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Varieties of Noisy Harmonic Grammar[1]

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BACKGROUND

1. Stochastic constraint-based grammar frameworks in modern linguistics

- These generally originate with Optimality Theory (Prince and Smolensky 1993) and represent an effort to **stochasticize** it:
    - Generate not a single winner but rather a **probability distribution** over GEN.
- The earliest frameworks of this sort were Partial Ordering OT (Anttila 1997) and Stochastic OT (Boersma 1997), but now there are a fair number of other approaches (below).
- The pursuit of such frameworks is motivated by several considerations:

2. Gradience in language is pervasive (Bod et al. 2003)

- Free variation outputs
- Gradient well-formedness intuitions
- Quantitative phonological patterns in the lexicon — often captured quantitatively by language learners and experimentally detectible ("Law of Frequency Matching"; Hayes/Zuraw/Siptár/Londe 2009, with lit. review)

3. Aside: is belief in non-gradience the consequence of traditional methodologies?

- Often, fieldworker and theorist never meet.
- The latter takes the former's best-estimate conjecture as the analytical target.
- Different results are obtained if the theorist is able to work with corpora (text and dictionary) or do experiments.

4. Stochastic grammar: analytic research is paired with learnability research

- I.e. efforts at computational modeling of human language learning.

[1] This talk develops material from a course at UCLA I co-taught with Kie Zuraw, to whom many thanks for help on this project. Thanks also to the members of the UCLA Phonology Seminar for listening to a rehearsal version of this talk and suggesting improvements.

- Typically each stochastic framework has coupled with it a computational procedure for learning its grammars.
- Stochastic frameworks are well adapted for this: isolated exceptions and speech errors don't faze them.

SOME CONSTRAINT-BASED STOCHASTIC GRAMMAR FRAMEWORKS

5. I will focus on a subset of them here

- I focus on the frameworks that performed well in the testing carried out by Zuraw and Hayes.
    - Kie Zuraw and Bruce Hayes (in press, *Lg.*), "Intersecting constraint families: an argument for Harmonic Grammar"
- Ability to predict the result when a single candidate competition responds to two independent constraint families strongly distinguishes the theories.
- The winners are all stochastic forms of …

6. Harmonic Grammar

- Legendre et al. (1990), Legendre et al. (2006), Potts et al. (2010), Pater (2016)
- Uses the same **GEN-cum-EVAL** architecture as Optimality Theory (Prince and Smolensky 1993).
- Constraints are not ranked but have numerical **weights**.
- For each candidate's row in the tableau: multiply all violation counts by corresponding constraint weights and add up the total across constraints = **Harmony**, a kind of penalty.[2]
- Winning candidate is the least penalized one.

/Input/	CONSTRAINT1	CONSTRAINT 2
weights:	2	1
Candidate 1		**
Candidate 2	*	*

/Input/	CONSTRAINT1	CONSTRAINT 2	Harmony
weights:	2	1	
☞ Candidate 1		** × 1 = 2	0 + 2 = **2**
Candidate 2	* × 2 = 2	* × 2 = 2	2 + 2 = **4**

7. Stochasticized versions of Harmonic grammar

- Two main approaches have been taken in "stochasticizing" this framework: NHG, and maxent.

[2] Different scholars deploy minus signs differently, so for some Harmony is a negative quantity. The nomenclature/choice of signs followed here is taken from Wilson (2006).

8. Noisy Harmonic Grammar

- Ref.: Boersma and Pater (2008/2016)
- NHG **stochasticizes** Harmonic Grammar:
  - at each "evaluation time," constraint weights are nudged upward or downward by a random amount drawn from a Gaussian distribution (bell curve) — the **noise**.[3]
  - other than this preliminary step, selection of winner works the same as in non-stochastic Harmonic Grammar

9. Classical Noisy Harmonic Grammar as a procedure, shown as a tableau

- We add in the noise factors at the very start — to the constraint weights.
- Calculating, we get noisy harmonies.

/Input/	CONSTRAINT1	CONSTRAINT 2	Harmony
weight:	$2 + N_1$	$1 + N_2$	
☞ Candidate 1		$2 \times (1 + N_2)$	$2 \times (1 + N_2)$
Candidate 2	$1 \times (2 + N_1)$	$1 \times (1 + N_2)$	$3 + N_1 + N_2$

- By iterating the noise-addition process, output probabilities of Candidate 1 and Candidate 2 can be estimated.

10. Varieties of Noisy Harmonic Grammar

- what I just described will be "Classical NHG"
- I will explore other variants, created by altering the procedure slightly:
  - Cell-specific noise
  - Addition of noise post violation-multiplication.[4]

11. NHG with cell-specific noise (Goldrick and Daland 2009)

- We put in a **fresh noise value for every cell** (cells with no violations can safely be skipped).

