Sharpening the empirical claims of generative syntax through formalization

Tim Hunter

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ESSLLI, August 2015
Part 1: Grammars and cognitive hypotheses
   What is a grammar?
   What can grammars do?
   Concrete illustration of a target: Surprisal

Parts 2–4: Assembling the pieces
   Minimalist Grammars (MGs)
   MGs and MCFGs
   Probabilities on MGs

Part 5: Learning and wrap-up
   Something slightly different: Learning model
   Recap and open questions
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Part 1

Grammars and Cognitive Hypotheses
Outline

1. What we want to do with grammars
2. How to get grammars to do it
3. Derivations and representations
4. Information-theoretic complexity metrics
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2. How to get grammars to do it
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4. Information-theoretic complexity metrics
Claims made by grammars

What are grammars used for?

- “Mostly” for accounting for acceptability judgements
- But there are other ways a grammar can figure in claims about cognition
What are grammars used for?

- “Mostly” for accounting for acceptability judgements
- But there are other ways a grammar can figure in claims about cognition

Often tempting to draw a distinction between “linguistic evidence” (where grammar lives) and “experimental evidence” (where cognition lives)

- One need not make this distinction
- We will proceed without it, i.e. it’s all linguistic (and/or all experimental)
Claims made by grammars

There’s a “boring” sense in which every syntax paper makes a cognitive claim, i.e. a claim testable via acceptability facts.
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  - Lingering externalism/Platonism?
  - Perhaps partly because it’s just relatively rare to see anything being tested by other measures
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- For another, we can incorporate grammars into claims that are testable by other measures.
  - This is the main point of the course!
  - The claims/predictions will depend on internal properties of grammars, not just what they say is good and what they say is bad
  - And we’ll do it without seeing grammatical derivations as real-time operations
Claims made by grammars

*If we accept — as I do — ... that the rules of grammar enter into the processing mechanisms, then evidence concerning production, recognition, recall, and language use in general can be expected (in principle) to have bearing on the investigation of rules of grammar, on what is sometimes called “grammatical competence” or “knowledge of language”.*

(Chomsky 1980: pp.200-201)

*[S]ince a competence theory must be incorporated in a performance model, evidence about the actual organization of behavior may prove crucial to advancing the theory of underlying competence.*

(Chomsky 1980: p.226)
If we accept — as I do — . . . that the rules of grammar enter into the processing mechanisms, then evidence concerning production, recognition, recall, and language use in general can be expected (in principle) to have bearing on the investigation of rules of grammar, on what is sometimes called “grammatical competence” or “knowledge of language”.  

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Evidence about X can only advance Y if Y makes claims about X!
What we will do:

- Put together a chain of linking hypotheses that bring “experimental evidence” to bear on “grammar questions”
  - e.g. reading times, acquisition patterns
  - e.g. move as distinct operation from merge vs. unified with merge

- Illustrate with some toy examples
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  - e.g. move as distinct operation from merge vs. unified with merge
- Illustrate with some toy examples

What we will not do:

- Engage with state-of-the-art findings in the sentence processing literature
- End up with claims that one particular set of derivational operations is empirically better than another
We’ll take pairs of equivalent grammars that differ only in the move/re-merge dimension.

- They will make different predictions about sentence comprehension difficulty.
- They will make different predictions about what a learner will conclude from a common input corpus.
Teasers

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- They will make different predictions about sentence comprehension difficulty.
- They will make different predictions about what a learner will conclude from a common input corpus.

The issues become “distant but empirical questions”. That’s all we’re aiming for, for now.
Outline

1. What we want to do with grammars
2. How to get grammars to do it
3. Derivations and representations
4. Information-theoretic complexity metrics
Interpretation functions

