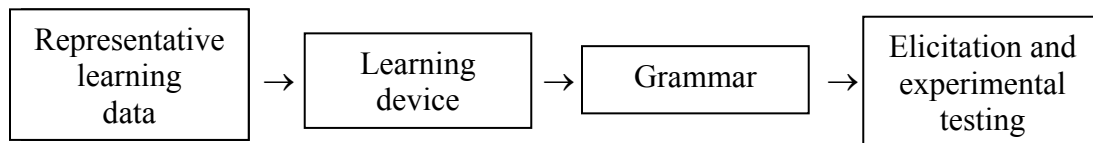


# Allomorph discovery as a basis for learning alternations

## BACKGROUND: LEARNING MODELS IN LINGUISTICS

### 1. The research program of generative grammar

- Chomsky (1965:24-26) and similar aspirational work, on human children:



### 2. What is in the “learning device”?

- Learning mechanisms of some sort
- Linguistic theory; often construed as principles of Universal Grammar (UG)

### 3. The role of computational learning models

- We can address the content of the learning device (as with other inaccessible natural systems) with computational modeling.
- That is, we model people by devising computational systems that perform the same way that people do — Emanuel Dupoux has called this “linguistics as reverse engineering.”
- What would perfect success for such a program (in the distant future) look like? Our models would make many **correct predictions**:
  - They will respond to elicitation identically to native speakers, including ambivalence.
  - They will perform identically in wug-tests and other experimental tasks.
  - Algorithms will occasionally learn patterns *incorrectly*, in the very same cases where people do (language change).

### 4. Phonology seems to be a good area for trying this sort of work<sup>1</sup>

- In Optimality Theory, we know how to rank constraints correctly, given a suitable set of inputs, winning candidates, and losers (Tesar and Smolensky 1993, 2000)
- Free variation doesn’t bother us, since our frameworks and algorithms can accurately learn such patterns as probabilistic grammars, that match frequency (Boersma and Hayes 2000, Goldwater and Johnson 2003, Boersma and Pater 2016, etc.)
- We can learn the phonotactics of a set of surface forms (Hayes and Wilson 2008).

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<sup>1</sup> But syntax, too! I have noticed Clark and Lappin (2011) and Abend et al. (2017). Morphology: Ryan (2010).

## 5. The focus here: alternations in paradigms (morphophonemics)

- Standard setup: data in rows and columns
  - rows are stems, columns are inflectional categories
  - the morphemes alternate, following the principles of the phonology
- When we analyze a data set of this kind, we
  - find the **underlying form** of each morpheme present
  - discover the morphological principles that order the morphemes linearly
  - **formulate and order rules** (Chomsky and Halle 1968), or **rank a constraint set** in Optimality Theory (Prince and Smolensky 1993), so as to derive the surface forms from the concatenated underlying forms.
- This sort of analysis is utterly central to phonology
  - ... and the basis for most problems sets used to train new participants in the field.

## 6. Can the alternation patterns of paradigms be learned by algorithm?

- **Computer scientists** are currently working on this problem, most effectively with neural networks — see e.g. Cotterell et al. (2017).
- I feel that **phonologists** should be participating in this enterprise.
  - Some of their theoretical ideas might be directly applicable to solving paradigm problems.
  - We also have a lot of data experience and typological knowledge.

## 7. What I've tried to do

- Invent a system for solving paradigm problems that makes maximal use of ideas from mainstream phonological theory

## 8. A cautionary note before going on: solving problem sets is an idealization

- A large body of research tells us that a standard problem set answer falls far short of what real learners know about phonological patterning — there is **much more detail**.
  - See Zuraw 2000, 2010, Albright 2002a, Albright and Hayes 2003, Ernestus and Baayen 2003, Hayes/Zuraw/Siptar/Londe 2009
- I have ideas on how to scale up the work here to handle detail, but will not address this here.

### BREAKING THE PROBLEM INTO STEPS

## 9. Stepwise solution of paradigm problems

- I think phonologists share, to some extent, an intuitive sense of how phonology problems can be most effectively solved.

- In my undergraduate textbook *Introductory Phonology* I tried to articulate this intuition explicitly, and will follow the outline given there — there are five steps.

### 10. Toy illustration: final consonants and clusters in Catalan

- Here is method as applied to five stems taken from Kenstowicz and Kisseberth's (1979:328) Catalan problem, (11) below.
- Go ahead and take a peek.
- Quick answer:
  - Stems are /ultim/, /plen/, /klar/, /profund/, /fort/
  - Fem. sg. and fem. plur. endings are /-ə/ and /-es/
  - Phonology: /n/ and /r/ are deleted finally, after which /t,d/ delete [ C \_\_\_\_ ], exposing new [n]'s and [r]'s to word-final position.