/Input/	CONSTRAINT1	CONSTRAINT 2	Harmony
weight:	2	1	
☞ Candidate 1		$2 \times (1 + N_2)$	$2 \times (1 + N_2)$
Candidate 2	$1 \times (2 + N_1)$	$1 \times (1 + N_3)$	$4 + N_1 + N_3$

- The difference is one of *scope*: at what level of calculation is the noise added in?
- We can think of (11) as displaying **cell-granularity** and (9) **constraint-granularity**.

[3] This distribution is given an arbitrary standard deviation, which is the same for all constraints.
[4] There are others; see Boersma and Pater (2016) for the option of letting weights go negative.

12. A completely different (non-NHG) approach to stochasticizing Harmonic Grammar: *maxent grammars*

- Sources: Goldwater and Johnson (2003), Wilson (2006), Jaeger (2007), Hayes and Wilson (2008)
- Again, carries forward the GEN-cum-EVAL architecture of OT.
- Again, carries forward Harmony, as calculated above, as the key basis of the theory.
- Intellectual ancestry: 19th century physics; 1980's cognitive science (Smolensky 1986)
- A simple procedure converts Harmony to output probabilities:

eHarmony:[5]	$e^{-\text{Harmony}(x)}$	(e to the minus Harmony of candidate x)
Z:	$\Sigma_j\, eHarmony(j)$	(sum eHarmony over all candidates)
probability:	$\frac{\text{eHarmony}(x)}{Z}$	(candidate x's share of Z)

- Intuitively:
  - Larger weights will have a greater role in lowering probability of violators.
  - Multiple violations will have more effect in lowering probability of violating candidate.

13. Where we are going

- We now have three stochastic theories in hand (2 NHGs and 1 maxent); more to come.
- Goal: locate cases in which these variant theories **behave differently**, in the hopes of connecting with empirical data that would bear on these differences.

HARMONIC BOUNDING

14. A very simple case of harmonic bounding

/Input/	CONSTRAINT1	CONSTRAINT 2
☞ Candidate 1		*
Candidate 2	*	*

- Candidate 2 has a proper superset of the violations of Candidate 1; i.e. Candidate 1 harmonically bounds Candidate 2.

15. Harmonic bounding in Classical OT

- Taught to beginners everywhere: harmonically bounded candidates never win under any ranking.

[5] This clear and useful term from Colin Wilson is, alas, also a joke (eHarmony is a dating site on the web).

16. Harmonic bounding in Classical Noisy Harmonic Grammar

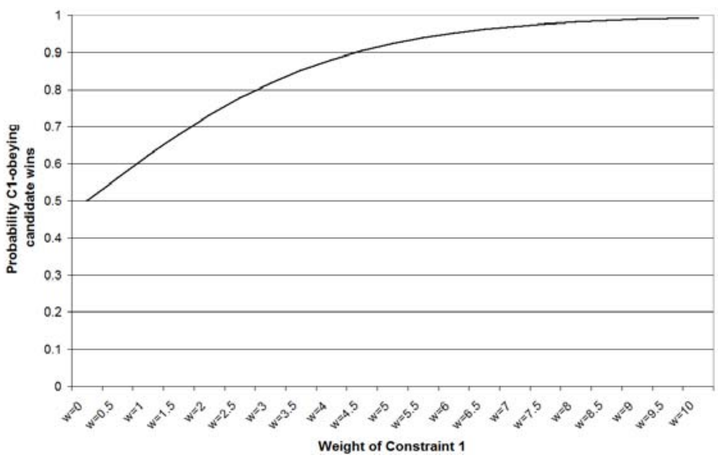
- Likewise, harmonically bounded candidates never win (frequency derived = zero).
- Why? In, e.g., (14) Candidate 2 must have a higher Harmony penalty than Candidate1 *on every evaluation time*, and *under any weighting*, since the only cases where violations don't match are to the disadvantage of Candidate 2.
  - Exception: if weight of Constraint 1 is zero you get a 50/50 tie.

17. Maxent

- Harmonically bounded candidates *can* win in maxent (Jesney 2007)
- Maxent imposes a **stochastic version of Harmonic Bounding**:
  - "A harmonically bounded candidate can never receive a higher probability than the candidate that bounds it."