- John loves Mary
- Everyone loves John
- Someone loves everyone

Caveats:
- Maybe we're interested in the finite specification of the set
- Maybe there's no clear line between observable and not
- Maybe some evidence is based on relativities among interpretations
Interpretation functions

\[
\begin{align*}
S & \\
NP & \text{John} \\
VP & V \\
      & \text{loves} \\
      & \text{Mary}
\end{align*}
\]

\[
\begin{align*}
S & \\
NP & \text{John} \\
VP & V \\
      & \text{loves} \\
      & \text{everyone}
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\]

\[
\begin{align*}
S & \\
      & \cdots \\
      & \cdots \\
      & \cdots \\
      & \cdots \\
\end{align*}
\]

\[
\begin{align*}
L(m)(j) & \\
\text{John loves Mary}
\end{align*}
\]

\[
\begin{align*}
\forall x L(x)(j) & \\
\text{John loves everyone}
\end{align*}
\]

\[
\begin{align*}
\exists y \forall x L(x)(y) & \\
\text{someone loves everyone}
\end{align*}
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$$\forall x \ L(x)(j)$$

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Telling grammars apart

So, what if we have two different grammars — systems that define different sets of objects — that we can’t tell apart via the sound and meaning interpretations?

(Perhaps because they’re provably equivalent, or perhaps because the evidence just happens to be unavailable.)
Telling grammars apart

So, what if we have two different grammars — systems that define different sets of objects — that we can’t tell apart via the sound and meaning interpretations?

(Perhaps because they’re provably equivalent, or perhaps because the evidence just happens to be unavailable.)

- Option 1: Conclude that the differences are irrelevant to us (or “they’re not actually different”).
- Option 2: Make the differences matter . . . somehow . . .
Morrill (1994) in favour of Option 1:

*The construal of a language as a collection of signs [sound-meaning pairs] presents as an investigative task the characterisation of this collection. This is usually taken to mean the specification of a set of “structural descriptions” (or: “syntactic structures”). Observe however that on our understanding a sign is an association of prosodic [phonological] and semantic properties. It is these properties that can be observed and that are to be modelled. There appears to be no observation which bears directly on syntactic as opposed to prosodic and/or semantic properties, and this implies an asymmetry in the status of these levels. A structural description is only significant insofar as it is understood as predicting prosodic and semantic properties (e.g. in interpreting the yield of a tree as word order). Attribution of syntactic (or prosodic or semantic) structure does not of itself predict anything.*
What are syntactic representations for?

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Where might we depart from this (to pursue Option 2)?

- Object that syntactic structure **does** matter “of itself”
- Object that prosodic and semantic properties are **not** the only ones we can observe
Interpretation functions

\[
\begin{align*}
S & \rightarrow \text{NP} \quad \text{VP} \\
\text{NP} & \rightarrow \text{John} \\
\text{VP} & \rightarrow \text{V} \quad \text{NP} \\
\text{V} & \rightarrow \text{loves} \\
\text{NP} & \rightarrow \text{Mary} \\
\end{align*}
\]

\[
\begin{align*}
S & \rightarrow \text{NP} \quad \text{VP} \\
\text{NP} & \rightarrow \text{everyone} \\
\text{VP} & \rightarrow \text{V} \quad \text{NP} \\
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Interpretation functions for “complexity”

What are some other interpretation functions?

- **number of nodes**
Interpretation functions for “complexity”

What are some other interpretation functions?

- number of nodes
- ratio of total nodes to terminal nodes (Miller and Chomsky 1963)
Ratio of total nodes to terminal nodes

Fig. 8. Illustrating a measure of structural complexity. $N(Q)$ for the $P$-marker (a) is $7/4$; for (b), $N(Q) = 5/4$.
Ratio of total nodes to terminal nodes

Fig. 8. Illustrating a measure of structural complexity. $N(Q)$ for the P-marker (a) is 7/4; for (b), $N(Q) = 5/4$.

Won’t distinguish center-embedding from left- and right-embedding

1. The mouse [the cat [the dog bit] chased] died.  
(center)

2. The dog bit the cat [which chased the mouse [which died]].  
(right)

3. [[the dog] ’s owner] ’s friend  
(left)

(Miller and Chomsky 1963)
Interpretation functions for “complexity”

What are some other interpretation functions?

