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# Determiners and the Logical Expressive Power of Natural Language


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## 0. Notional and Notational Preliminaries

We may consider that a major function of natural language is to enable us to raise questions and make assertions about the world. It is natural then to consider the precise semantic contribution to these assertions made by the various sorts of expressions a natural language provides. Here we are concerned with the expressive contribution of English Determiners (dets) and with compairing the expressive nature of dets with that of other categories.

We shall formulate our concerns within the model theoretic approach to Generalized Quantifiers expressed in various ways in Barwise and Cooper (1981), van Benthem (1982), Keenan (1981), Keenan and Moss (1984), Keenan and Stavi (1981), Linström (1966), Thijsse (1982), Westerståhl (1982), and Zwarts (1982).

On most of these approaches an extensional model for English is given by a non-empty set E of entities. And an extensional *property* is just a set of entities. The set P of all properties of the model is the denotation set for *common noun phrases* (CNP's). In general, given a model E, we use $D_C$ to denote the set of logically possible denotations (relative to E) of expressions of category C.

Proper nouns (e.g. *John*) denote sets of properties called *individuals*, where a set I of properties is an individual iff for some $e \in E$, $I = \{q \in P: e \in q\}$. To say that an individual I "has" some property q is just to say that $q \in I$. We commonly use the symbol n to refer to the number of individuals of a model, and we use $|q|$ for the number of individuals that have q. We use 1 for that property which all individuals have. And where p and q are properties, we use $(p \wedge q)$ for the property of being both a p and a q, that is, the property an individual has iff he has p, and he also has q.

We use $P^*$ to denote the set of all sets of properties. So each individual I is an element of $P^*$, and in fact, $P^*$ is the denotation set for full noun phrases (NP's). Finally we use $F_{P^*/P}$ for the set of functions from P into $P^*$. So this set provides the set of logically possible denotations for (one place) dets.

(We refer the reader to Keenan (1982) for a semantics equivalent to the above in which there is no E, P is a primitive, and individuals are defined directly as subsets of P satisfying certain conditions. On all views P is isomorphic to the power set of the set of individuals.)

## 1. Determiners and the Conservativity Constraint

We treat one place dets ($Det_1$'s) as expressions which combine with one CNP to form an NP. They include expressions such as *every, no student's, more of John's than of Mary's, at least two but not more than ten*, etc. We refer the reader to Keenan and Stavi (K&S) for a copious list of $Det_1$'s in English.

Semantically $Det_1$'s are interpreted by fuctions from P into P*. Earlier work, most exhaustively in K&S, established that in general the funcitons which can interpret elements of $Det_1$ are *conservative*, where:

Def 1: A funciton f from P into P* is *conservative* iff for all p,q∈ P, p∈ f(q) iff (p∧q)∈ f(q)

So to say that f is conservative is to say that f q's are p's iff f q's are both q's and p's. E.g. every student is a vegetarian iff every student is both a student and a vegetarian. We note that the functions which can interpret "non-logical" dets such as *John*'s and *more male than female* (as in *more male than female students failed the exam*) as well as "logical" dets are conservative.

## 2. Generalizing Determiners

The initial questions of expressive power we consider concern the relative expressive power of distinct subclasses of dets. We shall first extend the class of dets considered and then define the subclasses of interest.

We may think of k-place dets, $Det_k$'s, as expressions which combine directly with k CNP's to form an NP. They will be interpreted by functions from the set $P^k$ of k-tuples of properties into P*. Keenan and Moss (K&M) give several reasons for treating the italicized expressions in (1) below as $Det_2$'s:

(1) a. *more* students *than* teachers
 b. *every* author *and* critic
 c. *John's brown* cats *and* dogs
 d. *fewer French* workers *than* students
 e. *the 27* students *and* teachers

For example we want the two CNP's on the same level of structure so that they may both be under the scope of the adjectives in (1c) and (1d); equally they behave in similar ways with regard to number marking and the role they play in selectional restrictions. Further we can naturally represent the ambiguity in (1b) by treating *author and critic* as a conjunction of CNP's which combine with the $Det_1$ *every*, on the one hand, and by treating *every...and...* as a $Det_2$ which combines directly with (*author, Critic*) on the other hand. Additional sorts of reasons are given for treating *more...than...* (and other numerical comparatives such as *not more than twice as many...as...*) as a $Det_2$ (see also Napoli 1983). One such reason is given in Theorem 1 below, which uses the interpretation of *more...than...* given in Def 2:

Def 2: more s than t = {q: |s∧q| > |t∧q|}

So q is a property which more students than teachers have iff the number of students (s) with q is greater than the number of teachers (t) with q.