18. Relating constraint weights to probability of the harmonic-bounder candidate in maxent

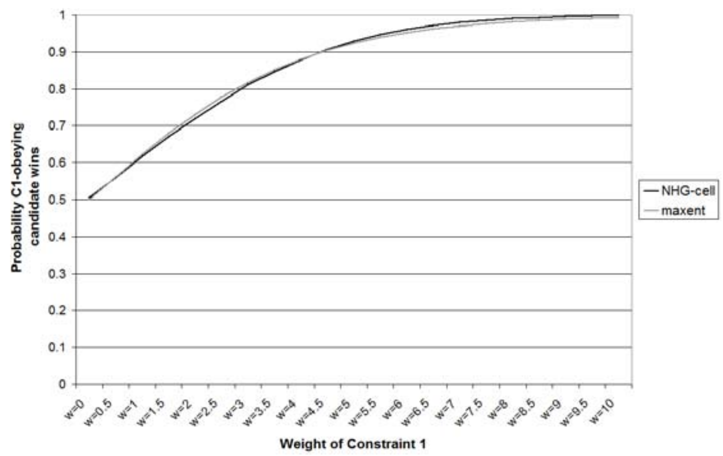
- Easy to work out the math on a spreadsheet.
- For (14), weight of Constraint 2 does not matter at all — it cancels out.
- If the weight of Constraint1 is zero, then candidates are 50/50.
- As the weight of Constraint1 approaches infinity, probability of Candidate1 **asymptotes at 1**.



19. Cell-granularity NHG

- Like maxent: harmonically bounded candidates are not shut out completely (Goldrick and Daland 2009)
- Because noise is computed separately for each candidate, in (14) Candidate 2 can sometimes win. These conditions must be met:
  - By chance, Candidate 1 is penalized on CONSTRAINT 2 more than Candidate 2 is, for this particular evaluation time.
  - And, the difference must exceed the weight of (randomly perturbed) CONSTRAINT 1.

- Working out the math, this too yields a rising curve asymptoting at 1.
- Maxent curve is given in gray for comparison. They are *almost* identical, when scaled appropriately.



- Again, it is not possible for the more-penalized candidate to get a higher probability than its harmonic-bounder.
- So Maxent and Cell-granularity NHG stand out as frameworks that implement the probabilistic, rather than absolute interpretation of harmonic bounding.

A SPECIAL CASE OF HARMONIC BOUNDING: LOCAL OPTIONALITY

20. Premise

- The same phonological process, guided by the same constraints, is applicable in more than one place in the input.

21. Literature

- Vaux (2003, 2008), Riggle and Wilson (2005), Kaplan (2011), Kimper (2011)
- There is a small pile of useful empirical cases to work with: French schwa deletion, Makonde vowel harmony, Bengali intonation phrasing, Pima reduplication ...
- AFAIK none of these has enough quantitative data to test the differences I will demonstrate.

22. I'll use a schematic example: intervocalic voicing of /p/ in long strings

/apapapapa/	IDENT(voice)	*VpV
[apapapapa]		****
[apapapa**b**a] et al.	*	***
[apapa**b**a**b**a] et al.	**	**
[apa**b**a**b**a**b**a] et al.	***	*
[a**b**a**b**a**b**a**b**a]	****	

- We see **double pyramids** of asterisks, since reducing Markedness violates Faithfulness.
- "Double pyramids" are characteristic, since multiple application will generally involve tradeoff of Markedness and Faithfulness violations.

23. Behavior of classical NHG

- Derives *only the extreme candidates*.
- Why? Harmonic bounding, as previous scholars have noted.

24. Multiple-violation patterns: what of the theories with probabilistic harmonic bounding?

- Maxent and Cell-granularity NHG *can* assign probability to the medial candidates.
- But let's take this further: what sort of *probability distributions* to the theories characteristically assign?

25. An abstract example to work with

- We scale up (22) to *six* /p/'s, with underlying /apapapapapapa/.
- Assume 50% probability of intervocalic voicing in all positions, with independent outcome for each locus.[6]

[6] I am aware that a real-life counterpart of this language, Warao (Osborn 1966), is claimed to have obligatory across-the-board-or-none application. On the slimness of the Warao data see Riggle and Wilson (2005).

- Here is a tableau for simulation/study, with the 64 equiprobable candidates.

Input	*Candidate*	*Freq.*	*VpV	IDENT(voice)
/apapapapapapa/	apapapapapapa	1	6	
	apapapapapaba	1	5	1
	apapapapabapa	1	5	1
	apapapabapapa	1	5	1
	apapabapapapa	1	5	1
	apabapapapapa	1	5	1
	abapapapapapa	1	5	1
	apapapapababa	1	4	2
	apapapabapaba	1	4	2
	(48 not listed)	…	…	…
	abababababapaba	1	1	5
	ababababababapa	1	1	5
	abababababababa	1	0	6