- number of nodes
- ratio of total nodes to terminal nodes (Miller and Chomsky 1963)
- degree of self-embedding (Miller and Chomsky 1963)
Degree of (centre-)self-embedding

A tree's degree of self-embedding is $m$ iff:
“there is ... a continuous path passing through $m + 1$ nodes $N_0, \ldots, N_m$, each with the same label, where each $N_i \ (i \geq 1)$ is fully self-embedded (with something to the left and something to the right) in the subtree dominated by $N_{i-1}$”

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- number of nodes
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- degree of self-embedding (Miller and Chomsky 1963)
- “depth” of memory required by a top-down parser (Yngve 1960)
Yngve’s depth

Number of constituents expected but not yet started:

```
S
  NP
    Det the
    N mouse
  VP
    V died
 NP
    Det the
    N cat
  S
    V chased
 NP
    Det the
    N dog
  VP
    V bit
```
Yngve’s depth

Number of constituents expected but not yet started:

```
S
  |NP
  |  |VP
  |  |S
  |  |VP
  |NP
  |Det the N
  |  |mouse
  |  |S
  |  |VP
  |NP
  |Det the N
  |  |cat
  |  |S
  |  |VP
  |NP
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  |  |dog
```

Unlike (center-)self-embedding, right-embedding doesn’t create such large lists of expected constituents (because the expected stuff is all part of one constituent). But left-embedding does.

Yngve’s theory was set within — perhaps justified by — a procedural story, but we can arguably detach it from that and treat depth as just another property of trees. (Yngve 1960)
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- minimal attachment, late closure, etc.? (Frazier and Clifton 1996)
Typically, arguments hold the grammar fixed and present evidence in favour of a metric.

\[
\text{complexity metric} \quad + \quad \text{grammar} \quad \rightarrow \quad \text{prediction}
\]
Reaching conclusions about grammars

complexity metric $+$ grammar $\rightarrow$ prediction

Typically, arguments hold the grammar fixed and present evidence in favour of a metric.

We can flip this around: hold the metric fixed and present evidence in favour of a grammar.

*If we accept — as I do — … that the rules of grammar enter into the processing mechanisms, then evidence concerning production, recognition, recall, and language use in general can be expected (in principle) to have bearing on the investigation of rules of grammar, on what is sometimes called “grammatical competence” or “knowledge of language”.*

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complexity metric + grammar → prediction

Example: hold self-embedding fixed as the complexity metric.
Reaching conclusions about grammars

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Example: hold self-embedding fixed as the complexity metric.

(4) That [the food that [John ordered] tasted good] pleased him.

(5) That [that [the food was good] pleased John] surprised Mary.

Grammar question: Does a relative clause have a node labeled S?
## Reaching conclusions about grammars

The general form:

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\text{complexity metric} + \text{grammar} \rightarrow \text{prediction}
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<th>(5) structure</th>
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<td>Yes</td>
<td>( \ldots [s \ldots [s \ldots ] \ldots ] )</td>
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Reaching conclusions about grammars

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Conclusion: The fact that (5) is harder supports the “No” answer.
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Derivations and representations

Question

But these metrics are all properties of a final, fully-constructed tree. How can anything like this be sensitive to differences in the derivational operations that build these trees? (e.g. TAG vs. MG, whether move is re-merge)
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- number of nodes
- ratio of total nodes to terminal nodes (Miller and Chomsky 1963)
- degree of self-embedding (Miller and Chomsky 1963)
- “depth” of memory required by a top-down parser (Yngve 1960)
- minimal attachment, late closure, etc.? (Frazier and Clifton 1996)
- “nature, number and complexity of” transformations (Miller and Chomsky 1963)
“nature, number and complexity of the grammatical transformations involved”

The psychological plausibility of a transformational model of the language user would be strengthened, of course, if it could be shown that our performance on tasks requiring an appreciation of the structure of transformed sentences is some function of the nature, number and complexity of the grammatical transformations involved.

(Miller and Chomsky 1963: p.481)
Derivations and representations

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The relevant objects on which the interpretation functions are defined encode a complete **derivational history**.
Derivations and representations

**Question**

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**Answer**

The relevant objects on which the interpretation functions are defined encode a complete derivational history.

e.g. The function which, given a complete “recipe” for carrying out a derivation, returns the number of movement steps called for by the recipe.
Full derivation recipes?

Are the inputs to these functions really full derivation recipes?

For minimalist syntax it’s hard to tell, because the final derived object very often uniquely identifies a derivational history/recipe.
Are the inputs to these functions **really** full derivation recipes?

