A.63

Chapter 2

Lexical Freedom and Large Categories*

Edward L. Keenan

SUMMARY

Grammatical categories of English expressions are shown to differ with regard to the freedom we have in semantically interpreting their lexical (= syntactically simplest) expressions. Section 1 reviews the categories of expression we consider. Section 2 empirically supports that certain of these categories are *lexically free*, a notion we formally define, in the sense that anything which is denotable by a complex expression in the category is available as a denotation for lexical expressions in the category. Other categories are shown to be not lexically free. Thus for those categories the interpretation of lexical expressions is inherently constrained compared to the interpretation of the full class of expressions in the category.

In section 3 we establish the principle generalization of this paper: *small categories are lexically free, large ones are not*, where the size of a category is formally defined in terms of the full range of extensional distinctions expressible by expressions in the category. In terms of this generalization we suggest an explanation for the distribution of lexical freedom established in section 2.

We conclude with a generalization of the notion *lexical freedom* and present some partial results, leaving certain problems open.

1. CATEGORIES CONSIDERED

Below we present, with suitable mnemonics, the categories of expression we consider. We shall assume that these categories of expression form part of a formal languae L, the precise nature of which is unimportant here. The language given in K&F (Keenan & Faltz, 1985) will do, as will the languages given in Montague (1970 and 1973) with trivial modifications.

CN or (*zero place*) *common noun phrase: man, tall man, man who Sue loves*

* At several points in this paper I have benefitted from discussion and comment from Johan van Benthem, Peter van Emde Boas, Dick de Jongh, and Larry Moss. In addition I wish to thank the Max-Planck Institute für Psycholinguistik, Nijmegen for having supported the research herein.

NP or *full noun phrase: John, every man, John and no other student, more students than teachers*

CN_1 or *one place (transitive) common noun phrase: friend (of), brother (of)*. They combine with NP's to form CN's: *friend of the President, brother of some senator*.

AP or *zero place adjective phrase: female, tall, who Sue loves*

AP_1 or *one place (transitive) adjective phrase: fond (of), jealous (of)*. They combine with NP's to form AP's: *fond of both John and Mary*, as in *Every student fond of both John and Mary came to their aid*.

Det_k or k-*place determiners:* they combine with k CN's to form an NP. Some Det_1's (or just Dets for short) are: *every, no student*'s, (as in *no student's cat*), *more male than female* (as in *more male than female students [passed the exam]*).
Some Det_2's are: more ... than ..., exactly as many ... as ..., as in *more students than teachers [attended the meeting]*.

P_k or k-*place predicates*; We identify P_0 with S (Sentence).
Some P_1's are: *walk, walk but not talk, walk slowly in the garden*. Some P_2's are: *hug, kiss, hug and kiss*. Some P_3's are: *show, give, show and give*. We do not consider L to have P_k's for $k \geq 4$.

PM or *predicate modifier: here, slowly, in the garden*. PM's combine with P_k's to form P_k's, $k \geq 1$. Some also combine with CN_k's to form CN_k's.

Prep or transitive PM's: *in, at, on*. They form PM's from NP's.

Remarks on the syntax

(i) We shall not treat PM's and Prepositions directly, though we shall treat CN_k's and P_k's formed from them. We shall not treat Det_k's for $k > 2$. The extensive list of Det_1's we consider is given in K&S (Keenan & Stavi, 1981 and to appear). The class of Det_2's considered is given in K&M (Keenan & Moss, 1984). In addition, certain subcategories of the categories noted above will be treated.

(ii) For C any of the categories considered, we write C_{lex} for the set of syntactically simplest expressions of category C. Elements of C_{lex} will be referred to as *lexical expressions of category* C. They normally coincide with the one word expressions in C, though in a few cases such as P_0 and Det_2 it may be that the syntactically simplest expressions are more than one word long (e.g. *John walks*, and *more ... than ... respectively*).

2. LEXICAL FREEDOM

We shall first illustrate the concept of lexical freedom by arguing that the category P_1 is lexically free and that the category NP is not lexically free. Then we give the category independent definition of lexical freedom.

Extensionally we think of (first order) P_1's as being true or false of individuals. E.g. in a world ("model") of six individuals $I_1, I_2, \ldots, I_6$ it might be the case that the P_1 *walk slowly in the garden* is interpreted in such a way as to be true of, say, I_1, I_2, and I_3 and I_5 (and fails of all the other individuals[1]). And in such a case it is logically possible that the lexical P_1 *whistle* is also true of just I_1, I_2, I_3 and I_5. That is, there is nothing about the meaning of *walk slowly in the garden* which prevents it from being the case that the individuals who are walking slowly in the garden are just those who are whistling. This and similar examples show that complex P_1's formed from PM's (Predicate Modifiers) such as *slowly* and *in the garden* do not associate truth values with individuals in ways which are inherently unavailable for lexical P_1's such as *whistle*.