### 11. Step 1: divide the word into its morphemes

u l t i m	'last'	p r o f u n	'deep'
u l t i m   ə	'last fem.sg.'	p r o f u n d   ə	'deep fem.sg.'
u l t i m   e s	'last fem.pl.'	p r o f u n d   e s	'deep fem.pl.'
p l e	'full'	f o r	'strong'
p l e n   ə	'full fem.sg.'	f o r t   ə	'strong fem.sg.'
p l e n   e s	'full fem.pl.'	f o r t   e s	'strong fem.pl.'
k l a	'plain'		
k l a r   ə	'plain fem.sg.'		
k l a r   e s	'plain fem.pl.'		

### 12. Step 2: Consulting the divisions made, list all allomorphs of each morpheme

'last':	[ultim]	f.sg.:	[-ə]
'full':	[plen] ~ [ple]	f.pl.:	[-es]
'plain':	[klar] ~ [kla]		
'deep':	[profund] ~ [profun]		
'strong':	[fort] ~ [for]		

### 13. Step 3: Consulting the allomorph list, find the segmental alternations

They are: n ~ Ø, r ~ Ø, d ~ Ø, t ~ Ø.

### 14. Step 4: Consider multiple hypotheses about underlying forms and reconstruct the derivations that they necessitate

- A good guess: *in Catalan nouns the feminine (prevocalic) forms always provide the right basis for the UR* (more on this below).
- So we provisionally adopt this idea and set up the “sketch derivations” that would be needed:

'full'	'full-f.'	'clear'	'clear-f.'	'deep'	'deep-f.'	
/plen/	/plen-ə/	/klar/	/klar-ə/	/profund/	/profund-ə/	hypothesized underlying forms
pleØ	—	klaØ	—	profunØ	—	what the phonology must do
[plen]	[plenə]	[kla]	[klarə]	[profun]	[profundə]	surface forms

- This set-up requires us to discover deletion at Step 5
- A full implementation of the approach would also try the other option, /ple/, /kla/, /profun/ with consonant insertion — and would fail at Step 5.

### 15. Step 5: Find a phonological grammar that will do what needs to be done

- For the above, in rule-based phonology, this would be:

#### N Deletion

$n \rightarrow \emptyset / \_\_\_ ]_{\text{word}}$

#### R Deletion

$r \rightarrow \emptyset / \_\_\_ ]_{\text{word}}$

#### Alveolar Stop Deletion

$\{t, d\} \rightarrow \emptyset / C \_\_\_ ]_{\text{word}}$  (must be ordered after N Deletion and R Deletion)

/plen/	/plen-ə/	/klar/	/klar-ə/	/profund/	/profund-ə/	hypothesized UR
pleØ	—	—	—	—	—	N Deletion
—	—	klaØ	—	—	—	R Deletion
—	—	—	—	profunØ	—	Alveolar Stop Deletion
[plen]	[plenə]	[kla]	[klarə]	[profun]	[profundə]	surface forms

- Success!
- Later we will do this problem again in Optimality Theory.

### 16. Can Steps 1-5, designed for humans, be made the basis of a learning algorithm?

- Step 1, “break up the words”, turns out to be hardest, and is the focus here.
- Steps 2-3, “find the alternations”, seems to be easier; see below.
- Step 4, “guess the UR’s” is trivial for **Albrightianists**, hard for others; see below.
- Step 5, “find the grammar that can achieve these mappings”, is easy if we are given OT constraints in advance, harder if we need to discover them.
- So let’s outline the five steps and then solve some phonology problems ...

### 17. One reason to favor an incremental strategy

- **Explosion of hypothesis space:** the set of possible UR’s coupled with grammars is incomparably vast for any decent-size problem.

- See Tesar (2014:§6.2), Jarosz (2015) for clear discussion on this point.
- My own system does have a big search-space bottleneck — Step 1 —but I think it’s not *too* big.

## STEP I: DIVIDING WORDS INTO THEIR MORPHEMES

### 18. The task for Step 1

- Given a glossed paradigm, as above, discover an appropriate division into morphemes, without yet knowing the phonology.

### 19. A warning before we even start

- The success of the “find morphemes first” strategy is not guaranteed in advance.
- Is it *really* possible to segment the words into morphemes without knowing the phonology?

### 20. A sobering thought experiment

- You hear:

[mapa]	‘cat-sg.’	[maparu]	‘cat-plur.’
[tapan]	‘dog.-sg.’	[tapanu]	‘dog-pl.’

- Is this /mapar/, with Final /r/ Deletion and suffix /-u/?
  - Could be, since Catalan works like this.
- Is this /mapa/, /tapan/ with suffix allomorphs /-u/ post-C, /-ru/ post-V?
  - Could be, since Japanese works like this.
- In fact, we will see that our model, equipped with full information about these languages, can make the right choices.

