-conditionalc ontent as entirely separate lexically specified components. I won't say more about this complaint because any theory that addresses the overgeneration problem discussed elsewhere will also avoid this issue.

[6] In particular, contextual restrictors on quantification are mentioned only peripherally, and nothing is said about issues related to the proviso problem. I also avoid explicit discussion of local accommodation, in large part because I suspect that local accommodation as a distinct phenomenon may disappear or become quite marginal once a good projection theory is combined with a good theory of context-specific domain restrictions. The phrasing of some of my examples has been selected with an eye towards controlling for these effects (in particular, to make context-specific domain restriction less likely, most of

2.1 Negation and Testing Presuppositions

To begin, the presuppositions of a sentence are preserved under negation - this is not so much an empirical fact as a part of the standard working definition of presupposition (see, for example, Keenan (1971)) - it is true that implicit in this working definition is the idea that being preserved under negation should be correlated with the major discourse properties of presupposition, and it is also true that some inferences we might not wish to treat as presuppositions (certain relevance implicatures, say) may share this property, but since the negation test is usually taken as the primary evidence of presupposition, any attempt to motivate it would have to deal with foundational questions beyond the scope of this work.

Simple negation tests often run into difficulty with complex sentences built around connectives, since in these cases simple negation with *not* is often syntactically questionable. A further possible confound for the negation test is that the English negation operator *not* participates in scope interactions, and in particular often scopes below certain modals and below subject position quantifiers (indeed, many quantifiers, when in subject position, are almost totally incapable of scoping under negation, requiring strongly biasing discourse context or special intonation that seems to indicate focus or metalinguistic negation). For this reason, simple negation data are not always available, and we must resort to other tests such as the formation of *yes/no* questions,[7] longer embedding structures that approximate the semantics of negation but display different scope bias[8], conditions for discourse deniability, and, as discussed in von Fintel (2004), availability of *wait a minute!* responses. The claim that simple expressions of negation or disagreement are linked to acceptance of presuppositions, while *wait a minute!* responses are used to call what has just been presupposed into dispute, suffers from the fuzziness associated with many discourse coherence tests, but nevertheless appears to have some basis. In, particular the responses marked with a # below are odd in a way that the unmarked ones are not:

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my quantified phrases have highly specific restrictors, often involving partitives or explicit mention of numbers of entities or narrow contexts), but I do not pretend to have conclusively ruled out all these effects, and none of these issues are systematically addressed here.

[7] *yes/no* question data are not used here because for complex and quantified sentences they encounter numerous scope-related and syntactic issues.

[8] Examples include the results of taking the sentence under investigation and embedding it under *it is not the case that ...*, *I doubt that ...*, or *I deny that ...*

(4) Smith: Watson knows that the Earth is flat.
Jones: Wait a minute! The Earth isn't flat.

(5) Smith: Watson knows that the Earth is flat.
Jones: # I disagree; the Earth isn't flat.

(6) Smith: Watson knows that the Earth is approximately spherical.
Jones: # Wait a minute! Watson believes that the Earth is cylindrical.

(7) Smith: Watson knows that the Earth is approximately spherical.
Jones: I disagree; Watson believes that the Earth is cylindrical.

One test of which we should be wary (for reasons discussed in von Fintel (2004)) is the direct elicitation of intuitions as to whether an untrue sentence is "false" or "neither true nor false" - to ask such questions is to ask for theory-laden rather than natural judgments,[9] and the judgments produced in such cases are known to sometimes be at odds with the results of other well-motivated tests of presupposition.

2.2 Binary Connectives

2.2.1 Conjunction

With negation out of the way, let's consider the less trivial case of *and*. To begin recall, the classic observation that (8) presupposes (9):

(8) The king of France is bald.

(9) France has a king.

These presuppositions need not be preserved under coordination, however, as we can see if we make a presuppositional sentence like (8) the second half of a conjoined sentence. So, for example, (10) does not presuppose (9):

(10) France is a monarchy and the king of France is bald.

The claim that a presupposition is absent here needs some motivation: (9) must be true for (10) to be true, and scoping simple negation of *and* is not really possible here, so whether the entailment from (10) to (9) is a

[9] More precisely, we might say that the English word *false* as a predicate of propositions, need not correspond precisely to the falsehood truth value 0 used in most semantic theories, where the main purpose of the system of truth values is to model entailment, informative content, and the like.

presupposition or merely an assertion can only be decided with the aid of other tests. First, let us try a paraphrased negation:

(11) I deny that France is a monarchy and the king of France is bald.

In uttering (11) the speaker in no way commits herself to the truth of (9). This can be seen in further detail by considering the longer discourse.

(12) The author of this leaflet describes France as a monarchy with a bald king. I deny that France is a monarchy and the king of France is bald. In fact, France is a republic.

Such examples do have the tendency to feel unwieldy and to take on a certain flavor of metalinguistic negation. However, the available responses to (10) also support the idea that there is no presupposition that France has a king:

(13) Jones: France is a monarchy and the king of France is bald.
Smith: # Wait a minute! France is a republic.

(14) Jones: France is a monarchy and the king of France is bald.
Smith: I disagree; France is a republic.

This contrasts with the case where the existence of the King is presupposed:

(15) Jones: The king of France is bald.
Smith: Wait a minute! France is a republic.

(16) Jones: The king of France is bald.
Smith: # I disagree; France is a republic.

Does (10) have any presuppositions at all that pertain to the French government? It turns out that it has some. Suppose in particular that France is a monarchy, but that it does not have a king (because its head of state is female, or, perhaps, because its head of state is male but is given some title not deemed equivalent to the English word *king*): here the sentence suffers presupposition failure. We get failure-like, rather than falsehood-like dialogue patterns in such cases, as can perhaps be seen more easily if we substitute actual monarchies without kings:

(17) a. Jones: The Netherlands is a monarchy and the king of the Netherlands is bald.
Smith: Wait a minute! The head of state in the Netherlands is female!

b. Jones: Japan is a monarchy and the king of Japan is bald.

Smith: Wait a minute! Japan has an emperor, not a king!

(18) a. Jones: The Netherlands is a monarchy and the king of the Netherlands is bald.
Smith: # I disagree; the head of state in the Netherlands is female.

b. Jones: Japan is a monarchy and the king of Japan is bald.
Smith: # I disagree; Japan has an emperor, not a king!

(10) appears to have its presuppositions met so long as either France is not a monarchy or France has a king. In cases where France is not a monarchy, we see above the sentence is deniable as false, and gives no sign of presupposition failure, but in cases where France is a monarchy, we find that it must have a King to avoid failure. The resulting disjunctive presupposition is approximated by the claim that if France is a monarchy, then the France has a king, which is the presupposition usually given. The literature usually claims that (10) has a conditional presupposition, which is true to the extent that the material conditional can be expressed by the standard conditional of English, but I am more comfortable with the disjunctive statement of the presupposition, because in my capacity as a semantic theorist I have a bit more faith in my understanding of disjunctions than in my understanding of conditionals.

When we reverse the order of the conjuncts, the situation seems very different:

(19) The king of France is bald and France is a monarchy.

A common view (adopted in, e.g., Heim (1983), Schlenker (2006)) is that this sentence presupposes that France has a King. This seems reasonable, since the following dialogue sounds inappropriate:

(20) # Jones: The king of France is bald and France is a monarchy.
Smith: I disagree; France is a republic.

If the inappropriateness of this dialogue is to be found in Smith's response, then it would appear that stronger presuppositions are at work in (19) then in (10) - however, it is not immediately clear that that is what is going on. (19) sounds noticeably more odd than (10), while modifying (19) to form the analogous true sentence (21) does nothing to reduce this sense of oddness:

(21) The king of Norway is bald and Norway is a monarchy.

It appears that the kind of redundancy involved in these examples creates problems independent of the issue of presupposition - indeed, non-presuppositional examples of a second conjunct that is entirely redundant with the first display the same awkwardness that we saw in (20):

(22) #Jones: Watson devoured the cinnamon bun and she ate it.
Smith: I disagree; Watson doesn't eat refined sugar.

Further, the problem is not alleviated by the substitution of *wait a minute!* for *I disagree*:

(23) #Jones: The king of France is bald and France is a monarchy.
Smith: Wait a minute! France is a republic.

It is clear that there is some source of asymmetry in conjunction, but it is not immediately clear whether the projection is itself asymmetrical, or whether in cases where the first conjunct is presupposition-laden the simple test examples are rendered problematic by some more general pragmatic effect.

2.2.2 Disjunction

Disjunctions also project presuppositions in a weakened or conditionalized form:

(24) France is a republic or the king of France is bald.

(24) is taken to presuppose that either France is a republic or it has a king. A *wait a minute!* response, but not a simple expression of doubt or disagreement, is warranted if France is governed by, say, a quasi-Maoist bureaucracy (so that it lacks a king but could not reasonable be called a republic), and the sentence is unobjectionably true if France is a republic, or if it has a bald king, but we only permit simple disagreement or doubt responses in the case where one accepts that France has a king but denies that he is bald. Unlike the case of conjunction, this projection behavior appears symmetrical[10] (although these symmetry claims are disputed in the literature) - everything said above holds true for (25):

(25) The king of France is bald, or France is a republic.

[10] At least, *or* appears symmetrical with the triggers I've been discussing here. For some others, especially *too*, we do see asymmetry effects.

This is easier to see if we consider a case where the presupposition involved is not known to be false, as with the following:

(26) The bathroom is well hidden, or there is no bathroom.

(27) Either Watson has stopped eating refined sugar, or she doesn't care for cinnamon buns.

If the asymmetry of *and* is a genuine asymmetry of presuppositions, then a good theory of projection will need to offer some explanation or excuse for the fact that order-dependence is seen with *and* but is often absent with *or*.

2.3 Quantifiers

2.3.1 Nuclear Scope

The presuppositions of quantified sentences are uncertain even in simple cases - even the truth conditions of sentences that include a presuppositional predicate in the scope of a quantifier are in dispute, and things are even more obscure when we attempt to determine which cases of untruth are cases of simple falsehood, and which of presupposition failure. One common claim is that presuppositions give rise to universal projections, so that (28) presupposes (29):

(28) Each of these six philosophers has stopped drinking.

(29) Each of these philosophers once drank.

The inference from (28) to (29) is unobjectionable, although it is not immediately clear whether all cases where (29) is false result in presupposition failure, or whether some result in failure and others in falsehood. More problematically, these universal inferences, whatever their character, are questionable in some cases. According to my intuitions, at least, the inference remains valid in the case of (30), but not for cases like (31), (32), and (33):

(30) None of these six philosophers has stopped drinking.

(31) At least one of these six philosophers has stopped drinking.

(32) Less than four of these six philosophers have stopped drinking.

(33) Exactly two of these six philosophers have stopped drinking.

For me (33) in particular is easily satisfied if the two who stopped drinking were the only two who ever drank. The exact presuppositions are unclear, however, and judgments are variable: Chemla (2007) finds that about half of French speakers report that universal inferences are suggested by various numerical quantifiers like those given above (with significantly higher rates for universal and *no*-like quantification), and my own (unsystematic) impression from conversations with others has been that judgments here vary a great deal between speakers and between judgment occasions. In light of the empirical uncertainty, any precisely articulated predictive theory is likely to make predictions that go well beyond the established data - this paper will not resolve the empirical questions, but I will make such fine-grained predictions, and they will be broadly compatible with my own intuitions on the matter.

2.3.2 Restrictors

Much less has been said about the role of presuppositions in the restrictor positions of quantifiers than has been said about their nuclear scope. This is in part because we must incur a significant degree of syntactic complexity even to place a presupposition trigger in a restrictor. Many of the easiest examples to produce involve relative clauses, and here we seem to get almost no presuppositional force at all:

(34) Every student in my category theory course who stopped drinking failed the midterm.

I judge sentence (34) to be quite true so long as the students in the class include at least one who used to drink but no longer does, and all the students with this property failed.[11] The presupposition appears to simply be narrowing the restrictor. A greater array of restrictor data would be desirable, since it is possible (as mentioned by Schlenker (2008a)) that this effect - where the presuppositions and assertions of a predicate are nearly indistinguishable when it is placed in the restrictor of a quantifier - is in fact

[11] The reference to the particular course is intended to reduce the potential impact of contextual domain restriction - we could go further with any of the following rather stilted examples, for which I have the same intuitions:

(i) a. My category theory course has forty-two students in it, and every student in that course who stopped drinking failed the midterm.
b. Every student in my forty-two student category theory course who has stopped drinking failed the midterm.
c. There are forty-two students in my category theory course, and every one of them who stopped drinking failed the midterm.

a feature of the projection behavior of relative clauses, and not of restrictors.