Theorem 1: In any model with at least two individuals the range of *more...than...* has larger cardinality than P itself. Hence, it cannot also be the range of any one place function (conservative or not) on P.



The theorem guarantees that *more...than...* cannot be paraphrased by any $Det_1$ and thus suggests that $Det_2$'s provide a non-eliminable part of the expressive apparatus of natural language.

k-place det denotations are also conservative, as per Def 3 below which is justified by Theorem 2:

Theorem 2: A function f from P into P* is conservative iff for all $p_1$, $p_2$, and q∈ P, if $p_1 \wedge q = p_2 \wedge q$ then $p_1 \in f(q)$, iff $p_2 \in f(q)$.

Theorem 2 says that the one place conservative functions are just those which cannot discriminate between properties $p_1$, $p_2$ which have the same meet with their argument q.

Generalizing below, we use $\bar{q}$ as a variable ranging over k-tuples of properties, the $i^{th}$ element of which is denoted $q_i$:

Def 3: A function f from $P^k$ into P* is *conservative* iff for all $p_1,p_2 \in P$ and all $\bar{q} \in P^k$, if $p_1 \wedge q_i = p_2 \wedge q_i$ for i = 1,2,...,k then $p_1 \in f(\bar{q})$ iff $p_2 \in f(q)$

So the k-place conservative functions are just those which cannot distinguish between properties $p_1$, $p_2$ having the same meet with each argument $q_i$.

We use $CONS_k$ for the set of k-place conservative functions and CONS for those functions in $CONS_k$ for some k.

## 3. Subcategories of Determiners

We shall here semantically subcategorize dets according as the functions which interpret them satisfy one or another condition. Specifically we shall distinguish "logical" dets such as *every, all but two, more than ten, more...than...*, etc. From "non-logical" or "real world" dets such as *John's, more French...than British..., all...but John*, etc. And secondly, among the logical dets we shall distinguish *cardinal* dets from non-cardinal ones.

Informally a function from P into P* will be called a *cardinal* function just in case whether it puts a property t in the set it assocates with q is determined by how many individual q's have t. For example the functions which interpret *more than ten, between five and ten*, and *infinitely many* are obviously cardinal functions, since if we know how many t's are q's we know whether infinitely many are, whether between five and ten are, etc.

Generalizing to k-place functions f, we say that f is a *cardinal* function just in case whether it puts t in the set it associates with a k-tuple $\bar{q}$ of properties is determined by how many individual $q_i$'s are t's, for each i between 1 and k. For example, if we know how many $q_1$'s are t's and how many $q_2$'s are t's then we know whether t is a property that more $q_1$'s than $q_2$'s have, so the funciton which interprets *more...than...* is a two place cardinal function. For the record:

Def 4: A function f from $p^k$ into P* is a *cardinal function* iff for all s,t∈ P and all k-tuples $\bar{p}$, $\bar{q}$ in $p^k$, if $|s \wedge p_i| = |t \wedge q_i|$, i = 1,2,...,k then $s \in f(\bar{p})$ iff $t \in f(\bar{q})$

$CARD_k$ will denote the set of k-place cardinal functions and CARD the set of functions which are in $CARD_k$ for some k. We note that for all k, $CARD_k$ is a proper subset of $CONS_k$ and that

CARD is a proper subset of CONS. So all (k-place) cardinal functions are (k-place) conservative functions. Finally of course we define a k-place det to be a cardinal det just in case its denotation always lies in $\mathrm{CARD}_k$.

Turning now to the larger class of logical dets, we shall find a property common to the denotations of those dets intuitively judged to be "logical" and then for each k, we define $\mathrm{LOG}_k$ to be the set of functions from $P^k$ into $P^*$ which have this property.

The intuition behind the definition of $\mathrm{LOG}_k$ is exemplified by the difference in truth conditions between (2a,b) on the one hand and (2c) on the other:

(2) a. Exactly one house is white.
b. Every house is white.
c. John's house is white.