### 21. Making it harder for realism’s sake: discontinuous allomorphs

- Let include cases where the surface forms are discontinuous, due to
  - **infixation**, as in Tagalog [*bago*] ~ [*b-um-ago*].
  - **metathesis** of segments belonging to separate morphemes [pama], /naj-pama/ → [*na-p-j-ama*], as in Yagua (Powlison 1962)
- For representations, we can’t just use hyphens; we must instead *coindex* every segment with the gloss of the morpheme it belongs to.

### 22. Sample of input data: part of my fictional “Suffix Fricatives” language

kuŋanpa	turtle-NOM	ruxiŋpa	dove-NOM	tuϕærpa	fox-NOM
kuŋanta	turtle-DAT	ruxiŋta	dove-DAT	tuϕærta	fox-DAT
kuŋanka	turtle-ACC	ruxiŋka	dove-ACC	tuϕærka	fox-ACC
piθoϕa	dog-NOM	ŋexexa	wolf-NOM		
piθoθa	dog-DAT	ŋexexa	wolf-DAT		
piθoxa	dog-ACC	ŋexexa	wolf-ACC		

- Pretty clear to any phonologist that we have:
  - stems: /kuŋan/, /ruxiŋ/, /tuϕæɾ/, /piθo/, /ŋexɛ/
  - suffixes: /-pa/, /-ta/, /-ka/
  - /ptk/ spirantize intervocalically to [ϕθx].

### 23. Correct intended algorithm-output for Suffix Fricatives

k <sub>4</sub> u <sub>4</sub> ŋ <sub>4</sub> a <sub>4</sub> n <sub>4</sub> p <sub>1</sub> a <sub>1</sub>	turtle <sub>4</sub> NOM <sub>1</sub>	r <sub>5</sub> u <sub>5</sub> x <sub>5</sub> i <sub>5</sub> ŋ <sub>5</sub> p <sub>1</sub> a <sub>1</sub>	dove <sub>5</sub> NOM <sub>1</sub>	t <sub>6</sub> u <sub>6</sub> ϕ <sub>6</sub> æ <sub>6</sub> r <sub>6</sub> p <sub>1</sub> a <sub>1</sub>	fox <sub>6</sub> NOM <sub>1</sub>
k <sub>4</sub> u <sub>4</sub> ŋ <sub>4</sub> a <sub>4</sub> n <sub>4</sub> t <sub>2</sub> a <sub>2</sub>	turtle <sub>4</sub> DAT <sub>2</sub>	r <sub>5</sub> u <sub>5</sub> x <sub>5</sub> i <sub>5</sub> ŋ <sub>5</sub> t <sub>2</sub> a <sub>2</sub>	dove <sub>5</sub> DAT <sub>2</sub>	t <sub>6</sub> u <sub>6</sub> ϕ <sub>6</sub> æ <sub>6</sub> r <sub>6</sub> t <sub>2</sub> a <sub>2</sub>	fox <sub>6</sub> DAT <sub>2</sub>
k <sub>4</sub> u <sub>4</sub> ŋ <sub>4</sub> a <sub>4</sub> n <sub>4</sub> k <sub>3</sub> a <sub>3</sub>	turtle <sub>4</sub> ACC <sub>3</sub>	r <sub>5</sub> u <sub>5</sub> x <sub>5</sub> i <sub>5</sub> ŋ <sub>5</sub> k <sub>3</sub> a <sub>3</sub>	dove <sub>5</sub> ACC <sub>3</sub>	t <sub>6</sub> u <sub>6</sub> ϕ <sub>6</sub> æ <sub>6</sub> r <sub>6</sub> k <sub>3</sub> a <sub>3</sub>	fox <sub>6</sub> ACC <sub>3</sub>
p <sub>7</sub> i <sub>7</sub> θ <sub>7</sub> o <sub>7</sub> ϕ <sub>1</sub> a <sub>1</sub>	dog <sub>7</sub> NOM <sub>1</sub>	ŋ <sub>8</sub> e <sub>8</sub> x <sub>8</sub> e <sub>8</sub> ϕ <sub>1</sub> a <sub>1</sub>	wolf <sub>8</sub> NOM <sub>1</sub>		
p <sub>7</sub> i <sub>7</sub> θ <sub>7</sub> o <sub>7</sub> θ <sub>2</sub> a <sub>2</sub>	dog <sub>7</sub> DAT <sub>2</sub>	ŋ <sub>8</sub> e <sub>8</sub> x <sub>8</sub> e <sub>8</sub> θ <sub>2</sub> a <sub>2</sub>	wolf <sub>8</sub> DAT <sub>2</sub>		
p <sub>7</sub> i <sub>7</sub> θ <sub>7</sub> o <sub>7</sub> x <sub>3</sub> a <sub>3</sub>	dog <sub>7</sub> ACC <sub>3</sub>	ŋ <sub>8</sub> e <sub>8</sub> x <sub>8</sub> e <sub>8</sub> x <sub>3</sub> a <sub>3</sub>	wolf <sub>8</sub> ACC <sub>3</sub>		

### THE PROPOSED APPROACH FOR DIVIDING UP WORDS

#### 24. Overall plan

- Let's use GEN + EVAL as in OT.
  - I.e., we lay out choices, then give formal criteria for picking a winner.
  - Such architectures are not just for linguistics, but are common in cognitive science as learning models.<sup>2</sup>

#### 25. GEN and its size

- Our GEN = all possible coindexations of segments with the morphemes of their word.