For minimalist syntax it’s hard to tell, because the final derived object very often uniquely identifies a derivational history/recipe.

- merge Y with RP
- merge the result with ZP
- merge the result with WP
- merge X with the result
- move ZP
Full derivation recipes?

A few cases reveal that (we must all be already assuming that) it’s full derivations/recipes that count.
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(6) *Which claim [that Mary$_i$ was a thief] did she$_i$ deny?

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---

What we want to do with grammars

How to get grammars to do it

Derivations and representations

Information-theoretic complexity metrics
Full derivation recipes?

Also:

- subjacency effects without traces
- compare categorial grammar
Full derivation recipes?

Also:

- subjacency effects without traces
- compare categorial grammar

And this is not a new idea!

[The perceptual model] will utilize the full resources of the transformational grammar to provide a structural description, consisting of a set of P-markers and a transformational history

Miller and Chomsky (1963: p.480)
Full derivation recipes?

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Miller and Chomsky (1963: p.480)
Full derivation recipes?

(4) the man who persuaded John to be examined by a specialist was fired.

The "transformational history" of (4) by which it is derived from its basis might be represented, informally, by the diagram (5).

(5)  

(1)  

$T_E - T_R - T_P - T_{AD}$

(2)  

$T_E - T_D - T_{io}$

(3)  

$- T_P$
Full derivation recipes?

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(5) (1) $T_E - T_R - T_P - T_{AD}$

(2) $T_E - T_D - T_{to}$

(3) $- T_P$

Differences these days:

- We’ll have things like **merge** and **move** at the internal nodes instead of $T_P$, $T_E$, etc.
- We’ll have lexical items at the leaves rather than base-derived trees.

(Chomsky 1965)
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Surprisal and entropy reduction

Why these complexity metrics?

- Partly just for concreteness, to give us a goal.
- They are formalism neutral to a degree that others aren’t.
- They are mechanism neutral (Marr level one).
- The pieces of the puzzle that we need to get there (e.g. probabilities) seem likely to be usable in other ways.
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John Hale, Cornell Univ.
What we want to do with grammars
How to get grammars to do it
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\begin{align*}
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& \downarrow \text{SEM} \\
& \downarrow \text{PHON} \\
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VP &\rightarrow \text{loves} \\
NP &\rightarrow \text{Mary} \\
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\[
\begin{align*}
L(m)(j) \\
& \downarrow \text{5} \\
\forall x \ L(x)(j) \\
& \downarrow \text{5} \\
\text{John loves everyone} \\
\end{align*}
\]

\[
\begin{align*}
\exists y \forall x \ L(x)(y) \\
& \downarrow \text{7} \\
\text{someone loves everyone} \\
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Surprisal

Given a sentence $w_1 w_2 \ldots w_n$:

$$\text{surprisal at } w_i = -\log P(W_i = w_i | W_1 = w_1, W_2 = w_2, \ldots, W_{i-1} = w_{i-1})$$
### Surprisal

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<td>John ran</td>
</tr>
<tr>
<td>0.15</td>
<td>John saw it</td>
</tr>
<tr>
<td>0.05</td>
<td>John saw them</td>
</tr>
<tr>
<td>0.25</td>
<td>Mary ran</td>
</tr>
<tr>
<td>0.1</td>
<td>Mary saw it</td>
</tr>
<tr>
<td>0.05</td>
<td>Mary saw them</td>
</tr>
</tbody>
</table>

What predictions can we make about the difficulty of comprehending ‘John saw it’?
Surprisal

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</table>

What predictions can we make about the difficulty of comprehending ‘John saw it’?

surprisal at ‘John’ = − log P(W₁ = John)
= − log(0.4 + 0.15 + 0.05)
= − log 0.6
= 0.74
Surprisal

What predictions can we make about the difficulty of comprehending ‘John saw it’?

surprisal at ‘John’ = − log \( P(W_1 = \text{John}) \)
= − log(0.4 + 0.15 + 0.05)
= − log 0.6
= 0.74

surprisal at ‘saw’ = − log \( P(W_2 = \text{saw} \mid W_1 = \text{John}) \)
= − log \( \frac{0.15 + 0.05}{0.4 + 0.15 + 0.05} \)
= − log 0.33
= 1.58
Surprisal