Moreover, what holds for P_1's formed with PM's holds for all ways of forming syntactically complex P_1's. That is, the various ways of building syntactically complex P_1's do not lead to P_1's which are true of individuals in ways which are in principle unavailable to lexical P_1's. And this is what we mean when we say that P_1 is *lexically free*. Any way of assigning truth values to individuals which are available for complex P_1's is also available for lexical P_1's.

To see that P_1 is in fact lexically free let us consider briefly the various ways in which complex P_1's may be formed. Modification with PM's has already been considered. Another way of forming complex P_1's is by taking boolean combinations in *and, or* and *not* (as well as *but, neither ... nor*, and a few others). Again, there is nothing about the meaning of, say, *walk but not talk* which prevents it from being the case that the individuals who are walking but not talking are just those who are smiling. So whenever *walk but not talk* is true of certain individuals it is logically possible that *smile* is also true of just those individuals. Third, consider P_1's formed from P_2's plus NP arguments, as *hug* and *kiss some student*. Obviously it is logically possible that the individuals who are both hugging and kissing some student are just those who are standing, so the lexical P_1 *stand* can in principle be interpreted in such a way as to be true of whatever individuals *hug* and *kiss some student* is true of. Fourth, consider P_1's formed by operations such as Passive, Reflexive, and Unspecified Object Deletion, as in *John was kissed, Mary admires herself*, and *Fred is reading*. Obviously, no matter which individuals were kissed, it is possible that just those individuals were *sleeping*, so the passive P_1 *was kissed* does not in principle hold of individuals which a lexical P_1 such as *sleep* could not hold of. Analogous claims hold for the P_1's *admire oneself* and *read*. As a last example consider P_1's formed from P_2's which take infinitival arguments, as in *want/begin/try* to *read this book*. Again it obviously could be the case that whatever individuals want to read this book are just the individuals who are humming, so such P_1's do not associate truth values with individuals in ways which are unavailable to lexical P_1's such as *hum*.

We conclude then that the category P_1 in English is lexically free. Moreover the informal arguments which support this claim are based on judgments which seem sufficiently banal as to make us wonder just how a category could really fail to be lexically free. There are however several such cases, among them the category NP considered below.

The category NP is not lexically free. Many complex NP's, such as *every student, more students than teachers*, etc. are built up from CN's (*student, teacher*), so just how such NP's are semantically interpreted will depend in part on how the CN's they are formed from are interpreted. We may (standardly enough) think of CN's as denoting *properties* of individuals, and we regard two CN's as extensionally distinct if, in some model, the individuals which have the property denoted by one are not exactly the same as those with the property denoted by the other. For example, *doctor* and *fat lawyer* are extensionally distinct since we can easily imagine a state of affairs in which the doctors and the fat lawyers are not exactly the same individuals. Up to isomorphism then, we may think of a CN as (extensionally) denoting a set of individuals.

Given this understanding of CN's, we may (up to isomorphism) think of full NP's as denoting sets of properties, i.e. sets of possible CN denotations. On this view, a sentence such as *Every doctor is a vegetarian* is true if the property denoted by *vegetarian* is an element of the set denoted by *every doctor*, and false otherwise.

Now, to show that NP is not lexically free, we will show that there are property sets denotable by complex NP's which are in principle undenotable by lexical NP's. The lexical NP's are largely just the proper nouns (*John, Mary*) and the singular personal pronouns (*he, she*). We might also include the demonstratives (*this, that*) as well as possessive deictics such as *mine, yours*. Let us further include (though most linguists would treat them as syntactically complex) the plural pronouns, such as *we, they, these, ours*.

Note that all these NP's denote property sets which are *increasing*, which we define as follows: A set K of properties is *increasing* iff for all properties p, q, if $p \in K$ and every individual with p is also one with q, then $q \in K$. An NP is *increasing* if it always denotes an increasing set.