For: [sui] 'pig<sub>1</sub>-VOC.<sub>2</sub>'  
 GEN is: { s<sub>1</sub>u<sub>1</sub>i<sub>1</sub>, s<sub>1</sub>u<sub>1</sub>i<sub>2</sub>, s<sub>1</sub>u<sub>2</sub>i<sub>1</sub>, s<sub>1</sub>u<sub>2</sub>i<sub>2</sub>, s<sub>2</sub>u<sub>1</sub>i<sub>1</sub>, s<sub>2</sub>u<sub>1</sub>i<sub>2</sub>, s<sub>2</sub>u<sub>2</sub>i<sub>1</sub>, s<sub>2</sub>u<sub>2</sub>i<sub>2</sub> }

- (23) shows the correct coindexation for the Suffix Fricative language; I calculate there are 634 octillion others.
- In general, the GEN needed is really big, which will be an issue below.

#### 26. Choice of constraint-based model

- I use **Harmonic Grammar** (Smolensky 1986, Pater 2009, Potts et al. 2010, etc.), so constraints are weighted, not ranked.
- For the computations I use the **maxent** variant of Harmonic Grammar (Smolensky 1986, Goldwater and Johnson 2003), for reasons to be made clear.

#### 27. How we extract a prediction from the model

- Maxent assigns a probability to every candidate in GEN.

<sup>2</sup> See e.g. Samut (2010), entitled "Learning as search", in a computer science encyclopedia.

- We will say that the intended division of the words into morphemes is the **highest-probability** candidate.

## 28. Constraints

- The learning system will consist of four constraints embodying what my experience indicates are properties shown by correct morpheme divisions.
- We will run through these constraints in what follows.

## 29. The more important basis for constraints: *similarity of allomorphs*

- Here is the right division for the Suffix Fricative language in (22):

A.	kuŋan-pa	turtle <sub>4</sub> NOM <sub>1</sub>	ruxiŋ-pa	dove <sub>5</sub> NOM <sub>1</sub>	tuφær-pa	fox <sub>6</sub> NOM <sub>1</sub>
	kuŋan-ta	turtle <sub>4</sub> DAT <sub>2</sub>	ruxiŋ-ta	dove <sub>5</sub> DAT <sub>2</sub>	tuφær-ta	fox <sub>6</sub> DAT <sub>2</sub>
	kuŋan-ka	turtle <sub>4</sub> ACC <sub>3</sub>	ruxiŋ-ka	dove <sub>5</sub> ACC <sub>3</sub>	tuφær-ka	fox <sub>6</sub> ACC <sub>3</sub>
B.	piθo-φa	dog <sub>7</sub> NOM <sub>1</sub>	ŋexex-φa	wolf <sub>8</sub> NOM <sub>1</sub>		
	piθo-θa	dog <sub>7</sub> DAT <sub>2</sub>	ŋexex-θa	wolf <sub>8</sub> DAT <sub>2</sub>		
	piθo-xa	dog <sub>7</sub> ACC <sub>3</sub>	ŋexex-xa	wolf <sub>8</sub> ACC <sub>3</sub>		

- The right division yields this allomorph list:

\*-pa ~ -φa      NOM  
 \*-ta ~ -θa      DAT  
 \*-ka ~ -xa      ACC

[kuŋan] ‘turtle’, [ruxiŋ] ‘dove’, [tuφær] ‘fox’, [piθo] ‘dog’, [ŋexex] ‘wolf’

- Here is a sample *wrong* division:

kuŋan-pa	turtle <sub>4</sub> NOM <sub>1</sub>	ruxiŋ-pa	dove <sub>5</sub> NOM <sub>1</sub>	tuφær-pa	fox <sub>6</sub> NOM <sub>1</sub>
kuŋan-ta	turtle <sub>4</sub> DAT <sub>2</sub>	ruxiŋ-ta	dove <sub>5</sub> DAT <sub>2</sub>	tuφær-ta	fox <sub>6</sub> DAT <sub>2</sub>
kuŋan-ka	turtle <sub>4</sub> ACC <sub>3</sub>	ruxiŋ-ka	dove <sub>5</sub> ACC <sub>3</sub>	tuφær-ka	fox <sub>6</sub> ACC <sub>3</sub>
*piθoφ-a	dog <sub>7</sub> NOM <sub>1</sub>	*ŋexexφ-a	wolf <sub>8</sub> NOM <sub>1</sub>		
*piθoθ-a	dog <sub>7</sub> DAT <sub>2</sub>	*ŋexexθ-a	wolf <sub>8</sub> DAT <sub>2</sub>		
*piθox-a	dog <sub>7</sub> ACC <sub>3</sub>	*ŋexex-a	wolf <sub>8</sub> ACC <sub>3</sub>		

