Statistics Fall 2005

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Assignments, Part 4.

Ex 4.1 Write R-functions that calculates for a given vector $\overline{x}$ and real number c an interval $[m-d, m+d]$ such that c is the confidence level that the actual mean falls into the interval. Base it on the definitions of the manuscript: with $\lambda = \delta\sqrt{n/\theta(1-\theta)}$, the interval is $[\theta - \lambda/2\sqrt{n}, \theta + \lambda/2\sqrt{n}]$ and the confidence is $1/\lambda^2$. Draw some samples and compute the results. Compare this with the result from the t-test (simply apply `t.test` to your vector). Warning: the formulae are valid for a Benoulli experiment only!

Ex 4.2 Given the probabilities $P_n(k)$, the function e^{-x^2} is approximated as follows: $e^{-x^2} \approx P_n(k)\sqrt{2\pi npq} = \binom{n}{k} p^k q^{n-k}\sqrt{2\pi npq} =: G(p, n, k)$. We can see how well this converges by making a plot for the right hand side. However, rather than plotting the values over k, we have to plot them over $x = (k - np)/\sqrt{npq}$. Thus we must define a function that runs through the values 0 to n and returns the pairs $\langle (k-np)/\sqrt{npq}, G(p, n, k)\rangle$. View the results for $p = .7$, and $n = 5, 10, 20, 50$. (A nice way of showing would be to plot them in one chart. One way to achieve this is to use `points`. This is tedious, but one can make R do it by writing a function.)

Ex 4.3 We shall try to get an idea of what the error is that the approximation is giving us. We have the function $f(x) = e^{-x^2}$, whose values we want to approximate by a function $g(x)$. The first question is: what is the maximum of $|g(x) - f(x)|$. Write an R-function that uses as input two vectors: one consisting of the f-values over a fixed set of points, and the other for the g-values; and it should return the maximum difference for all points. Next, we complicate this a little: we ask not about the difference in absolute terms but relative to the true value: thus we want the maximum of $|f(x) - g(x)|/|f(x)|$. Now apply this to the approximations of the previous exercise.