3 Background: Theory

This section sets out some conceptual and historical background. It is concerned primarily with reviving the strong Kleene trivalent logic of Kleene (1952) and related systems discussed in van Fraassen (1969) and Peters (1977) as viable approaches to the theory of presupposition projection. These systems have a long history in the logical and linguistic literature, predating the first reasonably general formalizations of dynamic semantics, but have remained relatively obscure as treatments of projection. Since these theories are the closest relatives of my own, I want to explore their predictive strengths and weaknesses in some detail. This section also attempts to familiarize the reader with the conceptual apparatus common to many of these theories, and to call attention to some important influences and related work.

3.1 A Word on Trivalence

Many of the theories presented below model presupposition with a third truth value #, which is assigned to declarative clauses that suffer presupposition failure, and treat the connectives *and* and *or* as truth functions. That is, when we do our projection calculations to see how # percolates up through the compositional system, we are able compute the failure conditions for a sentence. The presuppositions of a sentence are just the logical complement of its failure conditions. The discourse significance of presupposition and presupposition failure is left to the discourse model, presumably with a rule that it is inappropriate to utter a sentence the presuppositions of which you think another conversational participant might reasonably dispute. In any case, I will have nothing more to say on the pragmatic act of presupposing, but will concern myself entirely with the process of computing failure conditions.

It will be good to look at how to derive presuppositions from other semantic facts that implicitly characterize them. As already mentioned, if we know the conditions of presupposition failure for a sentence, then we negate a description of the failure conditions to get the presuppositions. If we know the truth conditions and falsehood conditions of a sentence, then the presuppositions of the sentence are exactly the disjunction of the truth and falsehood conditions. For the case of truth-functional connectives, we can state the above in terms of the truth tables: given a truth table for a trivalent connective, we can calculate its presuppositions by taking a disjunction of descriptions of all the lines where the lines where the connective outputs a value other than #, or, equivalently, we can take the conjunction of all the lines associated with #, and negate that. Of course, these rules will most often produce very unwieldy descriptions of the presuppositions, but it will usually be possible to paraphrase the resulting conditions more succinctly.

It is important to remember that the things we will directly see percolating up are truth values (possibly including the failure code #). From this behavior, we can compute the projection behavior, but trying to interpret cases where # "projects" up directly as corresponding to the projection of a presupposed proposition will lead to confusion and possible error in the cases where the system predicts conditional or disjunctive presuppositions.

3.2 Repair and Substitution

Most of the theories described here will be theories that deal with presupposition failure in an argument by considering repairs of the presuppositional element that avoid the failure, and evaluating by substituting those repairs for the original, problematic, value. The reason that presuppositions sometimes project is that sometimes more than one repair is considered, and in some cases, but not in others, substituting different repairs yields different outcomes - presupposition failure in larger constituents results from this kind of conflict between different repairs for the failures of smaller constituents. For illustrative purposes, let's consider a very informally how this might play out with a few simple examples.

First, in the case of presupposition failure in a sentential argument of a truth function, the truth value of # has two obvious repairs: 0 and 1. That is, if a sentential argument of a truth function suffers presupposition failure, we don't care so long as it doesn't matter whether the sentence is true or false. Consider first this familiar case:

(35) France has a king, and the king of France is bald.

Evaluating this sentence against the facts of the real world, we find that the first conjunct is false, and the second conjunct takes the truth value #. We therefore consider two repairs - we entertain pretense that we have a true second conjunct, and also the pretense that we have a false one. In each case, however, the sentence is false, since France does not have a king, and so the conjunction of *France has a king* with any sentence of any bivalent truth value will produce a false conjoined sentence. No matter what repair we choose, we get the same result, so this sentence is merely false. In the hypothetical world where France *does* have a king, matters are of course different. There, the truth value of the second conjunct very much matters, but in that case the second conjunct will never suffer presupposition failure, since if France has a king then an attribution of baldness to him will be either true or false. Thus, this sentence lacks in all cases the presuppositional force that would be associated with the second conjunct by itself.

We can also verify that, given these repair options, presuppositions are still unchanged under negation. Consider the following:

(36) Watson hasn't stopped drinking.

The case of failure for the sentence being negated is the case where Watson doesn't drink and never did. In this case, we want to take the negation of each repair of the negated #, where the two repairs are 0 and 1. Since the negation of 0 is 1 and the negation of 1 is 0, the two repairs do not produce agreement, so the negation gets the value # when the presuppositions of the negated sentence aren't met, and the sentence and its negation share the same presuppositions.

Now, let's look at one possible repair approach to a quantification example:

(37) Each of these three logicians has stopped eating refined sugar.

Suppose that there are three logicians under discussion - Kurt, Bert, and Rudy - and that none of them has ever eaten refined sugar. That is, for all $a \in \{[\![\text{Kurt}]\!],[\![\text{Bert}]\!],[\![\text{Rudy}]\!]\}$, $[\![\text{has stopped eating refined sugar}]\!](a)=\#$. In this case, we will always consider at least two repairs - the repair where all items mapped to # are instead mapped to 0 (i.e., we pretend that Kurt, Bert, and Rudy all persist in eating refined sugar), and the case where all items mapped to # are instead mapped to 1 (i.e., we pretend that Kurt, Bert, and Rudy all formerly ate refined sugar, but have stopped). Clearly, under the first repair, the sentence is false, while, under the second it is true. Because of this conflict between repairs, (37) suffers presupposition failure if none of the logicians has ever eaten refined sugar, as do the following sentences, assuming the same repairs:

(38) At least one of these three logicians has stopped eating refined sugar.

(39) None of these three logicians have stopped eating refined sugar.

Now suppose instead that Bert used to eat refined sugar and persists in the practice, while Kurt and Rudy have always avoided the stuff. In this case, it is false that Bert has stopped eating refined sugar, so the predicate maps him to 0, while Kurt and Rudy are still mapped to #. If we consider again just the repair where we replace all the #s with 1 and the one where we replace all the #s with 0, we see that both map Bert to 0, so in both cases it is false that each logician has the property characterized by the repaired predicate. Thus, given these repairs, we predict that (37) is false even though some of the logicians do not meet the presuppositions of the predicate.

Above, we often saw that it was helpful to frame repairs in terms of pretending that the situation was different in a way that avoided the value #; besides imagining repairs in terms of pretense, we can imagine them in terms of constituent substitution. In this case, the repair that differs ⟦has stopped eating refined sugar⟧ in that all entities that would be mapped to # are instead mapped to 0 is roughly equivalent to ⟦used to eat refined sugar but doesn't anymore⟧, while the repair that substitutes 1 for # is approximately equivalent to ⟦doesn't eat refined sugar⟧. Getting the same truth values for (37) substituting both repairs will be the same as insisting that, in the world as it is, we get the same truth values for both syntactic substitutions:

(40) a. Each of these three logicians used to eat refined sugar but doesn't anymore.
b. Each of these three logicians doesn't eat refined sugar.

Both the syntactic substitution approach and the pretense approach are, of course, only heuristics, but they can be quite helpful in reasoning through repair strategies.

To derive the full presuppositional behavior of any of the above sentences, we would have to work through every possible circumstance, and determine which ones cause the repairs to conflict - the proposition presupposed will be the set of worlds in which the repairs don't conflict. This task will be made more complicated in various ways, but in particular it will depend on how the set of repairs is determined (below, we will consider a number of theories that differ in how they select the set of repairs). The set of repairs will always depend on the item being repaired, and in some approaches (including the main one I finally adopt below) it will also depend on the context of evaluation, but one thing that will remain the same is that a repair for any presuppositional item will always agree with the item being repaired in those cases where presupposition failure does not come into play. If Kurt has never eaten refined sugar, we can entertain either the pretense that he is a current consumer of the substance or the pretense that he is a reformed one, but if Kurt used to eat refined sugar and has since sworn it off, we are forced to keep this constant in all pretenses we consider in the name of repair (or, on the syntactic substitution heuristic, we can only substitute other predicates that are also true of Kurt).

For the cases mentioned above, we seem to be deriving the right results. To build a theory around this intuition we will need to make precise the notion of there being or not being a difference between repairs, specify the point in the evaluation at which this not mattering is checked, and, as noted above, give explicit rules for computing the set of possible repairs (there is, of course, also the possibility of explicitly building in other components that manipulate presuppositions and repairs). The key to a successful theory is to define the notions of repairs and repair assessment in a way that gives good results for whatever presuppositions and structures we throw at it.

3.3 Influences and Related Work

The present theory will be easier to understand in the context of some related work, which helps both to motivate my particular decisions and to develop some intuitions for the general approach by exploring some easier-to-follow related formalisms. The discussion of the strong Kleene logic for connectives is assumed as background in later sections, but some of the other formal material, especially the Peters-Kleene system and the discussion of how strong Kleene may be generalized to quantifiers, is included to show how the approach I eventually adopt contrasts with earlier work and some natural generalizations thereof, and may be skimmed or skipped at the reader's discretion (although it will help to build familiarity with some tricks developed later).

The discussion of quantifiers here and in subsequent sections makes use of some new notations and formal devices. The most important of these is the idea of output substitution, which it will be best to define in advance. For function f, we define $f^{a/b}$ to be the function such that, for any argument sequence $\vec{u}$, $f^{a/b}(\vec{u}) = a$ if $f(\vec{u}) = b$, and $f^{a/b}(\vec{u}) = f(\vec{u})$ otherwise. The two relevant cases are those of $f^{0/\#}$ and $f^{1/\#}$. For f a unary predicate, $f^{0/\#}$ can usually be expressed in ordinary English by conjoining the assertions and presuppositions of f, and $f^{1/\#}$ can usually be expressed in ordinary English by taking the disjunction of some expression of $f^{0/\#}$ with the negation of the presupposition of f. Thus, if $f = [\![$knows that he is an idiot$]\!]$, $f^{0/\#} \approx [\![$is an idiot and knows it$]\!]$, and $f^{1/\#} \approx [\![$isn't an idiot or is an idiot and knows it$]\!]$.

Finally, a note is in order about my terminology - I call two systems below the strong Kleene and Peters-Kleene systems because the formulations below are inspired by systems developed by Kleene (1952) and Peters (1977), and because they coincide with these systems where commonly studied truth-functions are concerned. However, these systems are presented below in generalized forms and in quite different terms from the original descriptions, so the reader should refer to the original authors before attributing to them any specific view involving the systems named after them below.

3.3.1 Strong Kleene

The first system to consider is the strong Kleene trivalent logic of Kleene (1952). This logic can be though of as analyzing each instance of presupposition failure as an instance where the value is unknown. Because the value is unknown, we can only proceed if we are convinced that all possible values would yield the same result, so we will have a fairly expansive notion of what repairs need to be considered. If all the members of the repair set yield the same result, then we have proven to ourselves that that is the result, even if the value of a particular argument remains unknown. The application of this intuition to the study of presuppositions has a long but somewhat sparse history, going back at least to van Fraassen (1969) (where the formally rather different but similarly motivated theory of supervaluations was used).

In a bit more detail, a truth-functional strong-Kleene system has three truth values - besides the usual 0 and 1, there is the third value #, which is generated by failure and interpreted as uncertainty. To evaluate an n-ary truth function on a sequence of n arguments, the strong Kleene system evaluates every possible repair of the argument list, where a repair is a list of 0s and 1s that has 0s everywhere the original argument list has 0s and 1s everywere the original argument list has 1s, and may contain either a 0 or a 1 for any position where the original argument list had a #. If all possible repairs produce the same result, that is the value we get from the truth function. If different repairs produce different results, then the value of the whole term is #.

Since my system agrees with the strong Kleene system in the propositional case, it will be useful to flesh out the strong Kleene predictions. To begin, consider the case of negation. To evaluate $\neg\#$, we must consider two repairs: 1 and 0. Since $\neg 0 = 1$ and $\neg 1 = 0$ we have disagreement between these repairs, so $\neg\# = \#$ under the strong Kleene logic. For the other two possible arguments, no nontrivial repair occurs and as before $\neg 0 = 1$ and $\neg 1 = 0$. This is the trivalent truth function that corresponds naturally to negation in a presuppositonal system - it says that the circumstances that make a sentence fail are exactly the same as those that make its negation fail, so a sentence and its negation have the same presuppositions.

