C44
C.40

Compositionality: A Global Perspective

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Recent work, from diverse points of view—Lakoff (1987), Keenan (1993), Keenan and Stabler (1996), Kalman (1995), Zadrozny (1994) among others, has called into question the empirical force of *Compositionality* as a constraint on the interpretation of natural languages. There is even perhaps something of a consensus that Compositionality as standardly formulated is too weak, allowing too great a range of possible interpretations. But, as is clear from the detailed presentation in Janssen (1997), there is considerable difference as to precisely where the problems lie and precisely what modifications should be imposed.

Here we propose a modest strengthening of Compositionality, one that is, we feel, often assumed though not consciously intended. We call this strengthening Global Compositionality (GC). In §1 below we provide a formal statement of Standard Compositionality (SC). In §2 we review an extension to Compositionality proposed by Keenan and Stabler (1996), *Strong* Compositionality. We show precisely what kinds of semantic interpretations it rules out which nonetheless satisfy SC, and show that it properly generalizes SC, i.e. it entails but is not entailed by SC. In §3 we define GC and prove that GC properly generalizes Strong C. We exhibit semantic interpretations which satisfy Strong C but fail GC, thereby illustrating the sort of phenomena that GC excludes over and above what Strong C excludes.

1 Standard Compositionality

Compositionality is usually formulated as follows:

PRELIMINARY DEFINITION 1 *The (semantic) interpretation of a derived expression is a function of the interpretations of the expressions it is derived from plus how it is derived.*

Assumed here is that the language is given by a grammar; i.e. some expressions are derived from others, and some are *basic* (not derived). Preliminary Definition 1 imposes no conditions on underived expressions.

Let us spell out our preliminary definition more explicitly, making clear the sense in which Standard Compositionality is a "local" constraint. We shall, noncomittedly, think of a language as determined by a grammar G, where G

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consists essentially of a set Lex_G of basic (i.e. non-derived) expressions, sorted into categories, and a set F_G of *generating (structure building)* functions. An f in F_G maps tuples of expressions in specified categories to an expression of a specified category. L_G, *the language generated by G,* is the closure of Lex (subscripts are omitted when no confusion results) under the structure building functions F. That is, L_G is the set of (categorized) expressions that can be built from Lex by applying the structure building functions finitely many times. We illustrate these notions, as well as the notion of semantic interpretation, with a minimal language L. Our considerations apply to extensions of L to full type theoretic languages.

The basic symbols of L are some *individual constants* (ICs) *john, andy,* ... , some 1-place predicate symbols (P1's) *walks, talks,* ... , and the symbols & and $\neg$. The generating functions are PA, AND, and NOT. PA maps pairs $\langle p, c\rangle$, p of category P1 and c of category IC, to $c^\frown p$ of category S (Sentence). AND maps pairs $\langle s, t\rangle$ of category S to $(s^\frown \text{and}^\frown t)$ of category S. NOT takes unary sequences $\langle s\rangle$ of category S to $\neg s$ of category S.

Now consider the standard extensional interpretations for L. The type of object an expression denotes depends on its category. ICs denote objects in the universe of objects under discussion. P1's denote subsets of that universe. Different interpretations may differ both with regard to the choice of universe and, even holding the universe fixed, with regard to the objects denoted by the ICs and the subsets denoted by the P1's. Let us say then that a *model* for L is a pair (E, m_E), with E a universe and m_E an *interpretation* of L relative to E. Then m_E is a function with domain L satisfying:

DEFINITION 1

1. *for all sentences s,* $m_E(s) \in \{0, 1\}$
2. *for all individual constants c,* $m_E(c) \in E$
3. *for all P1's p,* $m_E(p) \subseteq E$
4. *for all P1's p and all ICs c,*

$$m_E(PA(p,c)) = 1 \quad \text{iff} \quad m_E(c) \in m_E(p)$$

5. *for all s, t of category S,*

$$m_E(AND(s,t)) = m_E(s) \wedge m_E(t) \quad \text{and} \quad m_E(NOT(s)) = \neg(m_E(s))$$

We think of '$\wedge$' here as a binary function on $\{0,1\}$ given by the standard truth table for conjunction; a comparable claim connects the use of '$\neg$' to the truth table for negation. We note that m_E may be required to map specific basic expressions to specific denotations. For example, if exists of category P1 is in Lex we may require that $m_E(exists) = E$.

We turn now to a proper characterization of Compositionality, illustrating both how interpretations of L above satisfy it and how variations on those interpretations fail it. We use the following notational simplification: if t is a sequence $\langle t_1, \ldots, t_n \rangle$ and m is a function with each t_i in its domain we write $m(t)$ for $\langle m(t_1), \ldots, m(t_n) \rangle$. Also if K is a subset of $\mathrm{Dom}\, m$ then by $m(K)$ is meant $\{m(x) \mid x \in K\}$.

