Statistics Fall 2005

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Assignments, Part 1.

You may email me the graphics instead of printing it out. Similarly for the answers to the assignments. Notice that in case you submit nonelectronically, you do *not* have to type them, I accept handwriting as well! When I ask you to use R, the answer will always at least consist in telling me what you told R to do, ie I want to see the code that you issued.

Ex 1.1 There is a lottery, where you have to guess 6 numbers between 1 and 49. What is the chance of getting 0, 1, 2, $\cdots$ 6 numbers right? How much more should one award to a person that gets 5 numbers right than to someone who gets 3 numbers right for this to be a fair reward scheme? Use R to generate a few sample bets.

Solution The chance of getting 6 numbers right is $\binom{49}{6} = 13983816$. To get 5 numbers right, proceed as follows. Let $x_1, x_2, \ldots, x_6$ be the 6 correct numbers. Now discard 1 of these numbers and replace it by any of the other numbers that have not been drawn: so you get $6 \cdot 43 = 258$ possibilities for each draw. To get 4 numbers right, discard 2 numbers ($\binom{6}{2}$ choices) and replace them by the other numbers ($\binom{43}{2}$ choices). The general formula is $\binom{6}{(6-i)} * \binom{43}{i}$ with values:

(1) 1 258 13545 246820 1851150 5775588 6096454

Now, the reward is given according to the odds, so you should get $246820/258 \approx 957$ times more. The code to get 6 samples with no repetitions is `sample (49, 6, F)`.

Ex 1.2 Write a program that plots the function $y = 20 - x^2$ and generate the ouput between 0 and 5 in a file. Overlay the following data points onto the graphics: $(0, 19.6), (2, 15), (3.5, 13), (4, 10), (5, .4)$.

Solution

(2)
```
> pdf (file="graph1.pdf")
> x <- c(0, 2, 3.5, 4, 5)
> y <- c(19.6, 15, 13, 10, .4)
> plot (x, y)
> plot (function(x) 20 - x * x, 0, 5, add=TRUE)
> dev.off ()
```

Ex 1.3 Draw the function $\binom{20}{i}$ using triangles as data points. Calculate the mean of this function and draw the mean as a line into the graphics. Can you give a short expression that calculates the mean other than adding up all the numbers?

Solution The function will be defined by issuing `f <- function (x) choose (20, x)`. The mean can be calculated as follows.

(3)
```
mean (choose (20, 0:20))
```

This will give you 49932.19, which is incidentally $2^{20}/21$. Check this by issuing `(2 ** 20)/21`. (The mean is the sum of all numbers divided by the length of the sequence, here 21! The sum is, as we have noted earlier, 2^{20}.)

For the drawing do the following.

(4)
```
> pdf (file="graph2.pdf")
> plot (0:20, choose (20, 0:20), pch=2)
> abline (h=49932.19)
> dev.off ()
```

Ex 1.4 Find a simple expression for $\binom{n}{k} + 2\binom{n}{k+1} + \binom{n}{k+2}$. *Hint.* Generate a few of these numbers and see if you can find a nice expression for them.

Solution The expression is $\binom{n+2}{k}$. To see this, do for example this:

(5)
```
x <- rep(0, 7)
x <- choose (8, 0:6) + 2 * choose (1:8) + choose (8, 2:8)
```

What you get is

(6)
```
[1] 45 120 210 252 210 120 45
```

Now issue

(7)
```
y <- rep(0, 7)
y <- choose (10, 2:8)
```

And you get the same value. Of course, a proof can also be given:

$$
\begin{aligned}
&\binom{n}{k} + 2\binom{n}{k+1} + \binom{n}{k+2} \\
=&\left(\binom{n}{k} + \binom{n}{k+1}\right) + \left(\binom{n}{k+1} + \binom{n}{k+2}\right) \\
=&\binom{n+1}{k+1} + \binom{n+1}{k+2} \\
=&\binom{n+2}{k+2}
\end{aligned} \tag{8}
$$