For conjunction, there are nine possible pairs of truth values. Four of these contain only 0 and 1, and so don't require us to consider alternative repairs and remain the same as in the bivalent case. Of the five remaining cases, four can be arranged into two symmetric pairs. First, $(0 \wedge \#) = 0$, since the two repairs to evaluate are $0 \wedge 0$ and $0 \wedge 1$, and both of these evaluate to 0, and by the same reasoning $(\# \wedge 0) = 0$. For $1 \wedge \#$, we try both repairs and find that $(1 \wedge 1) \neq (1 \wedge 0)$, so $(1 \wedge \#) = \#$, and so does $\# \wedge 1$. Finally, the four possible repairs for $\# \wedge \#$ do not all agree (for example, $(1 \wedge 1) \neq (0 \wedge 0)$), so $(\# \wedge \#) = \#$. The above considerations give us the following trivalent truth table for conjunction:

a	b	$a \wedge b$
1	1	1
1	0	0
1	#	#
0	1	0
0	0	0
0	#	0
#	1	#
#	0	0
#	#	#

It's worth taking a minute to understand what this truth table represents. The function $\wedge$ that we start with - the thing we ought to consider the lexical entry for *and* - is still the usual bivalent conjunction, definable by the accustomed four-line truth table. However, the strong Kleene system gives us a way to apply bivalent functions to trivalent arguments. Once one accepts the strong Kleene system, we are free to stipulate new bivalent truth functions, but the trivalent behavior we get for them is determined entirely by the strong Kleene rules.

Applying the truth table derived above to the concrete case of (10) (reproduced below as (41)), we find that if France is not a monarchy the sentence is false, and that if France is a monarchy the sentence is subject to failure if France has no king (since if France has a king then either he's bald or he's not, which will yield either truth or fasehood). So to avoid failure, what we require is that either France is not a monarchy or France is a monarchy with a king - this is exactly the disjunctive/conditional presupposition argued for earlier.

(41) France is a monarchy and the king of France is bald.

Perhaps more problematically, since the truth table is symmetrical, we produce the same predictions for the case where the order of the conjuncts is reversed.

Turning to disjunction, we reverse the pattern, with # percolating up in the presence of 0 but not of 1. As before, the four bivalent argument pairs create no need for repair and get their usual values. For $0 \vee \#$ (and likewise for $\# \vee 0$), we have $(0 \vee 1) \neq (0 \vee 0)$, so $(0 \vee \#) = \#$. On the other hand, since $(1 \vee 1) = (1 \vee 0) = 1$, we find $(1 \vee \#) = 1$ (and likewise for $\# \vee 1$). Finally, since (for example) $(0 \vee 0) \neq (1 \vee 0)$, and both of these are legitimate repairs for $\# \vee \#$, we find $(\# \vee \#) = \#$, so the strong Kleene system gives us the following truth-table for *or*:

a	b	$a \vee b$
1	1	1
1	0	1
1	#	1
0	1	1
0	0	0
0	#	#
#	1	1
#	0	#
#	#	#

Here we have a symmetrical disjunction, and it is associated with exactly the disjunctive presuppositions we want. This means that, in the case of (24) (reproduced below as (42)), we predict a presupposition that either France is a republic or France has a king.

(42) France is a republic or the king of France is bald.

3.3.2 The Peters-Kleene Connectives

Arguments in Karttunen (1973) attempted to show that trivalent semantic theories of projection were inadequate on empirical grounds. Peters (1977), took exception to this, pointing out that the possibility of specifying trivalent truth functions for the connectives that would produce the predictions of Karttunen (1973). The system of Peters (1977) appears to have been developed mainly as an exercise to prove a formal point. The truth functions associated with the connectives are not formally derived from any general principle, and extensions to other types are considered only very briefly. Nonetheless, if we draw up truth tables for the *and* and *or* of Peters (1977)[12] and compare them to the strong Kleene connectives above, a natural generalizing intuition presents itself. Here are the truth tables for conjunction and disjunction as given in Peters (1977):

a	b	$a \wedge b$	$a \vee b$
1	1	1	1
1	0	0	1
1	#	#	1
0	1	0	1
0	0	0	0
0	#	0	#
#	1	#	#
#	0	#	#
#	#	#	#

The reader may verify that these Peters-Kleene connectives agree with the strong Kleene connectives on instances of # in the second argument position, but always yield # when there is a # in the first argument position. This can be made into a general principle by framing things incrementally: for each argument, we get failure if there is any imaginable sequence of bivalent future arguments such that there are two distinct repairs for the present argument that would produce different outcomes with that sequence of future arguments. If, on the other hand, all repairs for the present argument (of which there is only one when the argument is non-presuppositional) yield the same results, given every fixed choice of a sequence of future arguments, then we can pick any repair arbitrarily and 'run with it'. The asymmetry arises because in the second argument position there are fewer future arguments left open as possibilities, and more past arguments that are already known quantities.

Note that the Peters-Kleene connectives give us the (at least superficially desirable) asymmetrical *and* but also give us a more questionable asymmetrical *or*. It is also noteworthy that in order to derive the Peters-Kleene connectives from the general principle stated here, we need to frame

[12]In Peters (1977), there is also a treatment of *if... then...* derived from the material conditional in the same way, but at present I wish to avoid discussion of conditionals, which are known to introduce a number of complications to any semantic discussion.

things in terms of left-to-right order - on most binary-branching analyses of coordination, the argument to which the function is first applied on a hierarchical traversal (i.e., the second argument in linear order - with which the conjunction is analyzed as forming a constituent) will be the wrong one to treat as the "first" argument for purposes of the notions of incremental asymmetry discussed here, since then we would get asymmetry in exactly the opposite of the direction we want. The degree to which one finds this distressing will depend on one's views of the notion of compositionality and the syntax of coordination, but, in any case, it is worth remembering that the notion of order involved would require clarification (such clarification is, of course, also needed in the strong Kleene case, but there the symmetry makes it a less pressing issue, and makes hierarchical order of arguments a feasible conservative choice).

The present work is in the spirit of Peters (1977) in at least one sense - just as Peters (1977) was developed as a counterexample to claims in Karttunen (1973) that certain data demanded a pragmatic account of presupposition, so this paper was developed as a response to a similarly anti-semantic outlook present in some of the discussion in Schlenker (2006, 2008a). It is probable that this origin is part of the reason the theory of Peters (1977) was not developed further, and was presented without any formalization of the fact that the trivalent connectives proposed could be derived from a unifying principle.

3.3.3 Related Systems in Programming Languages

It should be noted something equivalent to the Peters-Kleene approach to boolean connectives is already found in the "short circuit"[13] implementations of the boolean connectives in many computer programming languages.[14] In the computational setting, this approach is motivated less by a need to control the propagation of errors than by a desire to save processing time. When we wish to compute the disjunction or conjunction of two expressions, the computational resources required to evaluate each of the expressions may be substantial - the computer for this reason first devotes its resources to evaluating the first argument of the truth function, and, if this one is enough to determine the value of the larger expression (that is,

[13] The short circuit evaluation of connectives can be regarded as a special case of "lazy evaluation" associated with call-by-name and call-by-need in some languages.

[14] Experienced computer programmers will cringe at some of my simplifications, but I hope they will also appreciate my attempts to make the core intuitions available, and to recognize their community's early contributions in this area.

if it is 1 for disjunction or 0 for conjunction), the value is returned and the second argument is never evaluated. In systems where serial computations give rise to an error as soon as they encounter an expression that cannot be evaluated,[15] any error generated by the first argument (which is always evaluated) will result in an error when the truth-functional expression is evaluated. On the other hand, errors associated with the second argument will only be generated if the second argument is evaluated, which will only happen if the first argument's truth value does not give us enough information to compute the value of the whole with certainty. The applicability of this core insight (and of a more generalized idea of one primary argument "controlling" the evaluation of the function) to linguistic presupposition has been recognized for a while, and some variations are discussed in Kracht (1994). Although I have not conducted a detailed search, I'm not aware of anything along these lines in the computational literature that provides an improvement over the strong Kleene and Peters-Kleene approaches from a linguistic perspective, but this piece of history seems worthy of recognition, especially since it provided an early inspiration for my exploration of these issues before I became familiar with Kleene (1952), Peters (1977), and van Fraassen (1969).

3.3.4 Generalization to Quantifiers

The generalization of these kinds of trivalent systems beyond the truth-functional case has been explored on a number of occasions, but, although it can lead to fascinating fine-grained predictions for quantifiers, I am not aware of any serious attempts to apply such generalizations to presupposition projection of generalized quantifiers. In generalizing the strong Kleene logic and related systems beyond the truth-functional case, we have a number of options. If f is a function[16] that maps some possible arguments to #, what are the acceptable repairs of f? The two most straightforward generalizations of the strong Kleene intuition are as follows:

- First, we can treat each argument list mapped to # by f as a separate case of failure, to be independently repaired with either 0 or 1. To do this, let $E(f)$ be the set of all functions g of the same type as f such that, for every complete argument sequence $\vec{x}$, $g(\vec{x}) \in \{0, 1\}$, and, if $f(\vec{x}) \in \{0, 1\}$ then $g(\vec{x}) = f(\vec{x})$. We can then let the repairs of f be

[15] Many runtime errors, such as those arising from division by zero, could reasonably be regarded as a kind of presupposition failure.

[16] For simplicity, I consider only functions of types that "end in" `t`.

exactly those functions in $E(f)$. This is the approach most in keeping with the "uncertainty" intuition underlying the strong Kleene system, and it is equivalent to the traditional treatment of quantification in strong Kleene theories, producing results like those associated with the "supervaluation quantifiers" of van Eijck (1996). This is the option I assumed in George (2007) and George (2008b).

- Second, we can replace all cases of # in the output of f with either 0 or 1, so that the repairs of f are just $f^{0/\#}$ and $f^{1/\#}$. (A modified version of this option is pursued in the theory I adopt below. This choice is based less on any certainty that this option is superior than on the fact that the differences of prediction are subtle enough that I do not know of any good empirical reason to favor one or the other, and and that the computations are easier when only two repairs need to be considered.)

Both of the above options will produce 0 and 1 as repairs for # if we consider truth-values to be zero-ary functions.

The basic strong Kleene and Peters-Kleene approaches make some interesting and fine-grained predictions for quantifiers under either of the above options. Below, I want to consider quickly the predictions for universal, existential, and *exactly* quantification, as seen in sentences (28), (31), and (33), reproduced below as (43), (44), and (45):

(43) Each of these six philosophers has stopped drinking.

(44) At least one of these six philosophers has stopped drinking.

(45) Exactly two of these six philosophers have stopped drinking.

For the present examples, where pesuppositions only occur in the last argument (the nuclear scope), the Peters-Kleene and strong Kleene approaches will behave the same (this is because, for strong Kleene, the only thing to vary is the nuclear scope, and for Peters-Kleene, the need to consider any possible completion of the argument list becomes trivial with the last argument). We will thus need to consider only the two ways of generating a repair set (although these will also turn out to yield very similar predictions). In all cases, it will be helpful to separately assess the truth conditions, falsehood conditions, and failure conditions of each sentence:

- Sentence (43) with $\{f^{0/\#}, f^{1/\#}\}$: Here we're applying the function $g = [\![\text{each philosopher}]\!]$ to the function $f = [\![\text{has stopped drinking}]\!]$, by considering the two alternatives $f^{0/\#} \approx$ [[used to drink and doesn't at present]] and $f^{1/\#} \approx$ [[doesn't drink]].[17]

- When is (43) true? For (43) to be true, it is required that $g(f^{0/\#}) = g(f^{1/\#}) = 1$. Since $f^{0/\#}$ picks out a subset of the entities picked out by $f^{1/\#}$, and g represents universal quantification, this is the case iff $g(f^{0/\#}) = 1$, which is to say if each of the philosophers used to drink but doesn't anymore (so the sentence does yield an inference that each of the philosophers used to drink).
- When is (43) false? For (43) to be false, it is required that $g(f^{0/\#}) = g(f^{1/\#}) = 0$. Since we have a universal quantifier, we know that if $g(f^{1/\#}) = 0$ then $g(f^{0/\#}) = 0$, so the sentence is false iff $g(f^{1/\#}) = 0$, which is to say if at least one philosopher used to drink and still does.
- When does (43) suffer presupposition failure? The sentence gets the value # iff it is neither true nor false. That is, we get a presupposition failure iff none of the philosophers currently drinks, but not all of them are ex-drinkers. (Thus, we compute for the sentence the disjunctive presupposition that either at least one philosopher currently drinks or all of them are ex-drinkers.)

• Sentence (43) with $E(f)$: The case for the $E(f)$ repair set is not really different from the case of $\{f^{0/\#}, f^{1/\#}\}$, because of the monotonicity of the universal quantifier. That is, both of these repairs are in $E(f)$, every repair in $E(f)$ picks out a superset of the one picked out by $f^{0/\#}$ and a subset of the one picked out by $f^{1/\#}$, and if a universal quantification is true of the subset then it is true of all its supersets, and if it false of the superset then it is false of all its subsets.

• Sentence (44) with $\{f^{0/\#}, f^{1/\#}\}$: Here we're applying the function $g =$ [[at least one philosopher]] to the function $f =$ [[has stopped drinking]], by considering the two alternatives $f^{0/\#} \approx$ [[used to drink and doesn't at present]] and $f^{1/\#} \approx$ [[doesn't drink]].