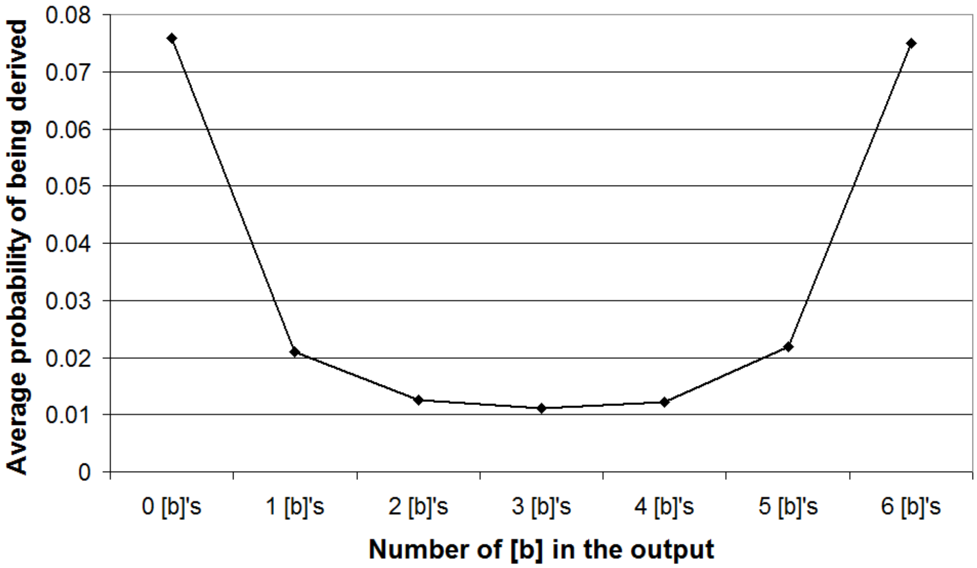
26. Maxent

- Under suitable training conditions,[7] we get the most minimal analysis conceivable.
  - Both constraints are weighted **zero**.
  - Equiprobability is predicted — a perfect fit.
- Indeed, this is why the theory is called maximum entropy!
  - = Minimal commitments in the absence of any sort of pattern in the training data.

27. Cell-granular NHG

- I have implemented Cell-granular NHG on a private copy of my "OTSoft" software,[8] with the Boersma-Pater learning algorithm.
- Curiously, probability is allocated to all candidates, but with *priority to the extremes*, and lowest probabilities to the midpoints.
  - The candidates are sorted here by number of /p/ → [b] changes; i.e. in order of descending Faithfulness.

[7] Specifically, the general penalty for constraint weights (Gaussian prior) often employed for maxent.
[8] www.linguistics.ucla.edu/people/hayes/otsoft/



28. Naming this pattern

- **Upsilonism**: the property of learning a U-shaped probability distribution when trained on flat data.
  - ➢ This is a property of cell-granular NHG (and no other theory I know of)

29. An intuitive explanation of upsilonism in cell-granular NHG

- Think of just the candidate [apapapapapapa].
- Suppose that the roll of the noise-dice is lucky for this candidate: the constraint that penalizes it, *VpV, happens to get a low weight.
- [apapapapapa] benefits *sixfold* from the lowness of this weight.
- [abababababa] likewise benefits sixfold when IDENT(voice) gets a low weight for this cell.
- "Medial" candidates benefit less (fivefold, fourfold, etc.) from their lucky moments — so they have trouble standing out; the more so the closer to the center they lie.

30. Flipping the terms of the deal: maxent deals with U-shaped training data

- Experiment: I trained a maxent grammar
  - ➢ same constraints, candidates, violations
  - ➢ frequencies were the very frequencies output by the upsilonic cell-granular NHG
- Result: flatline! — equal frequencies assigned across the board
- We might call this deviation-from-training-data **rectilinearism**.

31. A pattern that might someday lead to empirical testability

- Multiple-locus phonology
- Opposed constraints of Markedness and Faithfulness

- Training data is *overall* symmetrical between faithful and altered outputs.
- Maxent predicts rectilinearism; cell-granular NHG predicts upsilonism.

STILL MORE VARIETIES OF NHG:
POST-MULTIPLICATIVE VS. PRE-MULTIPLICATIVE NOISE ADDITION

32. The system of noise-addition for Classical NHG (repeated)

/Input/	CONSTRAINT1	CONSTRAINT 2	Harmony
weight:	$3 + N_1$	$1 + N_2$	
☞ Candidate 1		$2 \times (1 + N_2)$	$2 + (2 \times N_2)$
Candidate 2	$1 \times (3 + N_1)$	$1 \times (1 + N_2)$	$4 + N_1 + N_2$

- Notice that N_2 got doubled in the final Harmony computation for Candidate1.

33. Changing scope again: noise addition follows violation-multiplication

/Input/	CONSTRAINT1	CONSTRAINT 2	Harmony
basic constraint weight:	2	1	
select and remember a noise value:	N_1	N_2	
☞ Candidate 1		$\mathbf{(2 \times 1) + N_2}$	$2 + N_2$
Candidate 2	$(\mathit{1} \times 2) + N_1$	$(\mathit{1} \times 1) + N_2$	$4 + N_1 + N_2$

- We pick a noise value for each constraint.
- But add it in only after the violations have been multiplied by weights.
- The harmonies emerge as different: **unmultiplied** weights
  - ➢ Difference here: Harmony(Candidate1) is $2 + N_2$ instead of $2 + 2N_2$

34. Terminology

- Classical NHG has **premultiplicative noise**.
- (33) is **postmultiplicative noise**.