What we want to do with grammars  How to get grammars to do it  Derivations and representations  Information-theoretic complexity metrics

0.4  John ran
0.15 John saw it
0.05 John saw them
0.25 Mary ran
0.1 Mary saw it
0.05 Mary saw them

What predictions can we make about the difficulty of comprehending ‘John saw it’?

surprisal at ‘John’ = \( -\log P(W_1 = \text{John}) \)
\[ = -\log(0.4 + 0.15 + 0.05) \]
\[ = -\log 0.6 \]
\[ = 0.74 \]

surprisal at ‘saw’ = \( -\log P(W_2 = \text{saw} \mid W_1 = \text{John}) \)
\[ = -\log \frac{0.15 + 0.05}{0.4 + 0.15 + 0.05} \]
\[ = -\log 0.33 \]
\[ = 1.58 \]

surprisal at ‘it’ = \( -\log P(W_3 = \text{it} \mid W_1 = \text{John}, W_2 = \text{saw}) \)
\[ = -\log \frac{0.15}{0.15 + 0.05} \]
\[ = -\log 0.75 \]
\[ = 0.42 \]
Accurate predictions made by surprisal

(Hale 2001)
Accurate predictions made by surprisal

(8) The reporter [who ____ attacked the senator] left the room. (easier)
(9) The reporter [who the senator attacked ____] left the room. (harder)

(Levy 2008)
An important distinction

Using surprisal as a complexity metric says nothing about the form of the knowledge that the language comprehender is using!

- We’re asking “what’s the probability of \( w_i \), given that we’ve seen \( w_1 \ldots w_{i-1} \) in the past”.
- This does not mean that the comprehender’s knowledge takes the form of answers to this kind of question.
- The linear nature of the metric reflects the task, not the knowledge being probed.
Probabilistic CFGs

1.0  \( S \to NP \text{ VP} \)
0.3  \( NP \to John \)
0.7  \( NP \to Mary \)
0.2  \( VP \to ran \)
0.5  \( VP \to V \text{ NP} \)
0.3  \( VP \to V \text{ S} \)
0.4  \( V \to believed \)
0.6  \( V \to knew \)

\[
P(\text{Mary believed John ran}) = 1.0 \times 0.7 \times 0.3 \times 1.0 \times 0.3 \times 0.2 = 0.00504
\]
Probabilistic CFGs

1.0 $S \rightarrow \text{NP VP}$
0.3 $\text{NP} \rightarrow \text{John}$
0.7 $\text{NP} \rightarrow \text{Mary}$
0.2 $\text{VP} \rightarrow \text{ran}$
0.5 $\text{VP} \rightarrow \text{V NP}$
0.3 $\text{VP} \rightarrow \text{V S}$
0.4 $\text{V} \rightarrow \text{believed}$
0.6 $\text{V} \rightarrow \text{knew}$

$$P(\text{Mary believed John ran}) = 1.0 \times 0.7 \times 0.3 \times 0.4 \times 1.0 \times 0.3 \times 0.2$$

$$= 0.00504$$
Surprisal with probabilistic CFGs

**Goal:** Calculate step-by-step surprisal values for ‘Mary believed John ran’

surprisal at ‘John’ = $-\log P(W_3 = \text{John} \mid W_1 = \text{Mary}, W_2 = \text{believed})$
Surprisal with probabilistic CFGs

**Goal:** Calculate step-by-step surprisal values for ‘Mary believed John ran’

surprisal at ‘John’ = \(-\log P(W_3 = \text{John} \mid W_1 = \text{Mary}, W_2 = \text{believed})\)