- When is (44) true? This sentence is true iff $g(f^{0/\#}) = g(f^{1/\#}) = 1$. Noting that this is an existential quantifier, this is true iff $g(f^{0/\#}) = 1$, which is to say if at least one of the philosophers

[17] Here *doesn't drink* is used as a simpler approximate paraphrase of *never drank or used to drink and now doesn't*.

used to drink but now does not. Note that this does not warrant a universal inference.

  - When is (44) false? By the same reasoning, $g(f^{0/\#}) = g(f^{1/\#}) = 0$ iff $g(f^{1/\#}) = 0$, since it's always harder to make existential quantification false over a more inclusive nuclear scope. That is, (44) is false iff there is no philosopher who doesn't drink, which is to say if every philosopher drinks.
  - When does (44) suffer presupposition failure? Presupposition failure occurs in the gap case, so, here, when not every philosopher drinks and no philosopher is an ex-drinker - so we presuppose that either every philosopher drinks or some philosopher is an ex-drinker. Another way of framing this is that the sentence has basically existential presuppositions in positive instances but basically universal ones in negative instances. This prediction is a bit unexpected, but, I think, rather appealing.

- Sentence (44) with $E(f)$: As with the universal quantifier, with the existential we find no difference between the two options for repair sets discussed so far. If the existential quantification is true of $f^{0/\#}$, it is true of all the other (weaker) predicates in $E(f)$, if it is false of $f^{1/\#}$, it is false of all the other predicates, and if it disagrees on the two then, since both are in $E(f)$, we don't have agreement across all repairs and so get failure.

- Sentence (45) with $\{f^{0/\#}, f^{1/\#}\}$: Here we're applying the function $g = [\![\text{exactly two philosophers}]\!]$ to the function $f = [\![\text{has stopped drinking}]\!]$, by considering the two alternatives $f^{0/\#} \approx [\![\text{used to drink and doesn't at present}]\!]$ and $f^{1/\#} \approx [\![\text{doesn't drink}]\!]$.

  - When is (45) true? Yet again we ask when $g(f^{0/\#}) = g(f^{1/\#}) = 1$. Here we must consider each repair separately. $g(f^{0/\#}) = 1$ iff exactly two philosophers are ex-drinkers, and $g(f^{1/\#}) = 1$ iff exactly two philosophers are non-drinkers. Since every ex-drinker is a non-drinker, it follows from these two conditions both being met that the ex-drinkers are the non-drinkers, which is to say that each of the philosophers is either an ex-drinker or a current drinker, so we predict a universal inference that all the philosophers used to drink. This prediction is strongly at odds with (at least) my intuitions for (45).
  - When is (45) false? To make the sentence false, we need to ensure that $g(f^{0/\#}) = g(f^{1/\#}) = 0$. This is the case whenever

neither the number of non-drinking philosophers nor the number of formerly drinking philosophers is two.

  - When does (45) suffer presupposition failure? We have failure whenever the two repairs fail to agree on truth or falsehood. This occurs whenever there are either two philosopher ex-drinkers or two philosopher non-drinkers, and the number of philosopher never-drinkers is not zero. We thus compute the rather convoluted presupposition that either the number of philosopher ex-drinkers and the number of philosopher non-drinkers are both equal to two, or else neither number is equal to two.

- Sentence (45) with $E(f)$: Unlike the previous cases, for *exactly* quantification the use of the larger repair set $E(f)$ yields slightly different predictions. The truth conditions will be the same, since if the two extremes both contain exactly two philosophers then all the intermediate repairs will as well. The falsehood conditions, however, will be trickier, since now we will have one repair that gives us truth whenever the number two is (inclusively) in between the number of philosophers who are ex-drinkers and the number of philosophers who don't drink. So for the sentence to be false, it must be either that both numbers are greater than two or both are less than two. We get failure in all other cases, so we predict a presupposition that either the two numbers are both two, or both are less than two, or both are greater than two.

The observation made above for the monotonic quantifiers helps explain the similarity in predictions between the two repair set options, so it's worth restating generally. If we have a generalized quantifier that is upward or downward monotonic in its nuclear scope, with denotation f, then one of $f^{0/\#}$ and $f^{1/\#}$ will be the repair in $E(f)$ that has the "hardest" time making the quantifier true, and the other will be the one that has the "hardest" time making it false, we will have agreement across $E(f)$ iff we have agreement between these two extremes, hence:

Proposition 1 *For a predicate f in the nuclear scope of a non-presuppositional monotonic generalized quantifier, the strong Kleene system derives the same truth conditions and the same presuppositions when the repair set is $\{f^{0/\#}, f^{1/\#}\}$ as it does if the repair set is $E(f)$.*

The Peters-Kleene and strong Kleene approaches do diverge rather strikingly for restrictors. Since the strong Kleene approach is fully symmetrical for argument positions, its predictions for presuppositions in the restrictors of symmetrical (intersective) quantifiers will just be the mirror images of the nuclear scope predictions. Thus, on either of the above notions of repair set, (46) is associated with roughly the same truth conditions and presuppositions as (44), and likewise (47) will get the same predictions as (45).

(46) At least one person who has stopped smoking is among these six philosophers.

(47) Exactly two people who've stopped smoking are among these six philosophers.

For universal quantification, we do have asymmetry between the arguments, so we will of course get different predictions for restrictor presuppositions. Here, we should consider (34), reproduced below as (48):

(48) Every student in my category theory course who stopped drinking failed the midterm.

To get the strong Kleene predictions for (48), we ask the same three questions we've asked before. I will consider only the $\{f^{0/\#}, f^{1/\#}\}$ case, although the $E(f)$ case comes out the same. Here, $f = [\![$student in my category theory course who stopped drinking$]\!]$, $f^{0/\#} \approx [\![$student in my category course who is a drinker$]\!]$, and $f^{1/\#} \approx [\![$student in my category course who doesn't drink$]\!]$.

- When is (48) true? For (48) to be true, we need it to be the case both that every student in the course who doesn't drink failed and that every student in the course who is a former drinker failed, but the second of these is redundant with the first, so we get truth exactly when every nondrinking student failed.

- When is (48) false? For falsehood, we need there to be one nondrinking student who passed, and one formerly drinking student who passed. But the former is true whenever the latter is true, so we find that the sentences is false whenever there is a former drinker among the students who passed.

- When does (48) suffer presupposition failure? The sentence suffers failure whenever neither the truth nor falsehood conditions are met - that is, when every former drinker among the students failed, but some nondrinker passed. We thus get a presupposition roughly equivalent to a statement that if all the former drinkers failed, all the nondrinkers did.

The predicted presuppositions we get for restrictors under the strong Kleene theory are quite strange. They are also, at least in the case of the universal, probably too strong. They are, however, weaker than the universal and near-universal restrictor presuppositions that many other theories predict. This is an improvement if one rejects such universal presuppositions, but on this line it would be preferable to have presuppositions that are weaker still - the intuition that presuppositions can just be used to narrow down the restrictor does not correspond to the predictions we derive.

The Peters-Kleene theory predicts much stronger presuppositions in restrictors. Since the restrictor is the first argument considered, we predict failure unless every repair of the restrictor produces the same truth value for every possible nuclear scope predicate. With universal and (singular) existential quantification, this means we predict presupposition failure as long as the restrictor makes any presuppositions about the entities involved - for some cardinal quantification the results are slightly weaker, since the number of entities in even the largest repair of the restrictor can be small enough that all repairs make the restrictor true or false simply because to achieve one truth value we'd need more entities to work with. Since I think restrictor presuppositions are weak, I think this isn't a very good prediction, but it is at least a prediction with advocates and a certain kind of coherence, and it does reflect some kind of order-asymmetry. This is in contrast with the predictions of the basic strong Kleene system, which are symmetrical and, so far as I know, do not accord especially well with anybody's intuitions.

3.3.5 The Methodological Challenge of Transparency Theory

The genesis of the present system closely parallels that of the Peters-Kleene system. Schlenker (2006, 2008a) argues for a particular pragmatic approach (transparency theory) as preferable over dynamic semantics. As with Heim (1983)'s arguments for dynamic accounts of projection, methodological considerations play a key part. The particular objection raised here is that dynamic semantics *overgenerates* in the sense that the projection facts of a connective are coded into its dynamic meaning in a way largely independent of its truth-conditional (or at least truth-functional) contribution. A constraining theory, such as one that derives projection behavior from the normal truth-conditional semantics of words and their syntactic configurations, seems preferable. This work began as an exercise in arguing that a static semantic theory with this desirable property is possible and can make predictions similar to those of classical dynamic semantics and of transparency theory. The present goal, developed below, is to show how a certain refinement of the strong Kleene system can produce predictions that are different from, and, I hope, preferable to, those of these earlier theories.

3.4 Towards a Revised Approach

The strong Kleene and Peters-Kleene approaches, as described here, have two main components - a component that generates the repairs in the event of presupposition failure (above we considered $\{f^{0/\#}, f^{1/\#}\}$ and $E(f)$ options) and a component that computes truth values based on a calculated repair set (above we considered the strong Kleene approach, which lets us consider all arguments together, and the Peters-Kleene approach, which takes the arguments one at a time and must make snap judgments). Overall, the most questionable predictions appear to be those relating to restrictors and to *exactly* quantification. There are also some possible issues with the symmetry or asymmetry of various truth functions, and the fact that the "universal presuppositions" of certain quantifiers are derived only as entailments.

After introducing some technology to help with the more precise description of the system, I will outline describe a way to address the issues with *exactly* and with restrictors by keeping the strong Kleene approach to evaluating repairs, but introducing a new system for computing repair sets. This new approach to repair sets will make the repair set dependent on the function beng applied. The actual interaction of the function with a set of repairs will be symmetrical between argument positions, as in the strong Kleene system, but, since every time we apply a function to one argument we produce a new function of lower arity, we can end up using different repair sets in different argument positions, leading to asymmetrical projection predictions for symmetrical quantifiers. The influence of the function on the repair set is computed from the normal truth-conditional properties of the function, so the desirable predictive properties of the theory are preserved.

The key trick will be to define the repair set so that in certain contexts it is $\{f^{0/\#}, f^{1/\#}\}$, while in others it is just $\{f^{0/\#}\}$. That is, sometimes we are allowed to proceed simply by treating presupposition failure as if it were falsehood - the cases where this is allowed will include almost all restrictors, and also enough cases of the nuclear scope of *exactly* quantification to make the relevant quantified sentences true whenever there are the right number of verifying examples. To make this work, we need to identify a property of function-argument pairs that picks out the cases where we are allowed to use $\{f^{0/\#}\}$ as the repair set - if we allow this in too many cases, we may get undesirable predictions for other quantifiers. The key observations will be that, owing to the conservativity of natural language determiner denotations, the extension of the restrictor of a quantifier is often recoverable from a DP denotation, and that the relationship of a DP denotation to its restrictor can be understood as the same as the relation of a determiner denotation to the whole universe of entities. To describe the right criterion, we will need a notion of a set of values relevant to the evaluation of a quantifier, and a way of generalizing this notion to functions of a variety of different types, so that we cover, both DP and determiner denotations (and, for uniformity, truth functions as well). Before these notions can be made precise, though, it will be necessary to trudge through a certain amount of supporting formalism.

4 Background: Some Technology

Before I can describe my proposed refinement precisely, I will need some new formal machinery. In particular, it will be helpful to adopt a slightly deviant approach to manipulating functions and dealing with trivalence. This is because the standard, heavily curried functional calculus common in semantics will make the needed discussion of "possible outcomes" rather cumbersome, and require numerous disjunctive definitions for clearly analogous cases. A few definitions now will make things more concise later on. I will also be retreading some ground in order to make ideas already put forward a bit more precise.

4.1 Trivalence

Trivalence is implemented through a special error code value #. # is a special item distinct from all ordinary items, be they entities, worlds, truth values, events, indices, or functions or sequences of any types derived from these. Crucially, # is a value "outside" of the core semantics as seen by the lexicon - the domains of functions as defined in the lexicon do not include # or other presuppositional values (such as functions with # in their ranges), but will have their behavior in these cases derived by the operations of the semantics.[18] Functions in the lexicon may have # in their range if they are

[18]In the terms of Appendix C, denotations are all drawn from the set Δ_τ of generalized denotations of some type τ, which means that they are defined directly only over arguments

presupposition triggers.[19] Technically, we might wish to define a distinct $\#_\tau$ for every basic type τ, but for convenience I will write them all as #, and will in addition write # for any constant function that takes the value # at every argument sequence appropriate to its type and arity.

4.2 Sequences

Functions will be applied to argument sequences. Sequence variables are written with a small arrow symbol over them, so for instance $\vec{u}$ and $\vec{v}$ would be sequences, and $\vec{u}\vec{v}$ is the sequence that results from concatenating $\vec{u}$ with $\vec{v}$, and u_3 would be the third element of $\vec{u}$. Every sequence $\vec{u}$ has a length, written $|\vec{u}|$, which may be any non-negative integer. A sequence of length 1 is understood to be identical with its sole element, and the unique sequence of length 0 is written ε.