35. A scenario in which a difference can be observed

- Assume binary competitions in which two viable candidates, 1 and 2, compete for each input.
- Assume one constraint that always incurs one violation and penalizes Candidate 1.
- Assume an opposed **scalar** constraint, with a range of integer values, penalizing Candidate 2 depending on "how many" of something it has.

36. McPherson and Hayes's (2016) example: Tommo So vowel harmony

- The single-violation constraint is IDENT(feature) (Faithfulness)

- The scalar constraint is AGREE(feature) — for seven degrees of "morphological closeness"
  - Formalizing the "levels" of classical Lexical Phonology (Kiparsky 1982)
  - Inner levels give more violations of AGREE.

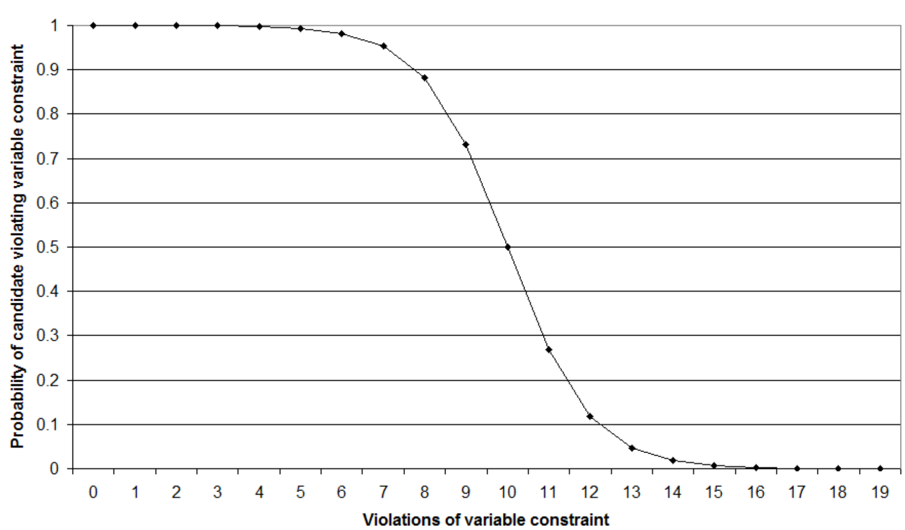
37. Toy example for the comparisons to follow

- We'll do a highly schematic tableau; real-life cases tend to be cut off owing to lack of the full range of input.

		VARIABLE CONSTRAINT	CONSTANT CONSTRAINT
Input 1	Cand1		1
	Cand2	0	
Input 2	Cand1		1
	Cand2	1	
Input 3	Cand1		1
	Cand2	2	
Input 4	Cand1		1
	Cand2	3	
…	…	…	…
Input 20	Cand1		1
	Cand2	19	

38. The essential plot — maxent version

- How does the probability of Candidate 1 go down as we increase the number of violations of VARIABLE CONSTRAINT?
- The following chart is for $\text{weight}_{variable} = 1$, $\text{weight}_{constant} = 10$

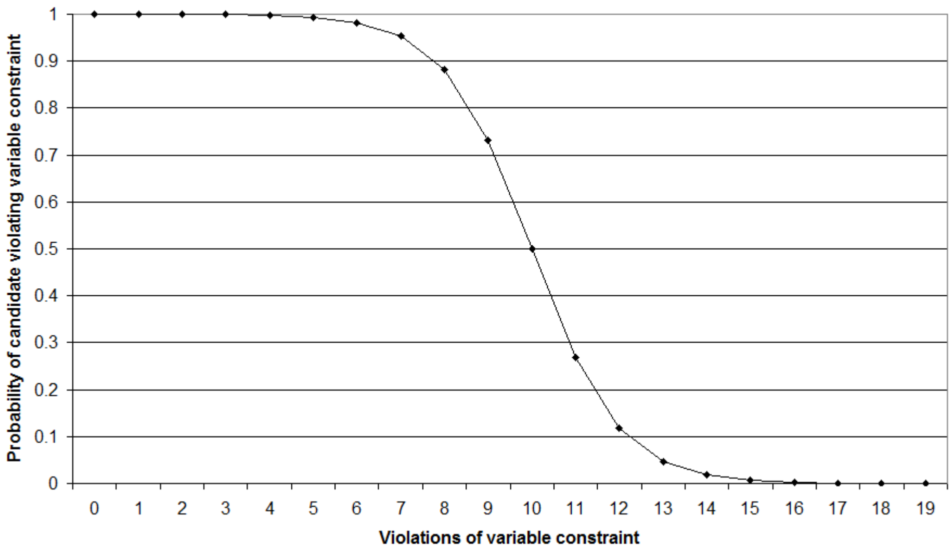


- This is a **sigmoid** curve.
- Mathematically it is a version of the **logistic function**.