The truth (or falsity) of (2a) and (2b) is determined once we have specified which objects have the house property and which objects have the property of being white. But to know whether (2c) is true we must in addition know which individual John is and which objects he "has". Thus, *John's* carries real world information in a way in which neither *exactly one* nor *every* does. More specifically, to say in a given state of affairs just which function from P into P* interprets *John's* we must be able to discriminate one individual from another. But to say which functions interpret logical dets such as *exactly one* and *every* we do not have to discriminate one individual from another in the sense of knowing which properties this individual has which that one doesn't. We may of course have to know the number of individuals with a certain property (as in the case of cardinal dets) but it doesn't matter which individuals they are. And we may have to assess the proportion of individuals with p that have q, as with dets such as *all, most, two-thirds of the*, etc., but again it doesn't matter which of the individuals with p have q as long as the proportion is satisfied.

More formally then we may define a function from $P^k$ into $P^*$ to be *logical* just in case it remains invariant under permutations of individuals. The formal statement of this definition is somewhat more "remote" than in the case of cardinal functions and space will not permit the length of discussion necessary to justify that the definition captures the intuition sketched above. The reader not concerned with the formal details may omit the formal definition (at least on first pass) noting only that we have found a property which descriminates the possible denotations of (pretheoretically judged) logical dets from non-logical ones. We may note further here that all cardinal functions are logical on the proposed definition, but the converse fails. E.g. dets like *every, most, all but two* denote in $\mathrm{LOG}_1$ but not in $\mathrm{CARD}_1$. Finally, we note that the definition of logical is not specific to det denotations but applies to the denotation set associated with any category. So it makes sense to ask what are the logical elements among CNP denotations? among two place predicate denotations? etc. And in the last section of this paper we compare, with interesting results, the nature of logical elements among Det denotations with those of many other categories.

*Defining "logical"*: A permutation of the individuals of a model is just a one-to-one function from the set of individuals onto itself. We shall refer to such functions $a$ as *basic automorphisms*. Each basic automorphism extends in a (mathematically) natural way to an automorphism on any denotation set for any category. (Here 'automorphism' is used non-trivially: the denotation sets in question normally have a rich boolean structure, and the extension of a basic automorphism is indeed a one-to-one function from the denotation set



onto itself which preserves that boolean structure.) The extension proceeds along the following lines: first, if $a$ has been extended to an automorphism on a set K then for each $S \subseteq K$, set $a(S) = \{a(s): s \in S\}$. This extends $a$ to an automorphism on the power set of K.[2] Similarly if $a$ has been extended to an automorphism on sets B and D then we extend it to an automorphism on the set of functions from B into D as follows: for each such function f, $a(f)$ is that function from B into D which sends each $a(b)$ to $a(f(b))$. And we may now define:

Def 5: An element d of any denotation set is ***automorphism invariant*** iff for all basic automorphisms $a$, $a(d) = d$

We now define $\mathrm{LOG}_k$ to be the set of those functions in $\mathrm{CONS}_k$ which are automorphism invariant. LOG denotes the set of those functions in $\mathrm{LOG}_k$ for some k. And an element of $\mathrm{Det}_k$ is *logical* iff it always denotes in $\mathrm{LOG}_k$.

We note further that in all models, $\mathrm{CARD}_k$ is a proper subset of $\mathrm{LOG}_k$, and in all but the model with just one individual, $\mathrm{LOG}_k$ is a proper subset of $\mathrm{CONS}_k$.

### 4. Questions of Expressive Power

(3) a. Suppose that English dets could take their denotations freely in the set of functions from P into P*. Is there any significant sense in which we could say more than we currently can?

b. Suppose that English had no non-logical dets. Would we suffer a significant loss of expressive power? E.g. can we always mimic the effect of possesive dets (*John's, every student's*, etc.), however ad hocly, with logical dets?

c. Can anything we can say using the full class of logical dets be paraphrased (however ad hocly) using only cardinal dets?

d. Does allowing k>1 place dets increase expressive power? E.g. can we in any interesting sense say more with $\mathrm{LOG}_2$ dets than with $\mathrm{LOG}_1$ dets?

e. Can we account for why there seem to be so many logical dets and so few logical expressions among most othercategories, e.g. the place predicates (VP's)?