4.3 Functions

Functions will be manipulated in an extremely un-Curried format, with the one-argument-at-a-time aspects of Currying put into the rules of function-argument interaction. Everything we say could be paraphrased in terms of the more common curried presentation, but the statements would be considerably more unwieldy, and many subcases of what is intuitively the same operation would have to be spelled out separately with awkward disjunctive definitions.

For our purposes, every function has an arity $n \geq 0$. An n-ary function is defined on a domain of sequences of length n (a function is still limited by the types of the elements in the argument sequences), and a codomain consisting of the set of items of some basic type combined with the error code value #. Items of basic types are understood as zero-ary functions.[20]

The kinds of functions denoted by various constituent types in this system are generally easy to compute by flattening the extensional types common in semantic analyses, and reformulating them in terms of sequences. Thus, unary predicates like verb phrases and count noun phrases will denote functions from unary sequences of entities to truth values, sentences will denote truth values (or, equivalently, functions from zero-ary sequences

in sets of non-presuppositional denotations D_ς, for various types ς.

[19] I do not mean to suggest that we should be content with a theory that treats presupposition triggering as arbitrary and lexical - only that I do not wish to address the triggering problem here.

[20] That is, the truth values 1 and 0 are manipulated as functions whose domain is $\{\varepsilon\}$, where it is important to remember that $\{\varepsilon\} \neq \emptyset$.

to truth values), determiners will denote functions from sequences of two unary predicate denotations to truth values, and DPs will denote functions from unary sequences of unary predicate phrases to truth values.

Note that, unlike the standard presentation, no function has a (non zero-ary) function type as its natural output type. Instead, we get functions to output other functions by putting the Currying into our rules of function application. Thus, where f is an n-ary function, and $0 \leq |\vec{u}| \leq n$, and where for any $1 \leq m \leq |\vec{u}|$, u_m is a non-presuppositional item of type appropriate to the m^{th} argument position of f, we define $f \backslash \vec{u}$, or f reduced by $\vec{u}$, as the unique $(n - |\vec{u}|)$-ary function g such that, for all $\vec{v}$ of length $n - |\vec{u}|$ and suitable type, $g(\vec{v}) = f(\vec{u}\vec{v})$. Considering the trivial cases, if $\vec{u} = \varepsilon$, $f \backslash \vec{u} = f$ (this is the only case for a zero-ary function), and if $|\vec{u}| = n$, $f \backslash \vec{u} = f(\vec{u})$.

As a quick example, let $\wedge$ be the usual (bivalent) function of boolean conjunction on truth values: we find $\wedge \backslash 0$ is the constant function g such that $g(0) = g(1) = 0$, $\wedge \backslash 1$ is the identity function I where $I(0) = 0$ and $I(1) = 1$, and $\wedge \backslash \#$ is not defined, since no sequence beginning with # is in the domain of $\wedge$.

4.4 Functions with Argument Alternatives

Repair theories understand presupposition as (at least sometimes) introducing cases where a function must be reduced not by a single argument value but by a set of competing repair values. To accommodate this, where f is an n-ary function for some $n > 1$, and is X a set of arguments of suitable type to fill the first argument position of f, we define $f \backslash\!\backslash X$, the strong Kleene reduction of f by X, to be the unique $(n-1)$-ary function g such that, for all $(n-1)$-ary sequences $\vec{v}$ of arguments of suitable type:

- If there is a such that for all $x \in X$, $f(x\vec{v}) = a$, then $g(\vec{v}) = a$.
- If there are $x, y \in X$ such that $f(x\vec{v}) \neq f(y\vec{v})$ then $g(\vec{v}) = \#$.

To see this in action, consider the case of $\wedge \backslash\!\backslash \{0, 1\}$. Since $\wedge$ is a binary function on truth values, this produces a function g on truth values. To define g we need only identify the values of $g(1)$ and $g(0)$. Now, $(0 \wedge 0) = (1 \wedge 0) = 0$, so by the first clause of the above $g(0) = 0$, but $(0 \wedge 1) \neq (1 \wedge 1)$, so by the second clause $g(1) = \#$.

Note that, although strong Kleene reduction operates one argument at a time, it actually exhibits no inherent asymmetry between argument positions (although of course it is possible to introduce asymmetry into a system that makes use of this operation). The corresponding asymmetrical operation is the Peters-Kleene reduction of a function f by a set X - written $f \mathrel{\backslash\!\!\backslash\!\!\!\!\!\!\rightarrow} X$ and defined it as follows:

- If there is g such that for all $x \in X$, $f \backslash x = g$, then $f \mathrel{\backslash\!\!\backslash\!\!\!\!\!\!\rightarrow} X = g$.
- If there are $x, y \in X$ such that $f \backslash x \neq f \backslash y$, then $f \mathrel{\backslash\!\!\backslash\!\!\!\!\!\!\rightarrow} X = \#$ (that is, the constant $(n-1)$-ary function that for any arguments of suitable types returns the value $\#$).

These definitions are just one set of formalizations of the strong Kleene and Peters-Kleene approaches discussed above. Here, as before, the difference between Peters-Kleene reduction and strong Kleene reduction as defined here is a difference between hasty evaluation with limited knowledge of the future, and cautious evaluation that waits to see what the future holds to pass judgment. To avoid failure, Peters-Kleene reduction requires that reduction by all alternatives in the set yield the same function (that is, that all imaginable (non-presuppositional) future sequences of arguments yield the same result with all the items between which we are uncertain) and yields failure if this requirement is not met. Strong Kleene reduction always gives us a function that, once it has seen the rest of the arguments, will give us failure unless, for those remaining arguments, the present uncertainty turned out not to matter. Peters-Kleene reduction gives up on any case of uncertainty unless it can convince itself, with only the information at hand, that the uncertainty is harmless, while strong Kleene reduction makes note of the uncertainty and its perils, and waits to see if it's really that much of a problem.

Proposition 2 *If x is a non-presuppositional argument of suitable type for the first argument position of f, then $f \mathrel{\backslash\!\!\backslash\!\!\!\!\!\!\rightarrow} \{x\} = f \backslash\!\!\backslash \{x\} = f \backslash x$.*

4.5 Relevant Sets

Functions will from time to time take other functions as arguments, but it is well known that a function is often only concerned with the behavior of its functional arguments at particular values. A set X is a *relevant set* for a function f iff, for all non-presuppositional functions g and h of the first argument type of f, if $g(\vec{u}) = h(\vec{u})$ for all $\vec{u} \in X$ then $f \backslash g = f \backslash h$.

To see an example of relevant set computation, let $f = [\![\text{every engineer}]\!]$. The relevant sets of f will be exactly those sets that contain all the engineers. That every set containing all the engineers is a relevant set for f is an immediate consequence of conservativity (see Keenan and Stavi (1986)), which guarantees that the action of the nuclear scope on the items in the restrictor is all we care about. To see that every relevant set contains all the engineers, suppose that X is any set that omits at least one engineer (call her Ada), and let $g = [\![\text{is an engineer}]\!]$ map every entity to 1, and $h = [\![\text{is an engineer but isn't Ada}]\!]$. Now, $g(x)$ and $h(x)$ for all $x \in X$, since g and h disagree only about Ada and Ada isn't in X, but $f(g) = 1$ while $f(h) = 0$ - so X cannot be a relevant set.

The idea is that a relevant set contains inputs to the first argument of a function that together serve as "all that matters" about the behavior of that argument. The notion of relevant set will mainly be relevant for the cases where the function is a determiner denotation or DP denotation - in the former case, the only relevant set will generally be the whole universe. In the latter case, for many well-behaved, nontrivial DPs over finite domains, the relevant set will be a way of recovering the restrictor from the DP denotation (because, for many such DPs, the set of relevant sets is exactly the set of supersets of the restrictor).

The behavior of a function on the relevant sets of the function of which it is an argument will be exactly what we need to look at to pick which repair sets to use, which will allow us to improve on the predictions of strong Kleene in the main problem cases identified above.

5 How to Combine Functions and Arguments

The deployment of a function on a presuppositional argument will have two steps: first, we will compute a set of non-presuppositional "repairs" of that argument, and second, we will use strong Kleene reduction to test the function on the repair set.

5.1 Computing Repair Sets

The repair set for an argument x will be considered with respect to the function f being applied to it. The repair set of x in f will be written $\mathcal{REP}_f(x)$ and will be defined as follows:

- If, for every Y a relevant set of f, there is $\vec{u} \in Y$ such that $x(\vec{u}) = 1$, then $\mathcal{REP}_f(x) = \{x^{0/\#}\}$.
- If there is Y a relevant set of f such that for all $\vec{u} \in Y$, $x(\vec{u}) \neq 1$, then $\mathcal{REP}_f(x) = \{x^{0/\#}, x^{1/\#}\}$.[21]

[21] An alternative worth exploring would be to let $\mathcal{REP}_f(x) = E(x)$ in this case.

These definitions will, of course, only be sufficient so long as we restrict attention to types "ending in `t`". Since some arguments are normally presumed to denote entities or other items not covered by the range of types considered here, this will not be sufficient in the general case, but it will do for most core quantifier and connective data.

Proposition 3 *If x is non-presuppositional, then $\mathcal{REP}_f(x) = \{x\}$.*

Proposition 4 *If f is a truth function, then $\mathcal{REP}_f(\#) = \{0,1\}$.*

This last proposition will guarantee that our results are the same as those of the strong Kleene logic in the case of truth functions.

Since we reduce the function by its arguments one at a time, by making the repair set contingent on the function, we've also made it contingent on the previous arguments. In particular, since for many quantifiers the class of relevant sets encodes the restrictor, this means that the system is checking against the restrictor when it computes the repair set for the nuclear scope.

As will be seen below, this more complex definition of repair sets, compared with the various simple repair set theories discussed in the section on strong Kleene logic, pays off primarily in two places: it weakens our predicted inferences from *exactly* quantification, and it explains why presuppositions in restrictors just serve to restrict, instead of projecting out.

5.2 Function Deployment

We can now define $f[x]$, the *deployment of f over x*, as follows:

$$f[x] = f \backslash\!\backslash \mathcal{REP}_f(x)$$

We can generalize this to multiple arguments, so that $f[x_1, ..., x_n] = f[x_1]...[x_n]$.

The deployment operation will be how linguistic functions combine with their arguments. Manners of combination that do not involve a simple function-argument relation will presumably require additional operations.

6 Deployment of some Linguistically Relevant Functions

6.1 Truth Functions

In the case of truth functions, we re-derive the strong Kleene logic. This is because, as observed, for every f a truth function, $\mathcal{REP}_f(\#) = \{0,1\}$, so we are just using the same old repairs we had before. We've already seen the strong Kleene predictions for *and* and *or*. To give a flavor of how these arguments go when framed with the new technology and repair system, I will sketch the analysis for negation below.

For $a \in \{0,1\}$, $\mathcal{REP}_\neg(a) = \{a\}$, since $a^{0/\#} = a^{1/\#} = a$. Thus $\neg[0] = \neg\backslash\!\backslash\mathcal{REP}_\neg(0) = \neg\backslash\!\backslash\{0\} = \neg\backslash 0 = \neg(0) = 1$, and in the same way $\neg[1] = 0$. On the other hand $\mathcal{REP}_\neg(\#) = \{\#^{0/\#}, \#^{1/\#}\} = \{0,1\}$, since the relevant set of $\neg$ contains just the empty sequence ε, and $\#(\varepsilon) \neq 1$, so we are forced to use this larger repair set. Since $\neg(0) \neq \neg 1$, we find that $\neg[\#] = \neg\backslash\!\backslash 0, 1 = \#$. That is, the presuppositions of a sentence are predicted to be the same as those of its negation.

In this way, we likewise replicate the behavior of the strong Kleene system for conjunction and disjunction, again getting a symmetrical disjunction (which is probably desirable) but also a symmetrical conjunction (which is much more questionable). The predictions that we would compute for the material conditional are not especially plausible, but natural language conditionals are widely believed to have semantic and syntactic complexities beyond those of the material conditional - until we settle on a theory of conditionals, and address the handling of intensionality in the present model, it will be hard to say whether the performance of this system with conditionals is acceptable.