- It is **symmetrical** about the point of 50% probability.
- It asymptotes at **one** and **zero**.
- It fits the Tommo So data pretty well; see McPherson/Hayes.

39. Same example, NHG with post-multiplicative noise

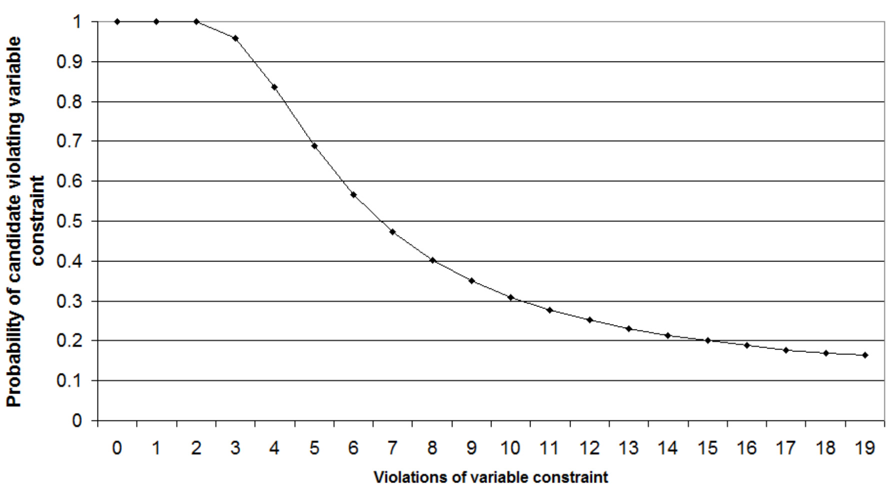
- The following chart is for $\text{weight}_{\text{variable}} = 2$, $\text{weight}_{\text{constant}} = 20$ (twice the maxent weights).



- This, too, is a sigmoid curve.
- Mathematically it is a version of the **cumulative normal distribution** — amazingly similar to the logistic function but of completely different mathematical origin.
- It too is symmetrical about the 50% point and asymptotes at one and zero.
- It also fits the Tommo So data pretty well.

40. Classical NHG (pre-multiplicative noise)

- Weights again 2 for constant constraint and 20 for variable constraint.



- This one is different:
  - Not symmetrical.
  - Asymptotes at one on the left, but **not at zero** on the right.
  - The asymptote at the right turns out to be about 0.07.

41. Why these differences? I: maxent

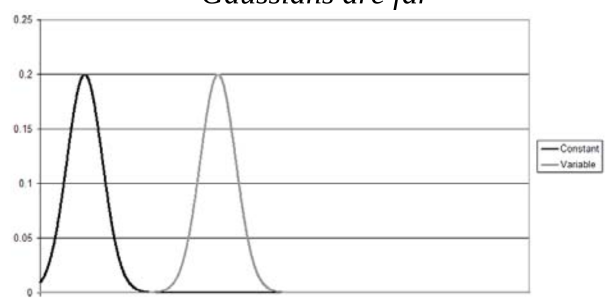
- Maxent: symmetricalness about 50% point and zero/one asymptotes are direct consequencies of the math.
  - Supplemental Materials for McPherson and Hayes (2016) gives all the details.[9]

42. Why these differences? II: post-multiplicative NHG

- Imagine two Gaussians;
  - the one for the variable constraint "slides rightward" as the number of violations of the variable constraint goes up.
  - They keep the same standard deviation, because noise is not multiplied.
  - Probability of a less-likely outcome ("reversed" harmony) will asymptote to zero, as the two Gaussians separate:

Gaussians are close

Gaussians are far



43. Why these differences? III: Classical (pre-multiplicative) NHG

- Again, sliding Gaussians for probability distributions of weights.
- But the rightward-sliding Gaussian also *expands its standard deviation* as it slides.
- So some "left tail" that has some reasonable chance of overlapping with the constant-constraint Gaussian remains in place, so probability eventually settles on an above-zero minimum.[10]