<table>
<thead>
<tr>
<th>surprisal</th>
<th>phrase</th>
<th>surprisality</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.098</td>
<td>Mary believed Mary</td>
<td></td>
</tr>
<tr>
<td>0.042</td>
<td>Mary believed John</td>
<td></td>
</tr>
<tr>
<td>0.012348</td>
<td>Mary believed Mary knew Mary</td>
<td></td>
</tr>
<tr>
<td>0.01176</td>
<td>Mary believed Mary ran</td>
<td></td>
</tr>
<tr>
<td>0.008232</td>
<td>Mary believed Mary believed Mary</td>
<td></td>
</tr>
<tr>
<td>0.005292</td>
<td>Mary believed Mary knew John</td>
<td></td>
</tr>
<tr>
<td>0.005292</td>
<td>Mary believed John knew Mary</td>
<td></td>
</tr>
<tr>
<td>0.00504</td>
<td>Mary believed John ran</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

There are an infinite number of derivations consistent with input at each point!
Surprisal with probabilistic CFGs

Goal: Calculate step-by-step surprisal values for ‘Mary believed John ran’

surprisal at ‘John’ = − log P(W_3 = John | W_1 = Mary, W_2 = believed)

<table>
<thead>
<tr>
<th>surprisal</th>
<th>string</th>
</tr>
</thead>
<tbody>
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<td>0.098</td>
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</table>

There are an infinite number of derivations consistent with input at each point!

surprisal at ‘John’ = − log P(W_3 = John | W_1 = Mary, W_2 = believed)

= − log \( \frac{0.042 + 0.005292 + 0.00504 + \ldots}{0.098 + 0.042 + 0.12348 + 0.01176 + 0.008232 + \ldots} \)
Intersection grammars

1.0  \( S \to NP\ VP \)
0.3  \( NP \to John \)
0.7  \( NP \to Mary \)
0.2  \( VP \to ran \)
0.5  \( VP \to V\ NP \)
0.3  \( VP \to V\ S \)
0.4  \( V \to believed \)
0.6  \( V \to knew \)

\[
\begin{array}{c}
0 \quad \text{Mary} \\
1 \quad \text{believed} \\
2 \quad * \\
\end{array}
\]

\( \cap \quad = \quad G_{2} \)
Intersection grammars

\[ 1.0 \quad S \rightarrow \text{NP VP} \]
\[ 0.3 \quad \text{NP} \rightarrow \text{John} \]
\[ 0.7 \quad \text{NP} \rightarrow \text{Mary} \]
\[ 0.2 \quad \text{VP} \rightarrow \text{ran} \]
\[ 0.5 \quad \text{VP} \rightarrow \text{V NP} \]
\[ 0.3 \quad \text{VP} \rightarrow \text{V S} \]
\[ 0.4 \quad \text{V} \rightarrow \text{believed} \]
\[ 0.6 \quad \text{V} \rightarrow \text{knew} \]

\[ \cap \]

\[ 0.3 \quad \text{NP} \rightarrow \text{John} \]
\[ 0.7 \quad \text{NP} \rightarrow \text{Mary} \]
\[ 0.2 \quad \text{VP} \rightarrow \text{ran} \]
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\[ 0.6 \quad \text{V} \rightarrow \text{knew} \]

\[ G_2 \]

\[ \cap \]

\[ 0.3 \quad \text{NP} \rightarrow \text{John} \]
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\[ 0.5 \quad \text{VP} \rightarrow \text{V NP} \]
\[ 0.3 \quad \text{VP} \rightarrow \text{V S} \]
\[ 0.4 \quad \text{V} \rightarrow \text{believed} \]
\[ 0.6 \quad \text{V} \rightarrow \text{knew} \]

\[ G_3 \]
Intersection grammars

\[
\begin{align*}
\text{surprisal at 'John'} &= - \log P(W_3 = \text{John} \mid W_1 = \text{Mary}, W_2 = \text{believed}) \\
&= - \log \frac{\text{total weight in } G_3}{\text{total weight in } G_2} \\
&= - \log \frac{0.0672}{0.224} \\
&= 1.74
\end{align*}
\]
Grammar intersection example (simple)