To check whether an NP is increasing (cf. Barwise & Cooper, 1981) check that it satisfies the entailment paradigm given below when substituted for X:

(1) Every doctor is a vegetarian
X is a doctor
Therefore, X is a vegetarian

The lexical NP's noted above are all obviously increasing, and it is tempting to conclude that all lexical NP's are increasing. There is however another class of NP's which at least are phonological words and might be considered by some to be syntactically simple. These are items often referred to in traditional grammars as "indefinite" pronouns, e.g. *all, none, someone, everyone, noone*. If we count these NP's as lexical, we must note that *none* and *noone* are not increasing. They are however *decreasing*, where a set K of properties is *decreasing* iff for all properties p, q, if $q \in K$ and every p is a q then $p \in K$. To check that X is a decreasing NP verify that the following argument is valid:

(2) Every doctor is a vegetarian
X is a vegetarian
Therefore, X is a doctor

A NP is called *monotonic* just in case it is either increasing or decreasing. Clearly the candidates for lexical NP's considered above are all monotonic, and as these exhaust our candidates we conclude that all syntactically simple NP's in English denote monotonic sets of properties. (In fact, not all monotonic sets can be denoted by these NP's, but the condition as it stands is sufficient for our purposes.)

Now to show that NP is not lexically free it is sufficient to find complex NP's which can denote non-monotonic sets. And in fact all major ways of forming syntactically complex NP's yield ones which may denote non-monotonic sets. Some examples formed from Det's plus CN's are: *exactly one boy, between five and ten boys, more male than female students, all but one student, every student's but not every teacher's bicycle*. Examples formed from Det_2 and two CN's are: *more students than teachers, fewer students than teachers*, and *exactly as many students as teachers*. Finally, examples formed by boolean combinations of NP's include: *John but not Mary, every boy but not every girl, either fewer than five students or else more than a hundred students*. (We invite the reader to test e.g. *exactly one boy* in the paradigms given in (1) and (2) to satisfy himself that this NP is neither increasing nor decreasing.)

Since lexical NP's in principle cannot denote non-monotonic sets we conclude that NP is not lexically free.

We may note however that if NP_{prop}, the set of proper nouns (*John, Mary*), can be distinguished on syntactic grounds as a (sub)-category, then it is lexically free. Indeed, some would doubtless say that there are no syntactically complex proper nouns, in which case it is trivial that anything which can be denoted by a complex proper noun can be denoted by a simple one. There are however certain syntactically complex expressions which we might consider to be proper nouns. For example, certain AP + proper noun combinations, as *Little John, Mighty Mo*. Similarly Proper Noun + family name expressions such as *John Smith, Ebeneezer Cooke*. Also Proper Noun plus epithet constructions, as *Eric the Red, Charles the Bald*. But in all cases these complex expressions denote individuals and so do not denote anything which is undenotable by simple proper nouns. So NP_{prop}, if a syntactically definable subcategory, is lexically free.

2.1 A general characterization of lexical freedom

The arguments above that P_1 is lexically free and that NP is not are ones based more or less directly on our judgments of entailment and logical equivalence among English sentences. In assessing the lexical freedom of other categories the arguments will have this same informal character, as they do not depend crucially on adopting one or another semantic formalism. It is nonetheless important to see that our considerations can be made formally precise. We sketch in this section one way of doing that. The reader who is satisfied with the informal treatment of lexical freedom already given may skip this section without loss of continuity.

The notion of lexical freedom basically compares the range of extensional distinctions which can be expressed by the lexical items in a category with the range expressible by the entire set of expressions in the category. Now informally we have regarded two expressions of a given category C as extensionally distinct if we can ultimately distinguish their meanings in terms of the properties possessed by individuals, and more generally the relations which individuals bear to one another. More formally, we shall measure the range of extensional distinctions expressible by a category C by the sets in which expressions of category C denote in an extensional model of our language L. Exactly what sets these are depends on which individuals there are. Given a set I of individuals, we shall write $Den_I C$ for the set in which expressions of category C denote. For example, given I, and chosing C to be P_1, we may think of $Den_I C$ as the set of functions from I into the set {T, F} of truth values. So a P_1 then will denote some function which assigns T (= **true**) to some of the individuals in I and F (= **false**) to the others. Similarly, chosing C to be P_2 we may take $Den_I C$ to be the set of functions from pairs of individuals in I into {T, F}. So two P_2's are extensionally distinct just in case for some universe I of individuals, they assign different truth values to at least one pair of elements of I. In general in what follows we shall introduce the denotation sets $Den_I C$ on a category by category basis.

Here it is only important to note that the $Den_I C$ are defined strictly in terms of the set I of individuals and the fixed set {T, F} of truth values. Since I can be any (non-empty) set, we cannot give the range of extensional distinctions expressible by, say, P_1 in absolute terms. It depends on what I is chosen, and can only be expressed as a function of I. We can of course give this function precisely. If I is chosen with n (possibly infinite) members, there will be 2^n functions from I into {T, F}. In section 3 we shall define the relative "sizes" of categories in terms of how fast their denotation sets grow as a function of the number of individuals in the universe.