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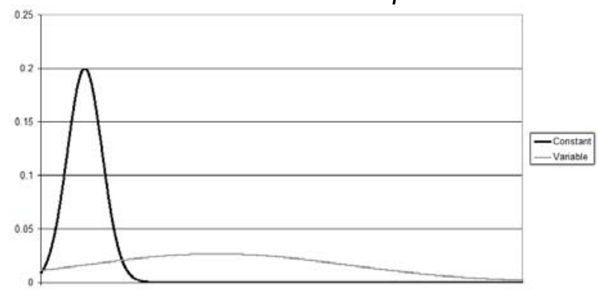
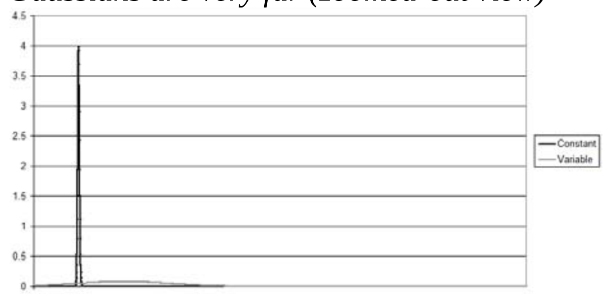
[9] www.linguistics.ucla.edu/people/hayes/papers/SupplementalFilesMcPhersonAndHayes2016.zip
[10] For the math involved here consult http://mathworld.wolfram.com/NormalDifferenceDistribution.html.

Gaussians are close

Gaussians are far



Gaussians are very far (zoomed-out view)



REVISITING UPSILONISM/RECTILINEARISM

44. Our tweaks are freely combinable

- It's perfectly feasible to set the noise afresh in every tableau cell, *and* add it in after weights are multiplied by violations, thus:

/Input/	CONSTRAINT1	CONSTRAINT 2	Harmony
weight:	3	1	
☞ Candidate 1		$(2 \times 1) + N_2$	$2 + N_2$
Candidate 2	$(1 \times 3) + N_1$	$(1 \times 1) + N_3$	$4 + N_1 + N_3$

- This creates yet another variety of Noisy Harmonic Grammar — post-multiplicative-noise, cell-granularity

45. The /apapapapapa/ language in post-multiplicative-noise, cell-granularity NHG

- Result:
  - Moving to post-multiplicative noise *cancels* upsilonism: output distribution is **flat**, just like in maxent.
  - This makes sense: the intuitive explanation for upsilonism in (29) depended on the multiplication of the noise.
- Rectilinearism:
  - Like maxent, post-multiplicative-noise, cell-granularity NHG has a rectilinearity bias: it outputs a flat distribution when trained on an upsilonic one.

46. Accidental twins?

- I have yet to find any qualitative differences between maxent and post-multiplicative noise, cell-granularity NHG; though their math is entirely different.[11]
- As noted, there *are* differences but they rest on the subtle difference between logistic vs. normal-distribution sigmoids — far beyond the resolution of any sort of empirical work being done today.

47. Summary: classifying the differences

Framework	*Effects of harmonic bounding*	*Upsilonism/ rectilinearism/ extremes only*	*Type of sigmoids derived*
Classical NHG	absolute	extremes only	asymmetrical with non-zero asymptote
NHG with cell-specific noise	probabilistic	upsilonism	asymmetrical with non-zero asymptote
NHG with post-multiplicative noise addition	absolute	extremes only	symmetrical with 1/0 asymptotes
NHG with cell-specific noise and post-multiplicative weight addition	probabilistic	rectilinearism	symmetrical with 1/0 asymptotes
Maxent	probabilistic	rectilinearism	symmetrical with 1/0 asymptotes

THE EMPIRICAL SIDE

48. What I have something to say about ..

- I have no data to bear on upsilonism/rectilinearism.
- I will address only the issues of
  - statistical harmonic bounding
  - sigmoid shape

49. Inventory theory

- Linguistics is filled with cases where derivations are of dubious value.
- I.e. we simply want to indicate the legal members of an inventory (possible words, possible sentences, possible metrical lines of verse)
  - Cf. phonology, where the task of unifying the members of a paradigm give us reason to derive surface forms from underlying forms.

[11] I just had time to check Jesney's (2007) case (CCVC can lose its coda or simplify its onset independently). Again they behave alike.