<table>
<thead>
<tr>
<th>Rule</th>
<th>Weight</th>
<th>Non-terminal</th>
<th>Non-terminal</th>
</tr>
</thead>
<tbody>
<tr>
<td>S → NP VP</td>
<td>1.0</td>
<td>S</td>
<td>NP VP</td>
</tr>
<tr>
<td>NP → John</td>
<td>0.3</td>
<td>NP</td>
<td>John</td>
</tr>
<tr>
<td>NP → Mary</td>
<td>0.7</td>
<td>NP</td>
<td>Mary</td>
</tr>
<tr>
<td>VP → ran</td>
<td>0.2</td>
<td>VP</td>
<td>ran</td>
</tr>
<tr>
<td>VP → V NP</td>
<td>0.5</td>
<td>VP</td>
<td>V NP</td>
</tr>
<tr>
<td>VP → V S</td>
<td>0.3</td>
<td>VP</td>
<td>V S</td>
</tr>
<tr>
<td>V → believed</td>
<td>0.4</td>
<td>V</td>
<td>believed</td>
</tr>
<tr>
<td>V → knew</td>
<td>0.6</td>
<td>V</td>
<td>knew</td>
</tr>
</tbody>
</table>
Grammar intersection example (simple)

1.0  \( S \rightarrow NP \ VP \)
0.3  \( NP \rightarrow John \)
0.7  \( NP \rightarrow Mary \)
0.2  \( VP \rightarrow ran \)
0.5  \( VP \rightarrow V \ NP \)
0.3  \( VP \rightarrow V \ S \)
0.4  \( V \rightarrow believed \)
0.6  \( V \rightarrow knew \)

NB: Total weight in this grammar is not one! (What is it? Start symbol is \( S_{0,2} \).
Each derivation has the weight “it” had in the original grammar.
Grammar intersection example (more complicated)

S → NP VP
VP → V NP
NP → DET
NP → DET N
NP → ADJ N
V → fish
V → damaged
DET → these
N → fish
ADJ → damaged

These fish damaged ...
Grammar intersection example (more complicated)

\[
\begin{align*}
S & \rightarrow NP \ VP \\
NP & \rightarrow DET \ NP \\
VP & \rightarrow V \ NP \\
V & \rightarrow fish \\
NP & \rightarrow DET \ N \\
DET & \rightarrow these \\
N & \rightarrow fish \\
NP & \rightarrow ADJ \ N \\
ADJ & \rightarrow damaged \\
\end{align*}
\]

\[
\begin{align*}
S_{0,3} & \rightarrow NP_{0,2} \ VP_{2,3} \\
NP_{0,2} & \rightarrow DET_{0,1} \ N_{1,2} \\
VP_{2,3} & \rightarrow V_{2,3} \ NP_{3,3} \\
DET_{0,1} & \rightarrow these \\
N_{1,2} & \rightarrow fish \\
V_{2,3} & \rightarrow damaged \\
\end{align*}
\]

\[
\begin{align*}
S_{0,3} & \rightarrow NP_{0,1} \ VP_{1,3} \\
NP_{0,1} & \rightarrow DET_{0,1} \\
VP_{1,3} & \rightarrow V_{1,2} \ NP_{2,3} \\
NP_{2,3} & \rightarrow ADJ_{2,3} \ N_{3,3} \\
V_{1,2} & \rightarrow fish \\
ADJ_{2,3} & \rightarrow damaged \\
\end{align*}
\]

\[
\begin{align*}
NP_{3,3} & \rightarrow ADJ_{3,3} \ N_{3,3} \\
NP_{3,3} & \rightarrow DET_{3,3} \ N_{3,3} \\
NP_{3,3} & \rightarrow DET_{3,3} \\
N_{3,3} & \rightarrow fish \\
DET_{3,3} & \rightarrow these \\
ADJ_{3,3} & \rightarrow damaged
\end{align*}
\]
Intersection grammars

1.0 $S \rightarrow NP\ VP$
0.3 $NP \rightarrow John$
0.7 $NP \rightarrow Mary$
0.2 $VP \rightarrow ran$
0.5 $VP \rightarrow V\ NP$
0.3 $VP \rightarrow V\ S$
0.4 $V \rightarrow believed$
0.6 $V \rightarrow knew$

