Computational Linguistics I, Winter 2006. Marcus Kracht

Solutions.

[A4.2] The strings of length at most 10 of (`a`*|`b`?)`ca` are:

`ca`,
`bca`,
`aca`,
`aaca`,
`aaaca`,
`aaaaca`, `aaaaaca`,
`aaaaaaca`,
`aaaaaaaca`,
`aaaaaaaaca`

The strings of length at most 10 of `ab`$^+$|`ba`$^+$ are

`ab`,
`abb`,
`abbb`,
`abbbb`,
`abbbbb`,
`abbbbbb`,
`abbbbbbb`,
`abbbbbbbb`,
`abbbbbbbbb`,
`ba`,
`baa`,
`baaa`,
`baaaa`, `baaaaa`,
`baaaaaa`,
`baaaaaaa`,
`baaaaaaaa`,
`baaaaaaaaa`

The strings of length at most 10 of $(\mathtt{aa}^*\mathtt{b})^2$ are

abab,
aabab,
abaab,
aaabab,
aabaab,
aaabab,
aaaabab,
aaabaab,
aabaaab,
abaaaab,
aaaaabab,
aaaabaab,
aaabaaab,
aabaaaab,
abaaaaab,
aaaaaabab,
aaaaabaab,
aaaabaaab,
aaabaaaab,
aabaaaaab,
abaaaaaab,
aaaaaaabab,
aaaaaabaab,
aaaaabaaab,
aaaabaaaab,
aaabaaaaab,
aabaaaaaab,
abaaaaaaab

[**A4.3**] Show that in general $L \cdot M = N$ iff (= if and only if) $L = N//M$ iff $M = L\backslash\backslash N$. **Solution.** ($\Rightarrow$) Assume that $N = L \cdot M$. Let $\vec{x} \in L$. Then for every $\vec{y} \in M$, $\vec{x}^\frown\vec{y} \in L \cdot M = N$, whence $L \subseteq N//M$, by definition. Furthermore, assume that $\vec{x} \in N//M$. Then there is $\vec{y} \in M$ such that $\vec{x}^\frown\vec{y} \in N = L \cdot M$, whence $\vec{x} \in L$, so the two sets are equal. ($\Leftarrow$) Assume that $L = N//M$. We aim to show that $L \cdot M = N$. Let $\vec{x} \in L$. Then for all $\vec{y} \in M$: $\vec{x}^\frown\vec{y} \in N$. $\vec{x}$ was arbitrary, and so $L \cdot M \subseteq N$. Now pick $\vec{x} \in N$. Then $\vec{x} = \vec{y}^\frown\vec{z}$ for some $\vec{y} \in L$ and $\vec{z} \in M$, showing $\vec{x} \in L \cdot M$. (The other equivalence is basically similar.)

[**A4.4**] Show that in general $(L^* \cdot M^*)^* = (L \cup M)^*$. **Solution.** We show first (a) $(L^* \cdot M^*)^* \subseteq (L \cup M)^*$. To establish this, it is enough to show that $(L^* \cdot M^*) \subseteq (L \cup M)^*$. To see this, note that $L \subseteq (L \cup M)^*$, whence $L^* \subseteq (L \cup M)^*$, and similarly $M^* \subseteq (L \cup M)^*$; finally, $L^* \cdot M^* \subseteq (L \cup M)^* \cdot (L \cup M)^* \subseteq (L \cup M)^*$. Next we show (b) $(L \cup M)^* \subseteq (L^* \cdot M^*)^*$. $L \cup M \subseteq (L^* \cdot M^*)$, since $L \subseteq L^* \subseteq L^* \cdot M^*$ (and similerly for M). Taking star on both sides preserves the inclusion. (We made use of the following principles: if $H \subseteq K$ then also $H^* \subseteq K^*$. It follows that if $H \subseteq K^*$ then $H^* \subseteq K^{**} = K^*$.)