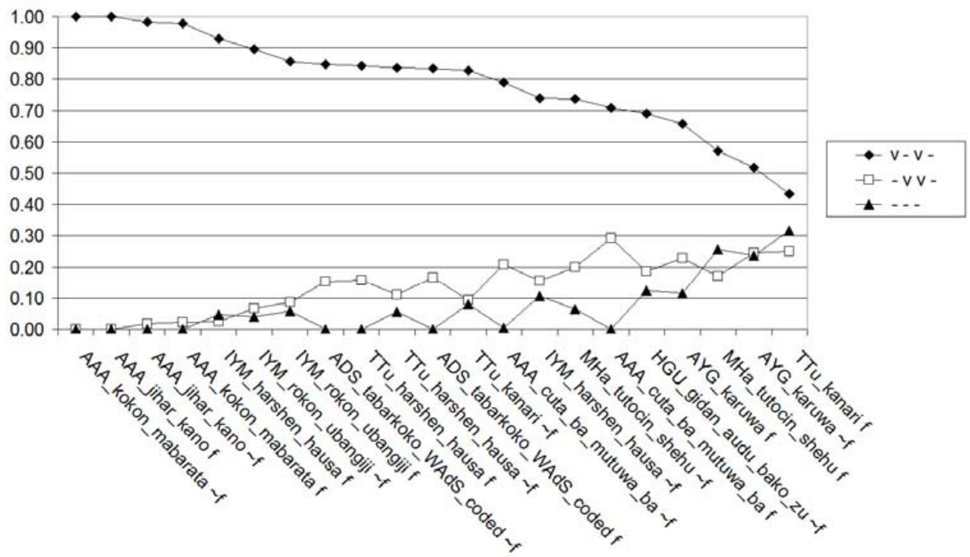
- The Rich Base concept does permit “pseudo-derivations” to underlie inventories, with a literature addressing this (Keer and Baković 1997, Hayes 2004, Prince and Tesar 2014, etc.)
- An alternative is simply to develop a Faithfulness-free stochastic grammar that assigns a probability to every member of GEN.
- Such a grammar would assign, I suspect, a lower probability to [pra] than [pa];
    - [pra] is harmonically bounded, and indeed should by any criterion receive a lower probability score than [pa].
    - Pseudo-derivational theories face the awkward fact that [pra] is more marked than [pa] *but not repaired* (other than in rare speech errors) to [pa].
- References on “evaluate-all-of-GEN” grammars: Hayes and Wilson (2008) on phonotactics; Hayes and Moore-Cantwell (2011) on metrics of Hopkins; Hayes, Wilson and Shisko (2012) on metrics of Shakespeare/Milton
- All such work must assign some probability to harmonically bounded candidates, and thus works only with some of the frameworks above.

50. An apparent case of statistical harmonic bounding

- Hayes and Schuh (in progress) evaluate three of the principal “metra” of the Hausa *rajaz* quantitative meter.

	STRONG POSITIONS MUST INITIATE HEAVY SYLLABLE	HEAVY SYLLABLE MUST INITIATE STRONG POSITION
x x x x x x x x ˘ — ˘ —		
x x x x x x x x — ˘ ˘ —	*	*
x x x x x x x x — — —	*	**

- A near-perfect pattern: no matter what the actual frequencies, the relative frequencies respect harmonic bounding relations for various poets/poems/stanza positions



- Why? because ◡ – ◡ – bounds – ◡ ◡ – and – ◡ ◡ – bounds – – –; and in the frameworks adopted here, subsets of violation implies *invariant differences of probability*; rather than outright absence of the candidate with superset violations.

51. The Goldrick/Daland theory of speech errors

- If Goldrick/Daland are right that a "misweighted" grammar occasionally outputs harmonically-bounded candidates.

52. Multiple locus cases

- Already discussed.
- To be sure, there are other remedies available. (Riggle-Wilson; Kaplan; Kimper)
- I think there is something appealing about getting multiple-locus cases — including, ideally, their fine-grained statistical detail — from the fundamental architecture of the theory (how probability distributions are computed)

CHARACTERISTIC SHAPE OF SIGMOIDS IN GRAMMAR

53. McPherson and Hayes's (2016) result

- We tried to fit the Tommo So data using classical NHG (asymmetrical sigmoids) and got a somewhat inferior fit to the data.
- Reason: our empirical sigmoids were quite symmetrical.

54. The world in general

- Zuraw and Hayes (in press) looked at a fair number of empirical sigmoids in phonology

- We repeatedly found symmetrical sigmoid curves asymptoting — if the empirically observed range permitted it — at zero and one.
- See Zuraw and Hayes (in press) for a brief survey of such sigmoids elsewhere in language.
  - speech perception
  - syntactic change (work of Kroch and colleagues)[12]
  - possibly even cognition in general

CONCLUSION

55. The research agenda

- Earlier research on stochastic constraint theories emphasized:
  - simple feasibility (ability to fit particular data sets; e.g. Anttila 1997; Boersma and Hayes 2001)
  - effectiveness of the associated learnability theory (e.g. Pater 2008)

- But it's also possible to treat such theories as **abstract characterizations of possible grammars**, which permit/favor only particular data patterns.
  - Zuraw and Hayes (in press) — only some theories capture the parallel sigmoid curves that characterize "intersecting constraint families"
  - Here: empirical picture less clear, but the theories are quite different in their predictions re. harmonic bounding, upsilonism/rectilinearism, sigmoid shape

- Empirical work can now keep its eyes open for data bearing on relatively abstract principles of grammatical organization.

[12] Indeed, I suspect that serious study of synchronic syntactic patterns from a stochastic constraint-based viewpoint would also reveal abundant sigmoidal phenomena.

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