Given a universe I of individuals there are typically many ways of assigning elements of $Den_I C$ to expressions of category C. A specification of one of these ways basically defines a model for our language. Formally, a *model for* L is a pair (I, m), where I is a non-empty set of individuals[2] and m is a denotation function, called an *interpretation of L relative to I*, which assigns to each expression d of category C some element of $Den_I C$. The acceptable interpreting functions m are required to satisfy two types of condition. The first is Compositionality. Namely, the value of m at a syntactically complex expression d is determined by the value of m at the syntactic parts of d. So an interpreting function has no freedom in assigning denotations to syntactically complex expressions. The second type of condition limits the freedom in assigning denotations to lexical (= syntactically simple) expressions. In the first place, some expressions such as *every, be*, and *true* are "logical constants" in the sense that given two models (I, m) and (I, m′) with the same universe, we require that m and m′ assign the same values to these expressions. E.g. given I, there is only one function from I × I into {T, F} which is an acceptable denotation for *be*. Moreover, even for lexical items which are not logical constants we find that an interpreting function may not assign values with complete freedom: some expressions are constrained in their interpretation relative to the interpretation of others. For example, if m(*kill*) is true of a pair of individuals (I_1, I_2) the m(*die*) must assign T to I_2.

Nonetheless, when all these conditions on acceptable interpreting functions are given, it is still the case that many lexical items exhibit considerable freedom in which elements of their denotation sets they may be interpreted as (= denote). And in fact for any non-empty universe I, there is always more than one interpretation of L relative to I. That is, there are at least two models (I, m) and (I, m′) with the same universe I but with different interpreting functions m and m′.

With these preliminaries, we may formally define lexical freedom as follows (where 'C' in the definition ranges over the categories of expression considered):

Def 1: a. For all universes I and all $K \subseteq C$,
$Den_I K = \{ m(d): d \in K \text{ \& } m \text{ is an interpretation of L relative to I} \}$
b. C is *lexically free* iff for all universes I, $Den_I C_{lex} = Den_I C$

Recall here that C_{lex} is the set of syntactically simplest expressions of category C. So trivially $C_{lex} \subseteq C$, whence $Den_I C_{lex} \subseteq Den_I C$, all I. So to show that a category C is not lexically free it is sufficient and necessary to show that for some universe I of individuals, $\mathrm{Den}_I C$ properly includes $\mathrm{Den}_I C_{lex}$. That is, for some universe I, there is something which can be denoted by some expression in C but which cannot be denoted by any lexical expression of C. Conversely, to show that C is lexically free we must show that for all universes I, anything that can be denoted by an expression of C can be denoted by a lexical expression of C.

Notice also a notational subtlety which actually raises an interesting theoretical question (though not one we pursue here). Namely, given a category C and a universe of individuals I, we write $\mathrm{Den}_I C$ for some set in which the expressions of category denote. $\mathrm{Den}_I C$ is defined strictly in terms of I and {T, F}. $Den_I C$ however, defined in Def 1 above, in addition refers to all ways m of interpreting expressions of category C. Trivially $Den_I C$ is a subset of $\mathrm{Den}_I C$. We might like to require as an adequacy condition on approaches to model theoretic semantics for natural language that the reverse containment also hold. Why should we put elements in $\mathrm{Den}_I C$ which can never be the denotation of an expression of category C under any interpretation of L? Nonetheless, several approaches, e.g. Montague (1973), do not satisfy the reverse inclusion. Further, while the definition of model in Montague (1973) could be appropriately modified, it is not obvious that there is a natural way to design the definition of the $\mathrm{Den}_I C$, all I and all C, so that inclusion does obtain. It might happen for example that the natural and normally very simple definitions guarantee the inclusion when I is finite but leave some elements of $\mathrm{Den}_I C$ inherently undenotable when I is infinite. See K&M for some relevant discussion.

2.2 *The distribution of lexically free categories*

CLAIM 1: The categories in A. below are lexically free, those in B. are not

A. CN, CN_1, P_0, P_1, P_2, P_3, NP_{prop}, AP_{abs}, AP_1
B. NP, AP, Det_1, Det_2

We consider first Claim (1.A.). P_1 and NP_{prop} (if it is a category) have already been argued to be lexically free. The arguments that P_2 and P_3 are L-free follow the same monotonous pattern as for P_1, noting that in general

the ways of forming complex P_2's and P_3's are somewhat less productive than the ways of forming complex P_1's. We should however make explicit two restrictions we have been assuming on the class of P_n's we consider. First, we only consider "first order" predicates – ones which predicate of individuals rather than sets of individuals or more complicated objects. Thus we do not consider P_1's such as *love each other, be the two students I know best*, etc. which are most naturally thought of as predicating of sets of individuals. Nor do we consider P_2's such as *outnumber* which relates sets of individuals, not mere individuals. Further, this first order restriction applies where appropriate to other categories. For example, among AP's we do not consider ones such as *neighboring* and *parallel* as in *neighboring villages, parallel lines*, nor do we consider these CN's, as they do not denote properties of individuals but rather properties of sets of individuals.