- The wrong division yields this allomorph list:

\*-pa ~ -a      NOM  
 \*-ta ~ -a      DAT  
 \*-ka ~ -a      ACC

kuŋan ‘turtle’                      ruxiŋ ‘dove’                      tuφær ‘fox’

\*piθoφ ~ piθoθ ~ piθox ‘dog’

\*ŋexexφ ~ nexexθ ~ nexex ‘wolf’

- These allomorphs are *far less mutually similar* than the allomorphs of the correct answer.
- N.B. Pressure toward allomorph-similarity is known elsewhere in phonology (historical change, Kiparsky (1982); elicitation from children, Jo (2017), Do (in press); artificial grammar learning studies (Wilson 2006; White 2013, 2014).

### 30. SIMILARITY stated intuitively

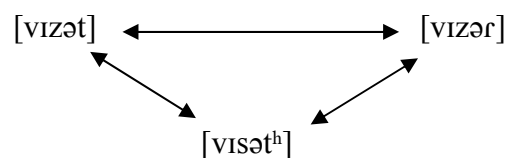
- “Penalize a candidate (paradigm with morphemic indices) to the extent that the allomorphs it implies fail to be mutually similar.”
  - What we actually want to compute is **dissimilarity**, since that is the basis for assigning constraint violations.

### 31. Assessing dissimilarity

- There is a substantial literature that can help; I’ve borrowed wholesale.
- I’ll cover just a quick outline, with references.

### 32. Proceed hierarchically, summing dissimilarity throughout

- Psycholinguistic experiments can be analyzed with maxent, yielding numerical values for the dissimilarity created by each **feature** (White 2012).
- The weights from feature differences can be added to obtain a metric of **segment dissimilarity** (Wilson and Obdeyn 2009).
- The segments of **two allomorphs** can be aligned in an optimal way (Sankoff and Kruskal 1999), such that their dissimilarity is the sum of the dissimilarity values of their aligned segments (Bailey and Hahn 2001, Albright and Hayes 2003).<sup>3</sup>
- For a parse of the data into morphemes, violations of SIMILARITY are calculated as the **summed dissimilarity of all allomorph pairs in the data**.
  - For example, English *visit* has three allomorphs in American English (in *visit*, *visitor*, *visitation*), so three comparisons would be made:



### 33. First refinement to SIMILARITY: the stem/affix distinction

- Languages widely rank Faithfulness constraints for stems higher than for affixes (McCarthy and Prince (1995), Casali (1997), Walker (2011:§2.5)
- So we might expect it to be useful to penalize stem-variation more harshly than affix-variation.

SIMILARITY<sub>STEM</sub>

<sup>3</sup> There is also a factor for “similarity to null”, calculated simply as best fit to the overall data set.



“Penalize candidates to the extent that the stem allomorphs they imply fail to be mutually similar.”

SIMILARITY<sub>AFFIX</sub>

“Penalize candidates to the extent that the affix allomorphs imply fail to be mutually similar.”

### 34. Second refinement to SIMILARITY: abstract away from size of data sample

- Stem similarity is normalized by dividing by the total number of stems.
- Affix similarity is normalized by dividing by the total number of affixes.

### 35. CONTIGUITY

- *Penalize a morpheme whose segments are not contiguous.*
- Invented (for phonology) by McCarthy and Prince (1995)
- In practice, this penalizes:
  - **Real-life cases** (as in (21) above). These candidates must win by performing better on the other constraints.
  - **Stupid candidates** we want to rule out (quite common). Consider an example modeled on real-life Lomongo, with intervocalic /b/ Deletion. Correct parse:

‘duck’	‘goose’	
molon-e	baram-e	NOM.
molon-o	baram-o	ACC.
pa-molon	pa-aram	GEN.
ti-molon	ti-aram	DAT.

— Allomorphs of ‘goose’ are [baram] ~ [aram].

Wrong parse:

‘duck’	‘goose’	
molon- <b>e</b>	<b>b</b> -aram- <b>e</b>	NOM.
molon- <b>o</b>	<b>b</b> -aram- <b>o</b>	ACC.
pa-molon	pa-aram	GEN.
ti-molon	ti-aram	DAT.

— Wrong parse: ‘goose’ is [aram], [-e] and [-o] have circumfix allomorphs [b- -e] and [b- -o]; CONTIGUITY discourages them.

### 36. VARIATION

- This constraint is the least like ordinary phonology.