DEFINITION 2 (STANDARD COMPOSITIONALITY) *Consider a grammar G with generating functions F_G. For all models (E, m_E), the interpretation m_E is* compositional *iff* $\forall f \in F_G$,

$$\{\langle m_E(t), m_E(f(t)) \rangle \mid t \in \mathrm{Dom}\, f\} \qquad \textit{is a function.}$$

Given f, we can identify this set of pairs as the function f^* so that

$$m_E(f(t)) = f^*(m_E(t)) \qquad \textit{all } t \in \mathrm{Dom}\, f.$$

It is easy to see that the interpretations of our minimal L above are compositional. For example, given a universe E and an interpretation m_E relative to E, the set of pairs in (1) is clearly a function.

(1) $\{\langle \langle m_E(p), m_E(c) \rangle, m_E(\mathrm{PA}(p, c)) \rangle \mid p \text{ a P1, } c \text{ an individual constant}\}$

(1) contains tuples like those in (2):

(2)
$$\langle \langle m_E(\text{walks}), m_E(\text{john}) \rangle, m_E(\text{john walks}) \rangle$$
$$\langle \langle m_E(\text{talks}), m_E(\text{andy}) \rangle, m_E(\text{andy talks}) \rangle$$

Now if it happens that m_E maps *walks* and *talks* to the same subset of E and it maps *john* and *andy* to the same element of E then it must map the S *john walks* to the same truth value it maps *andy talks* to. This follows directly from the conditions we imposed on each m_E.

To see what non-compositional interpretations might look like, imagine the following:

CONDITION 1 (PATHOLOGICAL) *We require interpretations m_E of L to satisfy the conditions that are like those in Definition 1 except that line 1.4 is replaced by (3):*

(3)
$$m_E(PA(p, c)) = \begin{cases} 1 & \textit{if } c \textit{ begins with a consonant,} \\ 0 & \textit{if } c \textit{ begins with a vowel.} \end{cases}$$

Then in an interpretation m (omitting subscripts) in which $m(\text{john}) = m(\text{andy})$ the set of pairs in (1) would contain ones like $\langle \langle m(\text{walks}), m(\text{john}) \rangle, 1 \rangle$ and $\langle \langle m(\text{walks}), m(\text{andy}) \rangle, 0 \rangle$ which fails to be a function since the left hand sides $\langle m(\text{walks}), m(\text{john}) \rangle = \langle m(\text{walks}), m(\text{andy}) \rangle$ are equal but the right hand sides are not. Observe, then, that given a grammar, SC is non-trivial; it admits some functions and not others.

2 Strong Compositionality

Standard compositionality rules out pathological cases of the sort just illustrated. But it does not block certain others we feel should be blocked. One reason is that it doesn't insist that the *class* of interpretations available for a given universe has a uniform character. Consider the following scenario:

CONDITION 2 (PATHOLOGICAL) *Let $\mathcal{M}_1$ be the class of models (E, m_E) given as in Definition 1. Let $\mathcal{M}_2$ be the class of models (E, m_E) such that for each E, the interpretation m_E is as in Definition 1 except that line 1.4 is replaced by:*

$$m_E(p, c) = 0 \quad \textit{iff} \quad m(c) \in m(p).$$

Now, consider the class of models $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$.

$\mathcal{M}$ has, in effect, contradictory members. Observe that for any particular model (E, m_E) in $\mathcal{M}$, m_E is a compositional function just as before. But this time, the nature of the interpretation for a particular universe is allowed to vary, with the result that an expression's truth value could change from one interpretation to another. This, we contend, is an odd situation which does not correspond to the generally conceived notion of compositionality.

A stronger form of compositionality which constrains the variation of interpretations for a particular universe was formulated by Keenan and Stabler (1996). We recast it here for uniformity:

DEFINITION 3 (STRONG COMPOSITIONALITY) *Consider a grammar G with generating functions F_G. For all universes E, $\forall f \in F_G$,*

$$\{\langle m(t), m(f(t))\rangle \mid (E, m) \textit{ a model for } L_G \;\&\; t \in \mathrm{Dom}\, f\} \qquad \textit{is a function.}$$

This formulation effectively constrains the class of models available; it is no longer a purely local condition on a single model, though it obviously does entail Standard Compositionality.