$\cap$

\[ \begin{array}{c}
0 \\
1 \\
2
\end{array} \]
Mary
believed

$\cap$

\[ \begin{array}{c}
0 \\
1 \\
2 \\
3
\end{array} \]
Mary
believed
John

\[ S \rightarrow NP\ VP \\
NP \rightarrow John \\
NP \rightarrow Mary \\
VP \rightarrow ran \\
VP \rightarrow V\ NP \\
VP \rightarrow V\ S \\
V \rightarrow believed \\
V \rightarrow knew \\
\]

surprisal at ‘John’ $= - \log P(W_3 = John \mid W_1 = Mary, W_2 = believed)$

$= - \log \frac{\text{total weight in } G_3}{\text{total weight in } G_2}$

$= - \log \frac{0.0672}{0.224}$

$= 1.74$
Computing sum of weights in a grammar ("partition function")

\[ Z(A) = \sum_{A \rightarrow \alpha} \left( p(A \rightarrow \alpha) \cdot Z(\alpha) \right) \]

\[ Z(\epsilon) = 1 \]

\[ Z(a\beta) = Z(\beta) \]

\[ Z(B\beta) = Z(B) \cdot Z(\beta) \quad \text{where} \quad \beta \neq \epsilon \]

<table>
<thead>
<tr>
<th>Production</th>
<th>Value</th>
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<tbody>
<tr>
<td>S \rightarrow NP VP</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>NP \rightarrow John</td>
<td>0.3</td>
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<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>V \rightarrow believed</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>V \rightarrow knew</td>
<td>0.6</td>
<td>-</td>
</tr>
</tbody>
</table>

\[ Z(V) = 0.4 + 0.6 = 1.0 \]

\[ Z(NP) = 0.3 + 0.7 = 1.0 \]

\[ Z(VP) = 0.2 + (0.5 \cdot Z(V) \cdot Z(NP)) \]

\[ = 0.2 + (0.5 \cdot 1.0 \cdot 1.0) = 0.7 \]

\[ Z(S) = 1.0 \cdot Z(NP) \cdot Z(VP) \]

\[ = 0.7 \]
**Computing sum of weights in a grammar (“partition function”)**

\[
Z(A) = \sum_{A \rightarrow \alpha} (p(A \rightarrow \alpha) \cdot Z(\alpha))
\]

\[
Z(\epsilon) = 1
\]

\[
Z(a\beta) = Z(\beta)
\]

\[
Z(B\beta) = Z(B) \cdot Z(\beta)
\]

(\text{where } \beta \neq \epsilon)

\[
Z(V) = 0.4 + 0.6 = 1.0
\]

\[
Z(NP) = 0.3 + 0.7 = 1.0
\]

\[
Z(VP) = 0.2 + (0.5 \cdot Z(V) \cdot Z(NP))
\]

\[
Z(VP) = 0.2 + (0.5 \cdot 1.0 \cdot 1.0) = 0.7
\]

\[
Z(S) = 1.0 \cdot Z(NP) \cdot Z(VP)
\]

\[
= 0.7
\]
Things to know

Technical facts about CFGs:

- Can intersect with a “prefix FSA”
- Can compute the total weight (and the entropy)

(Hale 2006)
Things to know

Technical facts about CFGs:

- Can intersect with a “prefix FSA”
- Can compute the total weight (and the entropy)

More generally:

- Intersecting a grammar with a prefix produces a new grammar which is a representation of the comprehender’s sentence-medial state
- So we can construct a sequence of grammars which represents the comprehender’s sequence of knowledge-states
- Ask “what changes” (or “how much changes”, etc.) at each step

The general approach is compatible with many very different grammar formalisms (any grammar formalism?) — provided the technical tricks can be pulled off.

(Hale 2006)
Looking ahead

Wouldn’t it be nice if we could do all that for minimalist syntax?

The average syntax paper shows illustrative derivations, not a fragment.

What would we need?

- An explicit characterization of the set of possible derivations
- A way to “intersect” that with a prefix
- A way to define probability distributions over the possibilities

This will require certain idealizations. (But what’s new?)
Part 1: Grammars and cognitive hypotheses
   What is a grammar?
   What can grammars do?
   Concrete illustration of a target: Surprisal

Parts 2–4: Assembling the pieces
   Minimalist Grammars (MGs)
   MGs and MCFGs
   Probabilities on MGs

Part 5: Learning and wrap-up
   Something slightly different: Learning model
   Recap and open questions