(Query (3e) arises in a natural way from the answers to (3a-d)).

To answer these questions we need a "significant" measure of the expressive power of the different classes of dets in question. Trivially for example if we had no non-logical dets in English we would lose the ability to denote conservative functions which were not automorphism invariant. But this loss hardly seems significant. We are not primarily interested in which functions from $P^k$ into $P^*$ we can name. Rather we think of the role of dets as being inherently ancillary—their "purpose" is to allow us to refer to sets of properties in terms of properties. A reduction in the class of dets of English then may be deemed significant if it results in a loss of sets nameable by NP's of the form Det+CNP. To make this notion precise:

Def 6: For $Q \subseteq P$ and $H$ any set of functions whose range is included in $P^*$, we say that $Q$ *is H-expressible* iff for some $h \in H$ and some $x$ in the domain of $h$, $Q = h(x)$. We let $E_H$ denote the family of H-expressible sets.

Note: the elements of $E_H$ vary with the choice of P.

Now the classes H of interest here are of course CARD, LOG, CONS, and for every k, $CARD_k$, $LOG_k$, and $CONS_k$. And for H and K any of these classes we may compare their expressive power by comparing $E_H$ and $E_K$. Unfortunately the results of the comparison sometimes depend on the choice of P. For example in a model with only one individual the $LOG_1$-expressible sets are the same as the $CONS_1$-expressible sets, whereas this identity fails in all models with more than one individual. To eliminate this dependency we define:

Def 7: For H and K any of our classes of interest, we write $E_H \subset E_K$ iff (i) and (ii) below hold:

(i) for all P, $E_H$ is a subset of $E_K$

(ii) for P sufficiently large, $E_H$ is a proper subset of $E_K$. (To say that P is sufficiently large is to say that for that P and all larger P, $E_H$ is a proper subset of $E_K$.)

We now answer our questions of interest:

Theorem 3: (a) $E_{CARD_1} \subset E_{LOG_1} \subset E_{CONS_1}$ and

(b) for all P, $E_{CONS_1} = P^*$

Theorem-3b answers query (3a) in the negative. For any P and any subset Q of P there is a one place conservative function f and a property s such that $Q = f(s)$. (We may in fact choose s to be the trivial property 1, the property which all individuals have). So restricting dets to conservative functions does not *in principle* restrict the sets we can refer to with full NP's.

Theorem-3a answers queries (3b) and (3c) in the affirmative for the case $k = 1$. Theorem 4 below shows that that result generalizes in a nice level by level way:

Theorem 4: (a) for all k, $E_{CARD_k} \subset E_{LOG_k} \subset E_{CONS_k}$

(b) $E_{CARD} \subset E_{LOG} \subset E_{CONS}$

We should note here that the examples of property sets which show that the inclusions above are proper are in practice expressible. For example, let q be an infinite property (one had by an infinite number of individuals, such as e.g. the property of being an even number). Then the set every (q) is not in $E_{CARD}$ and hence not in $E_{CARD_k}$ for any k. Since *every* denotes in $LOG_1$, that set is in $E_{LOG_1}$. Thus the expressive contribution of *every* cannot be mimicked by any cardinal dets, however ad hocly, since there are sets such as every(q) above which are not equal to f(s) for any k-tuple of properties s and any k-place cardinal function f.

Similarly we can show that among the acceptable interpretations of *some person's one or more things* are sets not in $E_{LOG}$ and hence not in $E_{LOG_k}$ for any k. *thing* can be interpreted as the trivial property 1. The algebra P however must be infinite and the possessor property



is infinite in our example and probably necessarily. We note that for any P, every finite set of properties is in $E_{LOG}$, whence if P is finite then every set of properties is in $E_{LOG}$.