6.2 Quantifiers

For quantifiers, the computation of repair sets is more involved. I will begin by considering restrictors in general, and then look at the nuclear scopes of some particular quantifiers:

6.2.1 Restrictors

Let us first consider the function $f_{every} = [\![\text{every}]\!]$. This function (like other determiner denotations) is a function from sequences of length 2, to truth values, where the argument sequences are sequences of unary predicates. Suppose we are evaluating f_{every} in a nonempty universe D_{e}. I claim that, for every relevant set X of f_{every}, $D_{\mathsf{e}} \subseteq X$. To see this, suppose (for the sake of deriving a contradiction) that we have X a relevant set of f such that $D_{\mathsf{e}} \not\subseteq X$. It follows that there is $a \in D_{\mathsf{e}}$ such that $a \notin X$. Now consider $g_{D_{\mathsf{e}}}$, the characteristic function of the universe of entities, and $g_{D_{\mathsf{e}}-\{a\}}$, the characteristic function of the set of all entities except in the universe a. Since $a \notin X$, and $g_{D_{\mathsf{e}}}$ and $g_{D_{\mathsf{e}}-\{a\}}$ disagree only at a, these two functions agree for all arguments drawn from X. If, as supposed, X is a relevant set for f_{every}, it should follow that $f_{every} \backslash g_{D_{\mathsf{e}}} = f_{every} \backslash g_{D_{\mathsf{e}}-\{a\}}$, but this is not the case, since $(f_{every} \backslash g_{D_{\mathsf{e}}}) \backslash g_{D_{\mathsf{e}}-\{a\}} = f_{every}(g_{D_{\mathsf{e}}}, g_{D_{\mathsf{e}}-\{a\}}) = 0$, while $(f_{every} \backslash g_{D_{\mathsf{e}}-\{a\}}) \backslash g_{D_{\mathsf{e}}-\{a\}} = f_{every}(g_{D_{\mathsf{e}}-\{a\}}, g_{D_{\mathsf{e}}-\{a\}}) = 1$. Thus, there can be no relevant set X that is not a superset of D_{e}. Note also that D_{e} is itself a relevant set of f by type-theoretic considerations, since any argument for f of suitable type will not even be defined outside of D_{e}. Now consider any function h - since D_{e} is a relevant set and is a subset of all relevant sets, there is $b \in D_{\mathsf{e}}$ such that $h(b) = 1$ iff, for every relevant set Y of f_{every}, there is $c \in Y$ such that $h(c) = 1$, since if there is such a $b \in D_{\mathsf{e}}$ then $b \in Y$ for all relevant sets Y, and if every relevant set contains a c, such that $h(c) = 1$ then D_{e}, being a relevant set, also contains such a c. That is, the $\mathcal{REP}_{f_{every}}(h) = \{h^{0/\#}\}$ iff h maps at least one entity in the universe to 1, and $\mathcal{REP}_{f_{every}}(h) = \{h^{0/\#}, h^{1/\#}\}$ otherwise.

This argument can be repeated, with minor modification, to show that D_{e}, if nonempty, plays the same role among the relevant sets of many other common determiner types:

Proposition 5 *Suppose that D_{e} is nonempty let $n < |D_{\mathsf{e}}|$, and let f be any of the following: ⟦each⟧, ⟦no⟧, ⟦at least n⟧, ⟦at most n⟧, ⟦more than n⟧, ⟦less than n⟧, or ⟦exactly n⟧. A set X is a relevant set of f iff $D_{\mathsf{e}} \subseteq X$.*

Note that this is not true for all possible determiner denotation - it fails in particular with *infinitely many*.

This is all rather abstract - let's consider the concrete case of (34), reproduced below as (49):

(49) Every student in my category theory course who stopped drinking failed the midterm.

Let $g =$ ⟦student in my category theory course who stopped drinking⟧ (so g is a trivalent predicate), and $h =$ ⟦failed the midterm⟧. We wish to evaluate $f_{every}[g]$, which is to say $f_{every} \backslash\backslash \mathcal{REP}_{f_{every}}(g)$ - as shown above, if any of the students in my category theory course is a former drinker, then g maps some entity (in particular that student) to 1, so, by the reasoning above, g maps some entity in every relevant set of f_{every} to 1, so $\mathcal{REP}_{f_{every}}(g) = \{g^{0/\#}\}$, so

$$f_{every}[g] = f_{every} \backslash\backslash \mathcal{REP}_{f_{every}}(g) = f_{every} \backslash\backslash \{g^{0/\#}\} = f_{every} \backslash g^{0/\#}$$

Now, on a standard analysis of *stop*, $g^{0/\#} \approx$ ⟦student in my category theory class who once drank but does not currently drink⟧. That is, so long as the presuppositional predicate is true of some entity, using it in a restrictor position with *every* (or any of the other quantifiers we've been considering) will produce a sentence with exactly the same truth conditions as we would get with a predicate that is true of the same entities but false of all other entities, and no presuppositions are introduced. On the assumption that there is a former drinker in the course (an assumption without which the sentence is pragmatically rather questionable), (49) is true if every former drinker in the course failed, and false if some former drinker in the course did not fail, and the students who never drank are simply irrelevant. If no student in the course is a former drinker, then the conditions presupposition failure will depend on our analysis of the relative clause, but failure will almost certainly be possible in, for example, the case where no student ever drank and not every student failed.[22] Still, the case where the restrictor predicate is true of some entity is more common, and here restrictor presuppositions serve only to restrict. This prediction of weak restrictor presuppositions is unusual, and is a point of contrast with both Schlenker (2006) and Heim (1983), and with the strong Kleene and Peters-Kleene systems outlined above. I think that, in light of examples like (49), it is a desirable prediction.

6.2.2 Nuclear Scope - *each*

Now we return to the sentence (28), reproduced below as (50):

(50) Each of these six philosophers has stopped drinking.

Let $f = [\![\text{each of these six philosophers}]\!]$, and $g = [\![\text{has stopped drinking}]\!]$. We seek to evaluate $f[g] = f \backslash\backslash \mathcal{REP}_f(g)$. To determine the value of $\mathcal{REP}_f(g)$ in particular circumstances, we need to identify the relevant sets of f, but it is easy to see that the relevant sets of f are exactly those sets that include all six of the philosophers, since by conservativity the action of the nuclear scope outside of the restrictor can never affect the outcome (so every set bigger than the restrictor is a relevant set), and the value of the nuclear scope of a universal quantifier at every point in the restrictor bears on the outcome

[22] This is because in this case the restrictor is true of no entity, since it is untrue of everybody not among the students, and # for all the students. Under the rules for calculating repair sets, we will have to consider the larger repair set for the restrictor, which will produce one option where the restrictor is empty (leading to vacuous truth) and one where it contains all the students (leading to falsehood, since the nuclear scope including one student who failed is true of all the students). I say it is "almost certain" to fail in this case because the argument I just sketched hinges implicitly on some issues with the projection properties of relative clauses and the truth conditions of universal quantification that are not entirely uncontroversial.

(so no set that fails to include the whole restrictor is a relevant set). That is $\mathcal{REP}_f(g) = \{g^{0/\#}\}$, where $g^{0/\#} \approx [\![\text{formerly drank, but now does not}]\!]$ iff at least one of the pilosophers is an ex-drinker, and otherwise (i.e., when every philosopher either still drinks or never drank) $\mathcal{REP}_f(g) = \{g^{0/\#}, g^{1/\#}\}$, where $g^{1/\#} \approx [\![\text{doesn't drink}]\!]$. With this background in mind, we again inspect the conditions for truth and falsehood:

- When is (50) true? If every philosopher is a former drinker, then of course the nuclear scope is true of at least one of them, so we have the smaller repair set $\mathcal{REP}_f(g) = \{g^{0/\#}\} = \{g\}$, so $f[g] = f\backslash\!\backslash\{g\} = f(g)$, and since g is true of all the philosophers $f(g) = 1$. On the other hand, if $g(a) = 0$ for any philosopher a, then $g^{0/\#}(a) = 0$, and since $g^{0/\#}$ must be in the repair set, just observing that in this case $f(g^{0/\#}) = 0$ is enough to verify that $f[g] \neq 1$. Likewise, if any philosopher b is mapped to $\#$ by g (that is, if any philosopher never drank) then we have $g^{0/\#}(b) = 0$ so $f[g] \neq 1$. Thus, all the philosophers must get mapped to 0 by g, and none of them can be mapped to anything else. We get truth iff all the philosophers are former drinkers. This means we get the commonly attributed "universal presupposition" as an entailment. This is not entirely unreasonable - that the inference from a sentence like (50) to the "universal presupposition" is a good inference is well-documented, but much less evidence has been offered that this inference has presuppositional force.

- When is (50) false? If any philosopher currently drinks, then that guarantees that $f(g^{0/\#}) = f(g^{1/\#}) = 0$, since then it is neither the case that each of the philosophers doesn't drink nor that each of them drank formerly but doesn't at present, so whatever repairs set we consider we find $f[g] = 0$. Since we already considered the case where all philosophers are former drinkers, the only cases left to consider are those where some are never-drinkers and the rest are former drinkers. In the case where some are former drinkers and some are never-drinkers, the former drinkers are mapped to 1 by g, so we get the smaller repair set $\mathcal{REP}_f(g) = \{g^{0/\#}\}$, which maps the never-drinking philosophers to 0, so the universal is false. In the case where we have only never-drinkers among the philosophers, then none of them are mapped to 1, so we have $\mathcal{REP}_f(g) = \{g^{0/\#}, g^{1/\#}\}$ - since this is the case where g maps all the philosophers to $\#$, the former repair is false of all of them and the latter is true of all of them, so the two repairs produce different results and so in this case we get a truth value of $\#$. Since this exhausts the possibilities, we find that the sentences is false if at least one philosopher is either a former or a present drinker, but not all are former drinkers.

- When does (50) suffer presupposition failure? We get presupposition failure only in the case where all the philosophers don't drink and never did, which is to say we avoid failure so long as at least one philosopher used to drink - we compute a simple existential presupposition for the universally quantified sentence.

6.2.3 Nuclear Scope - *none*

Now consider (30), reproduced below as (51):

(51) None of these six philosophers has stopped drinking.

As before, the relevant sets of the generalized quantifier are exactly those sets that contain the six philosophers, since any one philosopher can make the difference between none and some - now, where g is again the denotation of the nuclear scope, and f is the denotation of the subject DP:

- When is (51) true? For it to be the case that $f[g] = 1$, there can be no philosopher a such that $g(a) = 1$ (since then $f(g^{0/\#}) = 0$, ruling out the possibility of computing an answer of 1), so if the sentence is true then none of the philosophers is a former drinker. If this is the case, though, we find that g maps every element of the set of the philosophers (which is a relevant set) to values other than 1, so we are forced to consider the repair set $\{g^{0/\#}, g^{1/\#}\}$. So for the sentence to be true it must be the case both that none of the philosophers is a non-drinker and that none of the philosophers is a former drinker, which is to say that all of the philosophers drink (and have in the past). This means we again get a universal inference that all philosophers used to drink.

- When is (51) false? If there is one of the philosophers a such that $g(a) = 1$ (that is, a still-drinking philosopher), then the only repair we need to consider is $g^{0/\#}$, and since $g^{0/\#}(a) = 1$ and a is a philosopher, $f[g] = f(g^{0/\#}) = 0$. If there is no such a, then we are back to the case where we're forced to consider both repairs, and one of them will map all the philosophers to 0 and the other will not, so we get failure. Thus, the sentence is false so long as at least one philosopher is a former drinker.

- When does (51) suffer presupposition failure? We get presupposition failure in the remaining cases where none of the philosophers are former drinkers but not all of them are among those who used to drink and still do - we thus compute a presupposition that either all the philosophers are current drinkers or else at least one is a former drinker.

6.2.4 Nuclear Scope - *at least one*

For this quantifier, consider (31), reproduced here as (52):

(52) At least one of these six philosophers has stopped drinking.

Yet again, the relevant sets of the DP *at least one of these six philosophers* are exactly the sets containing all six philosophers - where we let f be the denotation of the subject DP and g be the denotation of the nuclear scope, we ask the familiar questions:

- When is (52) true? So long as there is at least one philosopher who used to drink but doesn't anymore, g maps at least one of the philosophers to 1, so $g^{1/\#}$ and $g^{0/\#}$ do as well, so $f(g^{0/\#}) = f(g^{1/\#}) = 1$, making the sentence true. On the other hand, if there is no philosopher who used to drink and now does not, then f maps no philosopher to 1, so $g^{0/\#}$ maps none of the philosophers to 1, so $f(g^{0/\#}) = 0$, which means that $f[g] \neq 1$ since $g^{0/\#}$ is always among the repairs. Thus, the sentence is true iff at least one of the philosophers used to drink and no longer does. We predict no universal inference.

- When is (52) false? If all of the philosophers are mapped to 0 by g, the they are also mapped to 0 by both possible repairs, so $f(g^{0/\#}) = g(g^{1/\#}) = 0$, hence $f[g] = 0$. If none of the philosophers are mapped to 1, but some are mapped to #, then the set of the philosophers is a relevant set the repair set contains both $g^{0/\#}$ and $g^{1/\#}$, but since g mapped some of the philosophers to #, $g^{1/\#}$ maps those philosophers to 1, so $f(g^{1/\#}) = 1$, and $g^{1/\#} \in \mathcal{REP}_f(g)$, so $f[g] \neq 0$. Thus, the sentences is false iff f maps all the philosophers to 0 - i.e., if all of the philosophers used to drink and still do. This means that the negation of sentence (52) will yield a universal inference, since for the negation to be true the sentence must be false, and for the sentence to be false so it must be that all the philosophers used to drink and still do, which is to say they all used to drink.