- Consider the **Prefix-Temptation** language: /kimen/, /kurat/, /petep/, /loran/, with prefixes /ni-/, /bi-/, /ri-/ undergoing vowel harmony (vowel changes to [u] next to stem [u]).<sup>4</sup>
- Here is the bad analysis we want to avoid:

n-ikimen	‘sing 1p.’	n-ukurat	‘swim-1p.’
b-ikimen	‘sing 2p.’	b-ukurat	‘swim-2p.’
r-ikimen	‘sing 3p.’	r-ukurat	‘swim-3p.’
n-ipetep	‘sit 1p.’	n-uloran	‘think-1p.’
b-ipetep	‘sit 2p.’	b-uloran	‘think-2p.’
r-ipetep	‘sit 3p.’	r-uloran	‘think-3p.’

- This analysis is *perfect* with respect to SIMILARITY but is probably wrong.
- The analysis is unlikely because every stem begins in either [i] or [u].
- Such analyses can be discouraged by requiring the stem inventory to be Variegated.

### 37. VARIEGATION (intuitively)

- “Disfavor candidate parses to the extent that the stem inventory is dominated by a single frequent initial or final segment.”
- I will skip formalization here.

### 38. Summary: the full constraint set

- SIMILARITY<sub>STEM</sub>
- SIMILARITY<sub>AFFIX</sub>
- CONTIGUITY
- VARIEGATION

### 39. Assigning weights to the constraints

- I looking at all 20 languages I was studying, all at once, with both the right parses and numerous wrong parses, and found weights that permitted the discovery of the right parse in all 20 languages.
- I did this with a large Excel spreadsheet and the Excel Solver add-in.
- The best weights found were:

STEM SIMILARITY	12.2
AFFIX SIMILARITY	4.9
VARIEGATION	240.0
CONTIGUITY	10.8

- But somewhat different weights also work.

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<sup>4</sup> Maltese Arabic, below, seems to be close to a real-life example where this danger arises.

#### 40. The status of this sort of weighting

- The weights are, in effect, a form of UG — me “designing” a version of humanity that is good at parsing out paradigms!
- It would be worth exploring in future work how the choice of weights could be made less stipulative.

#### 41. Searching for the best candidate morpheme-parse

- Now we have GEN (25) and EVAL (constraints and weights) — but how to find the winning candidate?
- Recall:
  - Candidates look like (23) (p. 6)
  - There are a huge number of them.
- Even worse, the search space is crammed with **local maxima** — the bane of learning-by-search.
- A few things that have proven useful are given in (42).

#### 42. Things that have helped

- Start by **finding nonalternating segments** and fixing their affiliations permanently.
- Then start the core search with random guesses.
- New candidates are found by **trying out edits on the old ones**.
- Use **beam search** — ten best candidates are kept in contention at once.
- Alternatingly search in various ways:
  - Search **small**: change affiliation of one segment at a time, or switch the affiliations of two segments.
  - Search **big**: re-conceive the search space as the *list of allomorphs*. Change an allomorph, and implement the change throughout the data.

### THE SIMULATIONS I TRIED AND THEIR RESULTS

#### 43. Choice of data sets

- 10 are made-up languages, meant to pose some particular challenge to the system, e.g.
  - the variegation language of (36).
  - Pseudo-Japanese, to test the Catalan/Japanese minimal pair described in (20).
- 10 are problem sets from Kenstowicz and Kisseberth (1979): Bizcayan, Chamorro, Catalan, Polish, Lamba, Maori, Maltese, Lomongo, Okpe, Modern Hebrew

#### 44. How does the system parse? Example from Catalan

- Data similar to (11): 27 adjectival stems, in masc. sg. (-Ø), masc. pl. ([-s]), f. sg. ([-ə]), f.pl. ([-es]).
- Sample issue:
  - We want [klarə] to be parsed [klar] + [ə], not \*[kla] + [rə] (like Japanese).

#### 45. Tableau

- Standard maxent calculations leading to probability values are given in bold.
- Candidates are parses of all 104 words in the problem, not just this stem.
- Just two candidates are presented; many others exist (they are bad, and lose)
- The bad candidate shown treats [r] as part of suffix, in Japanese fashion, in the three stems of the *klar-* pattern.

	SIMILARITY <sub>STEM</sub>	SIMILARITY <sub>AFFIX</sub>	VARIATION	CONTIGUITY			
<i>weights:</i>	12.2	4.9	240.0	10.8	<i>Harmony</i>	<i>eHarmony</i>	<i>p</i>
Correct parse	19.4	15.1	0.1481	0	347.8	$8.66 * 10^{-152}$	1
Parse treating stem-final [r] as if it were suffix	14.4	45.4	0.1481	0	435.3	$8.91 * 10^{-190}$	0

- Essential violations:
  - Correct analysis suffers from stem allomorphy: [kla] ~ [klar] etc., so worse on SIMILARITY<sub>STEM</sub>
  - Bad analysis suffers from affix allomorphy: [-ə] ~ [-rə] and [-es] ~ [-res], so worse on SIMILARITY<sub>AFFIX</sub>
- Intuitively, the stem violations are less salient because they occur only in a small subset of the total stem count (cf. (34)).