Query (3d) is mostly answered by Theorem 5 below:

Theorem 5: (a) $E_{CARD_k} \subset E_{CARD_{k+1}}$ for $k = 1,2$ and probably all k

(b) $E_{LOG_k} \subset E_{LOG_{k+1}}$, all k

(c) $E_{CONS_k} = E_{CONS_1} = P^*$, all k, all P

For example, we can show that for certain pairs of properties (p,q) the value of *more...than...* at that pair $\neq f(s)$, any property s, any $f \in CARD_1$. So with respect to cardinal and more generally logical dets the addition of $k > 1$ place dets does increase expressive power. But this is not the case for the full class of conservative functions. All sets are $CONS_1$ expressible so *in principle* we could not express more sets with $CONS_2$ functions. In practice however things might be otherwise. For while it is true that for any set Q of properties there is a $f \in CONS_1$ such that $f(1) = Q$, we have no general way to construct a determiner expression (however complex) in English which could be interpreted as this f. Moreover there appear to be serious constraints on the freedom with which we may interpret dets (of any degree). K&S prove an ineffability result concerning $DET_1$ which suggests that the size of the denotation set for $DET_1$ grows faster than the freedom with which we may interpret complex dets. More important for our purposes here, there are severe constraints on the interpretation of syntactically simple dets (mostly they denote increasing, automorphism invariant functions) and hence of the sets we can express with them. As a category then, Det provides many logical expressions and constrains the denotations of its lexically simple ones. Below we compare Det with other categories in these (and other) respects:

| | Category C | Size of $D_C$ | No. of AI elements | Lex Free? | Free? |
|---|---|---|---|---|---|
| large | $Det_1$ | $2^{3^n}$ | $2^{(n+1)(n+2)/2}$ | no | ?no |
| cats | NP | $2^{2^n}$ | $2^{(n+1)}$ | no | ? |
| | AP | $2^{n \times 2^{n-1}}$ | $2^n$ | no | ?yes |
| small | $P_3$ | $2^{n^3}$ | $2^5$ | yes | yes |
| cats | $P_2$ | $2^{n^2}$ | $2^2$ | yes | yes |
| | $P_1$ | $2^n$ | 2 | yes | yes |
| | CNP | $2^n$ | 2 | yes | yes |
| | $AP_{abs}$ | $2^n$ | 2 | yes | yes |
| | $NP_{prop}$ | n | 0 | yes | yes |
| | $S\ (= P_0)$ | 2 | 2 | n.a. | yes |

Space does not permit a serious discussion of the entries in the table. By way of minimal explanation of the entries we note: n is the number of individuals of the model and is assumed finite. $P_3$ is the category of 3-place predicate (ditransitive verb phrases); $P_2$ and $P_1$ are understood analogously. AP is adjective phrase, and $AP_{abs}$ is absolute adjective phrase, a subcategory of AP which includes e.g. *male* but not *tall*—so those AP's which actually determine properties. $NP_{prop}$ is the subcategory of proper NP's.

A category C is called *free* iff for every model, any element $d \in D_C$ is denotable, that is, there is an expression e in category C such that for some interpretation m of the language, m(e) = d. C is called *lexically free* if the element in question can always be chosen from among the lexical expressions of category C.

*Large categories* are ones the size of whose denotation sets is a hyperexponential function of the size of the universe of individuals. That is, the size of the denotation set increases as an exponential function of n. For *small categories* on the other hand the size of the denotation set increases merely as an exponential of a polynomial in n, or less.

We may now summerize the major generalizations from the table:

(4) Small categories are lexically free, large categories are not.

(5) For large categories the number of automorphism invariant elements in $D_C$ (so the number of possible denotations of logical constants of category C) increases as an exponential function of the size of the universe (and increases much faster for $Det_1$ than any other large category). For small categories the number of AI elements is fixed and independent of the size of the universe.

So from (4) we note that categories with large denotation sets constrain how lexical items in the category may be interpreted. E.g. lexical NP's in general denote individuals (and perhaps a handful of other sets, expressed by NP's such as *everyone, someone*, etc.)

And from (5) we have a partial answer to query (3c): No matter how big the universe there are only two distinct possible denotations for logical expressions of category $P_1$ (verb phrase). They are basically the denotations of *exist* and *not exist* (extensionally). So English cannot in principle provide more than two extensionally distinct logical expressions of category $P_1$. It can however provide infinitely many expressions of category $Det_1$ and (in an infinite universe) they can all denote differently.

## Footnotes

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1. We may then show that $f: P^k \to P^*$ is conservative iff $[p \in f(\bar{q})$ iff $p \wedge \bigvee_{i=1}^{k} q_i \in f(\bar{q})]$. This characterization of conservativity is comparable to the one orginally given for one place functions.

2. We use the same symbol *a* for the basic automorphism and its various extensions.



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