- When does (52) suffer presupposition failure? We have presupposition failure in the remaining case - the one where none of the philosophers are former drinkers but some of them never drank, so the presupposition is the negation of this - that either the philosophers all persist in drinking, or at least one is a former drinker.

6.2.5 Nuclear Scope - *exactly two*

Under the strong Kleene system, we predicted an undesirable universal inference from (33), reproduced below as (53):

(53) Exactly two of these six philosophers have stopped drinking.

With the new system, we get no such inference, as can be seen below (here $f = [\![$exactly two of these six philosophers$]\!]$ and $g = [\![$have stopped drinking$]\!]$) -

- When is (53) true? If g maps a number of the philosophers other than two to 1, then $g^{0/\#}$ maps the same number to 1, so $f(g^{0/\#}) = 0$, so $f[g] \neq 1$. If g maps exactly two of the philosophers to 1, then, since every relevant set of f includes all six of the philosophers, g maps something in any relevant set to 1, so $\mathcal{REP}_f(g) = \{g^{0/\#}\}$. Since g maps exactly two of the philosophers to 1, and $g^{0/\#}$ doesn't differ from g in which things it maps to 1, $g^{0/\#}$ maps exactly two of them to 1 as well. so $f[g] = f\backslash\!\!\backslash\mathcal{REP}_f(g) = f\backslash\!\!\backslash g^{0/\#} = f(g^{0/\#}) = 1$. Thus, the sentence is true iff exactly two of the philosophers are former drinkers, regardless of whether the others have any drinking experience.

- When is (53) false? If g maps a number of the philosophers other than zero or two to 1, then the repair set consists of just $g^{0/\#}$ which is true of some number of philosophers other than two, so the sentence is false. This leaves only the case where g maps none of the philosophers to 1. In this case, $\mathcal{REP}_f(g) = \{g^{0/\#}, g^{1/\#}\}$, and g maps some number of the philosophers to 0 and the rest to #. Here, $g^{0/\#}$ will map all of the philosophers to 0, so the sentence is false iff $f(g^{1/\#}) = 0$, which is to say $g^{1/\#}$ doesn't map exactly two philosophers to 1, which is to say g maps a number of other than two to #. So the sentence is false so long as some number of philosophers other than two are former-drinkers, and it is not the case that no philosophers are former drinkers and exactly two are never-drinkers.

- When does (53) suffer presupposition failure? In the case where g maps exactly two philosophers to # and the rest to 0, $f(g^{1/\#}) = 1$ but $f(g^{0/\#}) = 0$, so $f[g] = \#$. All the other cases are accounted for above, so this is the only case of failure.

The predictions above paint an encouraging picture: we get the desired very weak inferences for *exactly* quantification and restrictors, while retaining universal inferences for *each* and *no*.

7 Taking Stock

The theory presented above makes many desirable predictions about quantification, improving empirically on many competitors (including transparency theory and most versions of dynamic semantics) by introducing fine-grained distinctions between quantifiers that correspond reasonably well to our intuitions for simple examples, and improving on the strong Kleene theory by predicting appropriately weak projection from restrictors and from the nuclear scope of *exactly* quantification, where the strong Kleene system predicted over-strong presuppositions. It also makes plausible predictions for presuppositions in the second position of a conjunction and either position with a disjunction, making its predictions reasonably appealing overall.

A few of the predictions might give one pause. First, for commonly considered quantifiers, this theory never predicts universal presuppositions. However, it does predict that the statements described as "universal presuppositions" will be consequences of sentences with "each" and "none" quantification, which are the cases where universal inferences are most reliable. These patterns of entailment accord with my intuitions, while the question of whether these entailments are in fact presuppositions has not yet been settled empirically (although readers who think that they are will find a modification to address this issue discussed in appendix A).

If we are unconcerned about the fact that this theory predicts certain quantified sentences false in cases where other theories attribute to them presupposition failure (but where the data are at present unclear) we are left with two major points of empirical concern: the predicted symmetry of *and* and the fact that with many numerical quantifiers speakers are still willing to report universal presuppositions about half the time, while this theory predicts such presuppositions should never occur.

For the case of *and*, the best explanation from the perspective of this theory is that the asymmetry is the result of an independent constraint against a certain kind of redundancy, as previously discussed in the coverage of the data for *and*. One can reasonably maintain that the projection facts for *and* are in fact quite symmetrical, but that simple examples with a presupposition in the first conjunct are ill-formed because they make the second conjunct violate a linear redundancy prohibition (one that also seems to apply in examples with assertions). The difference between (54) and (55) supports the idea that some kind of triviality effect is making some examples sound worse than they would otherwise be,[23] although it is not at all clear that (55) is free of presuppositions, which is what my theory predicts.

(54) # Watson has stopped drinking, and she used to drink.

(55) Watson has stopped drinking, and she used to guzzle a mug of absinthe before every lab meeting.

Although this explanation of the conjunction data is reasonable, the question of the projection behavior of triggers in the first argument of conjunction is far from settled; it is not even clear that (55) is free of presuppositions, which is what my theory predicts. If we decide the data demand an asymmetric *and*, there are variations on the theory available that will provide one, as discussed in appendix A.

For the data in Chemla (2007) about universal inferences being reported about half the time with numerical quantifiers, we should first note that most competing theories make a single prediction for projection facts, and so are equally at a loss to account for this variation. The present theory has an advantage over many competitors in that it makes significantly different predictions for different quantifiers, and predicts universal inferences only for those where universal inferences are most frequent. If forced to make a guess about the universal inferences sometimes seen with various numerical quantifiers, I would argue that these inferences are in fact not presuppositions or entailments, but conversational implicatures (in fact, the study described in Chemla (2007) involved giving subjects examples of implicatures as instances of the kinds of inferences being asked for, so this is a very real possibility). Conversational implicatures are often quite fragile, so this half-and-half behavior would be unsurprising on such an account. This does, however, raise the question of where such implicatures would come from, since it seems that they must be bound up in the presupposition system somehow. To see one way that such an implicature might arise, note that ensuring the truth of universal presuppositions would be a natural production heuristic, since calculating the precise presuppositions of a

[23] It must be admitted, of course, thar (55) is not the best of sentences, and that a pair of examples with a stronger contrast would be more compelling.

quantified sentence is somewhat involved, but the present theory guarantees that the presuppositions predicted will never be stronger than the universal presupposition. Making sure the common ground satisfies the universal presupposition guarantees the speaker felicity (so far as the nuclear scope presuppositions are concerned) without requiring him or her to even think about which quantifier she is using - if the "universal presuppositions" are checked for a production heuristic, it is only natural that hearers should exploit the existence of the heuristic to make additional inferences about the beliefs of speakers, giving rise to exactly the implicature we need. Obviously, a more developed theory of these kinds of production heuristics and their associated implicatures would be needed to make this line of reasoning really convincing, but some additional pragmatic resource will be needed to account for this variation, and I think that the approach sketched here at least shows that such variation could plausibly be accounted for while maintaining that the presupposition projection predictions of the present theory are accurate.

In spite of these open empirical issues and the limited coverage of the current formulation of the theory (which is limited by the lack of an account of intensionality or of the evaluations of functions with codomains not consisting of truth values), the present approach is among the few theories of projection currently available that make significant distinctions between different quantifiers,[24] and the distinctions predicted are on the whole quite appealing. This theory is also one of a handful of predictive theories of presupposition projection[25] - rather than freely stipulating the projection behavior of each function in its lexical entry, with little or no dependence on its truth conditional contribution, this theory allows only one projection behavior for each possible bivalent function and syntactic configuration, so the overgeneration problem discussed by Soames (1989), Heim (1983), and Schlenker (2008a) is avoided. Besides the merits of its new predictions and its methodological value as a theory that avoids overgeneration, this theory serves to demonstrate that current challenges in the study of presupposition projection can still be fruitfully confronted within a static, trivalent

[24]The other theories with fine-grained quantifier predictions that I'm aware of are that of Chemla (2008), the various strong Kleene and Peters-Kleene variations discussed here, some other variants on the repair approach explored in George (2008b) and in appendix A, and some other strong Kleene and supervaluation variants that Danny Fox and Philippe Schlenker have looked at.

[25]Other predictive theories include various strong Kleene and supervaluation variants, the transparency theory of Schlenker (2006, 2008a), the similarity theory of Chemla (2008), the constrained dynamic semantics of LaCasse (2008), and the local context version of dynamic semantics developed in Schlenker (2008b).

semantics, and in fact with tools similar to those of some very old work in trivalence, demonstrating that it is premature to cite projection as phenomenon that forces us to abandon a static semantics or to reject a semantic account of presupposition.

A Possible Modifications

It is possible to combine the pieces set out above in a number of different ways. We could, in particular, substitute the Peters-Kleene for the strong Kleene reduction with the revised repairs sets, or we could change our revised notion of repair set so that the bigger repair set possibility is $E(f)$. Since $E(f)$ produces very similar results to $\{f^{0/\#}, f^{1/\#}\}$ in the cases considered so far, there is not likely to be a strong empirical case for the distinction between them that is not based on some class of functions beyond those considered here. The Peters-Kleene case is a bit more interesting, as it would allow us to trade in asymmetric versions of *and* and *or* for symmetric ones, should that turn out to be desirable. Beyond these minor variations, we can consider more radical revisions that involve adding entire new components to the system. One of these previously explored in George (2007, 2008a,b) - is considered below. This new component introduces significant additional complexity, but also has a credible claim at substantially improving predictions, since for quantifiers it strengthens several universal inferences to universal presuppositions, and for connectives it predicts asymmetry for *and* but not for *or*.

A.1 Strengthening Presuppositions - A Disappointment Constraint

This disappointment constraint can be thought of as separating presuppositions into "preconditions for truth" and "preconditions for falsehood", and treating the former as more important than the latter for projection purposes. The idea is that we want to be forced to resort to attributing falsehood to an utterance by virtue of the assertive content of something, and that if an argument rules out truth simply by virtue of its presuppositions, that is enough to make the incremental composition process give up and declare presupposition failure - the system is biased in favor of truth and against disappointment. This can be incorporated into any variant on the system, and in each case it serves to strengthen some presuppositions, by converting some cases of falsehood into cases of failure. As presented here, this component is incremental and local, but a non-incremental version would not be hard to formulate, and in a less compositional version of the theory a global variant could also be described.

Here are the steps we introduce: whenever the presupposition failures associated with an argument, by themselves, rule out the possibility of truth as an outcome (but truth was a possible outcome before we encountered that argument), we generate failure. This process is explicitly incremental - the semantics moves through the argument list of a function from left to right, and, for each new argument it first considers the impact of its presuppositional aspects, and only goes on to consider the rest of it if these presuppositional aspects aren't sufficient to disappoint any hope of an eventual outcome of truth.

This gives us an asymmetric *and*, for the following reason: if we encounter # in the first argument position of *and*, we do not, under the function deployment defined above, know whether the outcome will be 0 or #, but we know there is no second argument which will cause us compute a value of 1 for the conjoined expression. Thus, before we saw # we had hope of truth (since there is a possible pair of argument values for *and* that makes things true). However, the presuppositional issues by themselves ruled this out, so that, evaluating $\wedge$ on the sequence $\#, 0$ we now get an output of #. On the other hand, when we consider the sequence $0, \#$, we find that nothing about the *presuppositional* content of 0 rules out our hopes of truth, but the fact that it is 0 does dash our hopes, so we don't have an issue with disappointing presuppositions, but we come to the second argument with no more hope to disappoint, so here we compute an output of 0. The above is vague - it can be made precise with a few more formal tricks.

First, we will need the notion of presupposition-equivalence - this will allow us to identify what is true of a function by virtue of its presuppositional part, since it will be exactly what is true of all presupposition-equivalent things. For functions f and g of the same type, f and g are presupposition-equivalent, written $f \backsimeq g$, iff for every sequence $\vec{u}$ of the same arity as f and g and of suitable type, $f(\vec{u}) = \#$ iff $g(\vec{u}) = \#$. Note that, in the case where f and g are zero-ary functions (that is, for present purposes, truth values), $f \backsimeq g$ iff either $f(\varepsilon) = f = \# = g = g(\varepsilon)$ or else $f \in \{0, 1\}$ and $g \in \{0, 1\}$.