Moreover, we find that Strong Compositionality rules out the pathological scenario in Condition 2. Let us use the models of $\mathcal{M}$ above for our language, and consider two models $(E, m_1) \in \mathcal{M}_1$ and $(E, m_2) \in \mathcal{M}_2$. Let $m_1(\text{john}) \in m_1(\text{walks})$ and $m_2(\text{john}) \in m_2(\text{walks})$ and $m_1(\text{john}) = m_2(\text{john})$ and $m_1(\text{walks}) = m_2(\text{walks})$. Then $m_1(\text{john, walks}) = m_2(\text{john, walks})$ but $m_1(\text{john}\frown\text{walks}) = 1$ while $m_2(\text{john}\frown\text{walks}) = 0$. This result contradicts Strong Compositionality. In consequence:

PROPOSITION 1 *Strong Compositionality entails but is not entailed by SC.*

3 Global Compositionality

Is Strong Compositionality sufficient? We think not. While it does constrain the variation of interpretations for a particular universe, Strong Compositionality says nothing against making the interpretation dependent on properties of the universe in its model—any properties. This can have unfortunate consequences:

CONDITION 3 (PATHOLOGICAL) *We require that for each E, interpretations m_E of L are as in Def. 1 except that line 1.4 is replaced by:*

(4) $$\text{if } 5 \in E \text{ then } m_E(p,c) = 1 \text{ iff } m(c) \in m(p) \text{ and}$$
$$\text{if } 5 \notin E \text{ then } m_E(p,c) = 0 \text{ iff } m(c) \in m(p)$$

We observe that for any given E, either 5 is in E or it isn't. Thus for each E, m_E is provably a compositional function. Moreover, the entire class of functions available to a particular E obeys Strong Compositionality. But clearly whether *john walks* is true under a given interpretation depends on more than just the denotation of *john* and of *walks*; it also depends on whether 5 is an element of the universe. Thus let E be a non-empty set $\{a, b, \dots\}$ which lacks 5 and set $E' = E \cup \{5\}$. Let $m_E(\text{john}) = m_{E'}(\text{andy}) = b$ and let $m_E(\text{walks}) = m_{E'}(\text{talks}) = \{a, b\}$. But clearly $m_E(\text{john walks}) \neq m_{E'}(\text{andy talks})$, even though m_E interprets the constituents of this sentence exactly the same as $m_{E'}$.

Variations on this pathological case are easy to come by. We might, for example, condition how m_E interprets a derived expression according as E was finite or not, or had an even number of elements or not, rather than according to whether a given object was an element of E. It is pathological conditions like these that we rule out with Global Compositionality.

DEFINITION 4 (GLOBAL COMPOSITIONALITY) *Consider a grammar G with generating functions F_G and model class $\mathcal{M}_G$. $\forall f \in F_G$,*

$$\{\langle m(t), m(f(t))\rangle \mid (E, m) \in \mathcal{M}_G \;\&\; t \in \mathrm{Dom}\, f\} \qquad \textit{is a function.}$$

That GC does entail Strong C is immediate from the comparison of Definition 4 with Definition 3. GC also blocks the pathological Condition 3, whereas Strong C does not; this shows:

PROPOSITION 2 *Global Compositionality entails but is not entailed by Strong Compositionality.*

Note that GC does not prevent us from making the interpretation of an expression dependent on the universe of the model. As we have noted, it would be natural to require of the P1 *exists* that each m_E interpret it as E. Equally, enriching our minimal L with P2's in the obvious way, it would be natural to require that for all E, $m_E(\text{is}) = \{\langle a, a\rangle \mid a \in E\}$. Similarly, as van Benthem (1986, Ch. 3) notes, a P1-level negation such as *non-* in *non-student* is universe-dependent; we might have two universes with the same students but different non-students.

4 Concluding Reflections

It is explicit in our definition of Compositionality that it is a relation between semantic interpretation and syntactic derivation. A given set of expressions, or even categorized expressions, may be generated by many different grammars and a fixed semantic interpretation may be compositional with respect to some of these but not others. Hence on our view the question of whether a given interpretation of a set of (categorized) expressions is compositional is not well-defined. We can only ask this relative to a grammar which generates these expressions, and different choices of grammars will give rise to different results (c.f. Janssen's 1997 example involving non-arithmetic versus arithmetic interpretations of digit sequences like '007').

Of course we may ask a different question: Does a given set of expressions have a grammar that can be compositionally interpreted? But here the answer is a trivial "yes." (To construct a compositional interpretation of the expressions L_{G_1}, define a grammar G_2 where $\text{Lex}_{G_2} = L_{G_1}$ and $F_{G_2} = \emptyset$; interpret the expressions in any fashion whatsoever.) Whether a given interpreted grammar (G, m) is compositional is a non-trivial question; more interestingly, whether a given G admits of a compositional interpretation is also non-trivial (Janssen, as we have shown above).

In Linguistic practice the syntactic functions are given in partial independence from semantic interpretation. In accord with this, a grammar G with generating functions F_G has no semantics built in. A semantic interpretation provides each syntactic f with a corresponding f^*, and it is of these "semantic generating functions" that questions of compositionality should be raised.

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