#### 46. How well did the system do in general?

- All 10 made-up languages, plus 8 real languages: the outcome of the search is the correct answer.
- As we hoped, Catalan and Pseudo-Japanese (20) are each given their own correct analysis.
- For Okpe and Hebrew, search fails, landing on a bad parse. But this parse is far less harmonic than the linguist's parse, which I conjecture to be the true-but-unfindable optimum.

### THE REMAINING STEPS TO PHONOLOGY

#### 47. Back to Step 2: list all allomorphs of each morpheme

- These are trivially read off the indexed representations used here, e.g. for (23) above:

k <sub>4</sub> u <sub>4</sub> ŋ <sub>4</sub> a <sub>4</sub> n <sub>4</sub> p <sub>1</sub> a <sub>1</sub>	turtle <sub>4</sub> NOM <sub>1</sub>	r <sub>5</sub> u <sub>5</sub> x <sub>5</sub> i <sub>5</sub> ŋ <sub>5</sub> p <sub>1</sub> a <sub>1</sub>	dove <sub>5</sub> NOM <sub>1</sub>	t <sub>6</sub> u <sub>6</sub> ϕ <sub>6</sub> æ <sub>6</sub> r <sub>6</sub> p <sub>1</sub> a <sub>1</sub>	fox <sub>6</sub> NOM <sub>1</sub>
k <sub>4</sub> u <sub>4</sub> ŋ <sub>4</sub> a <sub>4</sub> n <sub>4</sub> t <sub>2</sub> a <sub>2</sub>	turtle <sub>4</sub> DAT <sub>2</sub>	r <sub>5</sub> u <sub>5</sub> x <sub>5</sub> i <sub>5</sub> ŋ <sub>5</sub> t <sub>2</sub> a <sub>2</sub>	dove <sub>5</sub> DAT <sub>2</sub>	t <sub>6</sub> u <sub>6</sub> ϕ <sub>6</sub> æ <sub>6</sub> r <sub>6</sub> t <sub>2</sub> a <sub>2</sub>	fox <sub>6</sub> DAT <sub>2</sub>
k <sub>4</sub> u <sub>4</sub> ŋ <sub>4</sub> a <sub>4</sub> n <sub>4</sub> k <sub>3</sub> a <sub>3</sub>	turtle <sub>4</sub> ACC <sub>3</sub>	r <sub>5</sub> u <sub>5</sub> x <sub>5</sub> i <sub>5</sub> ŋ <sub>5</sub> k <sub>3</sub> a <sub>3</sub>	dove <sub>5</sub> ACC <sub>3</sub>	t <sub>6</sub> u <sub>6</sub> ϕ <sub>6</sub> æ <sub>6</sub> r <sub>6</sub> k <sub>3</sub> a <sub>3</sub>	fox <sub>6</sub> ACC <sub>3</sub>
p <sub>7</sub> i <sub>7</sub> θ <sub>7</sub> o <sub>7</sub> ϕ <sub>1</sub> a <sub>1</sub>	dog <sub>7</sub> NOM <sub>1</sub>	ŋ <sub>8</sub> e <sub>8</sub> x <sub>8</sub> e <sub>8</sub> ϕ <sub>1</sub> a <sub>1</sub>	wolf <sub>8</sub> NOM <sub>1</sub>		
p <sub>7</sub> i <sub>7</sub> θ <sub>7</sub> o <sub>7</sub> θ <sub>2</sub> a <sub>2</sub>	dog <sub>7</sub> DAT <sub>2</sub>	ŋ <sub>8</sub> e <sub>8</sub> x <sub>8</sub> e <sub>8</sub> θ <sub>2</sub> a <sub>2</sub>	wolf <sub>8</sub> DAT <sub>2</sub>		
p <sub>7</sub> i <sub>7</sub> θ <sub>7</sub> o <sub>7</sub> x <sub>3</sub> a <sub>3</sub>	dog <sub>7</sub> ACC <sub>3</sub>	ŋ <sub>8</sub> e <sub>8</sub> x <sub>8</sub> e <sub>8</sub> x <sub>3</sub> a <sub>3</sub>	wolf <sub>8</sub> ACC <sub>3</sub>		

- For the morpheme glossed as NOM., index 1, we get the allomorph set { [-pa], [-ϕa] }

#### 48. Step 3: Consulting the allomorph list, find the segmental alternations

- We use the standard procedure of string-alignment-by-similarity; already used in calculating SIMILARITY violations ((**Error! Reference source not found.**)).
- For our current example, good alignment yields (a), not (b):

a.	p	a		not:	b.	p	a	∅
	ϕ	a				∅	ϕ	a

- Therefore:

[p] ~ [ϕ] is an attested alternation.  
 [p] ~ ∅, [a] ~ [ϕ], [a] ~ ∅ are not attested alternations.