Now that we have a definition of presupposition-equivalence, we can define disappointment on the basis of presuppositions. For f an n-ary function, x an item of the first argument type of f, x is presuppositionally disappointing in f iff the following two conditions are met:

- There is a sequence $\vec{v}$ of suitable type for f such that $f(\vec{v}) = 1$
- For all y such that $y \backsimeq x$, all $\vec{w}$ an $(n-1)$-ary sequence of non-presuppositional items such that $x\vec{w}$ is of suitable type for f, $f[y](\vec{w}) \neq 1$.

Disappointment has two components - we must have hope of truth going into the argument (otherwise, it was that argument in particular that disappointed us), and we must be able to infer, considering only the question of where the argument takes the value #, that there will be no hope of truth when we are done with it. The semantics is willing to compute outputs other than 1, but only if it concludes this from evaluating the assertions, instead of being pushed into it by the presuppositions.

We now turn to the notion of $f{:}(x){:}$, f deployed on x with concern for disappointment, defined as follows:

- If x is disappointing in f, then $f{:}(x){:} = \#$ (where # is here shorthand for the constant function of whatever type and arity is suitable that takes the value # for any input).
- If x is not disappointing in f, then $f{:}(x){:} = f[x]$.

As already noted, use of deployment with concern for disappointment produces the asymmetric Peters-Kleene *and*, and so agrees as well with the presuppositions given to *and* by Heim (1983) and Schlenker (2006). In contrast with *and*, the incremental disappointment criterion leaves *or* symmetric. This is because if we try to compute $\vee{:}(\#){:}$, we find no disappointment, since $\# \backsimeq \#$ and $\vee[\#](1) = 1$, so # in the first argument position is not disappointing. # is likewise not disappointing in the second argument position unless the first argument is 0, and since $\vee[0, \#] = \#$ already, nothing changes.

Another promising prediction of the disappointment approach is that any quantifier that gives rise to universal inference with respect to presuppositions under the basic system will now give rise to universal presuppositions. We can see this by proving a slightly more general result. Suppose that g is a function that takes a function type as an argument, and there is f such that $g(f) = 1$, and that X is a set such that, for all f', if $g(f') = 1$ then $f'(x) \neq \#$ for all $x \in X$. Now consider an h such that there is a $y \in X$ for which $h(y) = \#$. h is disappointing in g, since for all $h' \backsimeq h$, $h'(y) = \#$ and so $g(h') \neq 1$. Thus $g{:}(h){:} = \#$, which is to say $g{:}(h){:}$ suffers presupposition failure whenever $h(x) = \#$ for some $x \in X$, thus:

Proposition 6 *Where g is a unary function such that* 1 *is in the range of g, and X is a set such that, for all f, if $g[f] = 1$ then $f(x) \neq \#$ for all $x \in X$, it is the case that, for all functions h of suitable argument type for g, if there is $y \in X$ such that $h(y) = \#$ then $g{:}(h){:} = \#$.*

But if a quantifier g gives rise to the universal inferences, this means exactly that the restrictor is a set such that the quantifier is only true of functions that never suffer presupposition failure on that set. So, whenever g is a quantifier that gives rise to the universal inferences under the deployment defined above, and g is true of some predicate, g gives rise to universal presuppositions under deployment with consideration for disappointment. This in particular covers the case of quantification with *each* and *none*, which means that the new system renders the universal inferences we predicted before as presuppositions.

In fact, the above reasoning works in both directions; if for f, x, g, and X, such that $x \in X$, $f(x) = \#$, and $g[f] = 1$, we know, since $f \backsimeq f$ that f is not disappointing in g, so $g{:}(f){:} = g[f] = 1 \neq \#$, so all the quantifiers that lacked universal inferences under standard deployment will still lack universal presuppositions under deployment with concern for disappointment.

The disappointment system is one possible incremental filter on the presuppositional content of arguments. The mechanism of introducing such filters has great power and should be used cautiously. This one, at least, has a short, somewhat intuitive statement, and is limited in its impact because it does not affect truth conditions - it only takes some cases of of falsehood and turns them into presupposition failure.

B Notions of Argument Order

Most of the systems discussed above depend, to one degree on another, on a notion of the order in which the arguments of a function are read in. All the theories with predicted order asymmetries for quantifiers assume that the order of arguments places the restrictor before the nuclear scope, and all the theories that predict asymmetric connectives assume that the left argument of a connective is read in before the right one. For quantifiers, at least two notions of order will work - hierarchical order (in which a function first combines with the arguments that form smaller minimal constituents, as in the semantics of Heim and Kratzer (1998)) or linear order at LF (in which the argument of a function that is leftmost after QR is read in first). For connectives, prominent accounts of the syntax involve either ternary branching (in which case there is no hierarchical order to differentiate the arguments) or a binary branching structure in which the connective forms a constituent with its rightmost argument. In either of these cases, the asymmetric theories (like the Peters-Kleene system and the asymmetrical disappointment condition) will not be able to get the needed order out of the hierarchical relations alone, at least not in the absence of some significant additional assumptions.

The system presented in the main body of this text does not have such issues - since it doesn't predict asymmetrical connectives, it can work fine with hierarchical or linear order. This may be an advantage, as hierarchical order of evaluation is closely associated with most formulations of strong compositionality - I think that the weakening of compositionality needed to introduce linear order would probably be harmless, but it would certainly be contentious, and it would require some care to make sure that the natural order could be read off the syntactic structure and the word order facts.

It is, of course, possible to combine the hierarchical and linear conceptions of order in various ways - we could, for example, say that the arguments of a function are grouped into hierarchically defined regions (roughly, we'd want to distinguish between the semantic arguments that are also "structural arguments" like restrictors, and those where the structural argumenthood relationship is reversed, like nuclear scopes - notions of dependency and natural projection could be used to formalize this idea in various ways, depending on one's preferred syntactic theory), and to say that the argument regions are evaluated in hierarchical order, but that within each region linear order is maintained. If we want left-to-right-order for connectives, the adoption of a hybrid system might allow us to get a restrictors-first order for all quantifiers without a need to appeal to the notion of linear order at LF.

A possible advantage of hierarchical order, in either its pure or hybrid form, is that, to the extent that we can get reasonable predictions for conditionals, they should allow us to get the same predictions for postposed *if* clauses - this is an area where a linear order theory encounters difficulties, but the two examples appear to have the same presuppositions:

(56) If France is a monarchy, the king of France is bald.

(57) The king of France is bald, if France is a monarchy.

Of course, in the absence of a particular semantics of conditionals, it is hard to evaluate this. The various theories making use of strong Kleene (as opposed to Peters-Kleene) reduction all associate rather weak projections with presuppositions in the *if* clause under simple material and quantifier-restricting theories of conditionals. A theory along the lines of Stalnaker (1968), on which *if* clauses denote possible worlds, situations, or the like might fare better, but this would require an extension of the theory to functions of types not ending in `t`, which I have not developed here (although the generalization of, say, strong Kleene to arbitrary types is reasonably straightforward).

In any case, the system where our function application is the $f[x]$ application (that is, a system based on strong Kleene reduction with the repair sets dependent on relevant sets but without disappointment) gets as good results as it can get at all with hierarchical order (although it does fine with linear order too), in contrast with the theories of Schlenker (2006) and Schlenker (2008b), which depend on linear order.

C Type Theory

The system presented here has taken some of the Currying associated with typical semantic theories out of the types and put it into the combinatorics. This section contains a brief sketch of the implicit type theory for the technically inclined. I assume throughout that singletons are identical with sequences of length one and with zero-ary functions.

First, the system will have some set of basic types, including at least the type $\mathtt{t}$ of truth values. With each basic type τ is associated a nonempty domain D_τ and an error code value $\#_\tau \notin D_\tau$. The whole system of types is defined as the smallest set making the following true:

- For every basic type τ, τ is a type.
- For every $\vec{\varsigma}$ a sequence of types $\langle \vec{\varsigma} \rightarrow \tau \rangle$ is a type.

Note that in this system the "end" type of any functional type is always exposed, and the notion of filling in all the arguments of a function is always just filling in a sequence of types associated with its type.

Every type τ is associated with two domains - the non-presupposotional domain D_τ and the expanded domain Δ_τ. It will also be useful to associate with every nontrivial sequence of types $\vec{\varsigma}$ associated domains $D_{\vec{\varsigma}}$ and $\Delta_{\vec{\varsigma}}$. For τ a basic type and $\vec{\varsigma} = \varsigma_1, ..., \varsigma_n$, a nonempty sequence of types:

- D_τ is given by the model.
- $\Delta_\tau = D_\tau \cup \{\#_\tau\}$.
- $D_{\vec{\varsigma}} = D_{\varsigma_1} \times ... \times D_{\varsigma_n}$.
- $\Delta_{\vec{\varsigma}} = \Delta_{\varsigma_1} \times ... \times \Delta_{\varsigma_n}$.
- $D_{\langle \vec{\varsigma} \rightarrow \tau \rangle} = {D_\tau}^{D_{\vec{\varsigma}}}$.
- $\Delta_{\langle \vec{\varsigma} \rightarrow \tau \rangle} = {\Delta_\tau}^{D_{\vec{\varsigma}}}$.

The mix of Ds and Δs in the last line of the above is intended - it says that denotations are permitted to be presupposition triggers, but never to specify how they filter or manipulate the presuppositional content of arguments. A constituent of type τ will always have a denotation in Δ_τ, and so the normal application will be defined only on arguments in the D sets, not the Δ sets. Wherever in the above I refer to non-presuppositional arguments (including all cases of considering possible arguments or argument lists in the definitions of disappointment and the function operations), I mean arguments drawn from D_τ or $D_{\vec{\varsigma}}$ for an appropriate $\vec{\varsigma}$ or τ.

In the references to # throughout this paper, # is a shorthand for $\#_\tau$, for whatever τ is appropriate, where the $\#_\tau$s for the basic types are assumed, and for the function types we say that $\#_{\langle\vec{\varsigma}\to\tau\rangle}$ is the unique function in $\Delta_{\langle\vec{\varsigma}\to\tau\rangle}$ such that, for all $\vec{u} \in D_{\vec{\varsigma}}$, $\#_{\langle\vec{\varsigma}\to\tau\rangle}(\vec{u}) = \#_\tau$.

D Nation-States Mentioned

name in English	form of government	current head of state English title / bald?
France	Republic	President / not bald
Norway	Constitutional Monarchy	King / bald
The Netherlands	Constitutional Monarchy	Queen / not bald
Japan	Constitutional Monarchy	Emperor / not bald

E Notation

E.1 Variable Name Conventions

names	sort
f, g, h, f', et c.	functions
$\vec{u}$, $\vec{v}$, $\vec{w}$, $\vec{w}'$, et c.	sequences
x, y, z, x', et c.	sequences of length 1
X, Y, Z, X', et c.	sets
a, b, a', et c.	zero-ary functions
m, n, n', et c.	non-negative integers
τ, ς, τ', et c.	types
$\vec{\tau}$, $\vec{\varsigma}$, $\vec{\tau}'$, et c.	sequences of types
φ, ψ, φ' et c.	sentences, formulae, or other linguistic expressions

E.2 Relations, Operations, and Constants

notation	interpretation
$\emptyset$	empty set
ε	empty sequence
$\#$	"error code" value - denotes presupposition failure (or $\#_\tau$ for any type τ)
$f\backslash\vec{u}$	f reduced by $\vec{u}$ (generalization of Curried function application)
$f\backslash\!\!\backslash X$	the strong Kleene reduction of f by X
$f \not\backslash\!\!\backslash X$	the Peters-Kleene reduction of f by X
I	identity function
$f^{a/b}$	f with output substitution of a for b
$E(f)$	the set of bivalent functions extending the bivalent part of f
$\mathcal{REP}_f(x)$	the repair set for x in f
$f[x]$	f deployed on x (a presuppositional generalization of function application)
$f{:}(x){:}$	f deployed on x with concern for disappointment
ς_n, u_n, et c.	the n^{th} elements of sequence $\vec{\varsigma}$, $\vec{u}$, et c.
$X_1 \times ... \times X_n$	the set of n-ary sequences $\vec{u}$ such that when $1 \leq m \leq n$, $u_m \in X_m$.
X^Y	the set of functions from Y into X.
$\langle\vec{\varsigma} \to \tau\rangle$	the $\vec{\varsigma}$ to τ function type.
D_τ	the set of non-presuppositional denotations for type τ
Δ_τ	the set of (possibly presuppositional) denotations for type τ
$\#_\tau$	the failure value of type τ (If τ is basic, this is given by the model. If τ is a function type, this is a suitable constant function.)
$\lvert\vec{u}\rvert$	length of $\vec{u}$
$\lvert X\rvert$	cardinality of X
$[\![\varphi]\!]$	the extension of φ
$x \approx [\![\varphi]\!]$	it is probably harmless to act as if $x = [\![\varphi]\!]$
$\vec{u}\vec{v}$	$\vec{u}$ concatenated with $\vec{v}$

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