#### 49. Step 4: Consider general hypotheses about underlying forms and reconstruct the derivations needed

- For present purposes let's all be Albrightianists.
- There is one privileged slot in the paradigm from which the UR of a stem is taken; the child makes this choice early and sticks with it for life.
- See Albright (2002a,b; 2005; 2008a,b; 2012), Jun and Albright (2017), Do (in press) for supporting evidence.
- Albrightianism is fantastic for learnability: There are so few phonologically-distinct paradigm slots that it is quick and easy for an algorithmic learner to try them all.
- Catalan: we try just two hypotheses: word final allomorph and prevocalic (feminine) allomorph.
  - Only the latter will work since it encodes the essential underlying distinctions in stem-final consonants.

#### 50. Step 5: Find a phonological grammar that will do what needs to be done

- Let's do it with OT.

#### 51. We need a GEN

- Hooray, we know all the segmental alternations (Step 3), and we also have our candidate underlying forms.

- No surface allomorph can exist that is not derived from a UR by attested alternations.
- So, if we simply apply every alternation in every possible location, we will have a GEN that is in a sense “complete” and sufficient for learning.<sup>5</sup>

## 52. GEN example: German phonology

- Classical data: stems contrasting in final obstruent voicing, neutralized to voiceless in final position

[rat] ‘wheel-nom.’ [rad-əs] ‘wheel-gen.’  
 [rat] ‘advice-nom.’ [rat-əs] ‘advice-gen.’

- We segment morphemes, find allomorphs, and find one alternation, the [t] ~ [d] of ‘wheel’.
- Activate GEN: using the extracted alternation [t] ~ [d], we replace the [t] of [rat] ‘advice’ with [d], obtaining a new allomorph \*[rad].
  - This allomorph is completely impossible, yet it turns out to be good, because when we use it as a candidate, it tells us how to rank phonological constraints (Pater et al. 2012).

/rat-əs/ ‘advice-gen.’	IDENT(VOICE)	*INTERVOCALIC VOICELESS
☞ rat-əs		*
rad-əs	*	

- This shows that German is a final devoicing language, *not* an intervocalic-voicing language like Korean.

## 53. Constraints

- For now, I’m just being an unreconstructed classical OT person, typing in a rather thorough set of universal constraints (Markedness, Faithfulness).
- I am optimistic that these constraints can ultimately be learned from the data.

## 54. For each choice of UR’s, do this:

- Concatenate the aligned allomorphs (either attested or GENERATED) appropriately to form words, combining them in all possible ways to create a candidate set for each input.
- So far, I’ve found that this fits on a spreadsheet, not more than a few thousand rows...

## 55. The last step

- Add to your spreadsheet the constraints and their violations, and perform OT constraint ranking with any one of the many reliable ranking or weighting algorithms (see (4) above).

<sup>5</sup> This must be adjusted for epenthesis, which must be given a modest context to keep it from applying everywhere, or indeed an infinite number of times.

- Since no variation is present, we can conveniently use the venerable Recursive Constraint Demotion algorithm (Tesar and Smolensky 1993 et seq.).
- If ranking converges, then the UR's you are testing are sufficient.

### 56. Sample from a machine-generated tableau for Catalan

- Choice of Albrightian UR for the stems: either of the Feminine forms (/ \_\_\_ -ə, -es).
- Candidates from GEN:
  - Catalan has [r] ~ [Ø] alternations like [klar-ə] ~ [kla], so GEN freely substitutes Ø for /r/.
  - Catalan has [k] ~ [ɣ] alternations elsewhere in the system, so GEN freely substitutes [ɣ] for /k/.

/klar/	ID(VOICE)	*CODA R	MAX
☞ kla			*
klar		*!	
ɣla	*!		*
ɣlar	*!	*	

### 57. Phonology problems solved so far in this way

- Bizcayan, Catalan, Chamorro, Lamba, Lomongo, Okpe, Polish, also Maori (not KK).
- Not Maltese, nor my concocted pseudo-Yagua (cf. KK 73-74), since both of these have metathesis and my GEN so far is can only concatenate morphemes.

## CONCLUSIONS

### 58. Key point: incrementalism

- We keep the search space size — otherwise quite fatal — under control as we proceed through the Five Steps toward an answer.
  - Key to this was parsing the allomorphs *before* we knew the phonology.
  - This work suggest that this is feasible.

### 59. Future work

- Remove the idealization given above in (8): scale up to include the highly detailed environments, used by human learners.
- With this done, we can take this project out of the cradle:
- Do full-scale empirical work, with wug-testing:
  - Test the predictions of the learned grammars against intuitions of adult native speakers.
  - Test the predictions of grammars learned on child-size dataset on the productions and intuitions of children.

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