Second, we limit ourselves to expressions which can be treated extensionally. E.g. we do not consider P_2's such as *seek* and *need*, nor do we consider AP's such as *skillful, fake*, and *apparent*. (Note that *skillful* and many other though not all, scalar AP's are not extensional in the sense that if the extensions of say *doctor and lawyer* are the same, i.e. the doctors and the lawyers are the same individuals, it does not follow that the extensions of *skillful doctor* and *skillful lawyer* are the same, i.e. the skillful doctors and the skillful lawyers may still be different individuals. We refer the reader to K&F (Keenan & Faltz, 1985) for a more thorough characterization of non-extensional subcategories of the categories we consider here.

Turning to the other categories mentioned in Claim (1.A), we have that P_0 is trivially L-free since clearly the simplest expressions of that category may be either true or false, so all elements of $\mathrm{Den}_I P_0$ are denotable by elements of $(P_0)_{lex}$.

The claim that CN is L-free is slightly more interesting. We have earlier referred to CN denotations as *properties* (of individuals). Extensionally a property may be thought of as a set of individuals (i.e. those which have the property) and we may take, to use the formalism of the previous section, $\mathrm{Den}_I CN$ to be the power set of I, the set of individuals of the model. Now to show CN to be L-free we must show that any way of forming syntactically complex CN's does not result in CN's which may denote properties inherently undenotable by lexical CN's. The most productive way of forming complex CN's is by modification with AP's, relative clauses, and PM's. But clearly, given any universe I of individuals, those who have the property expressed by *tall and handsome doctor* could be just those with the *bachelor* property, so CN's formed by AP modification do not take us outside the set of properties denotable by lexical CN's. Analogous claims hold for CN's such as *doctor who Susan kissed* and *child on the floor*, so CN's formed by relative clause and PM modification do not lead to anything extensionally new. Similarly, CN's formed from CN_1's plus NP arguments, as *friend* *of every senator*, are also not extensionally new. Clearly the individuals who are friends of every senator could be just the doctors. Finally, CN's formed by boolean combinations (not as productive as with many other categories) are not extensionally new. It is logically possible that the individuals with the property expressed by *non-doctor and non-lawyer* are just those with the vegetarian property. As this appears to exhaust the ways of forming complex CN's we conclude that the category CN is lexically free.

Consider now the less well studied case of CN_1's such as *friend (of)*. Extensionally we may think of them as denoting functions from individuals to properties (CN denotations). E.g. semantically *friend (of)* associates with each individuals J a property, friend of J. K&F investigate two ways of forming complex CN_1's: boolean combinations and modification. It is at least reasonable to consider that *friend and colleague (of)* is a complex CN_1 formed by conjoining *friend (of)* with *colleague (of)*. But it is clearly possible that for each individual J, the friends and colleagues of J are just the brothers of J, so *friend and colleague (of)* does not denote a function in principle undenotable by lexical CN_1's such as *brother (of)*.

Concerning modified CN_1's, K&F suggest that the classical ambiguity in *old friend of the President* may be represented by the scope of the modifier *old*. On the analysis on which *old* combines with the CN *friend of the President* it yields the property an individual has iff he is a friend of the President and he is old. In the case of interest, it combines with the CN_1 *friend (of)* to yield the CN_1 *old friend (of)*, which semantically maps each individual J to the property an individual has iff he has been a friend of J for a long time. But again there is no reason why, for each J, the old friends of J in this sense could not be the brothers of J, so *old friend (of)* does not denote any function which is undenotable by lexical CN_1's such as *brother (of)*. And as we can think of no further ways of forming complex CN_1's we conclude that CN_1 is a lexically free category.

The remaining categories listed in Claim (1.A) are adjectival and will be considered under the more difficult but more interesting Claim (1.B), to which we now turn.

NP has already been shown not to be lexically free. Consider Det (= Det_1). We may represent dets extensionally as functions from properties to sets of properties. E.g. *every* will denote that function which associates with each property p the set of those properties q common to all individuals with p. Now, to show Det to be not L-free we must find some property of lexical det denotations not shared by det denotations in general. (3) below, adapted from K&S, is intended as an essentially exhaustive list of lexical dets:

(3) every, each, all, both, neither, most, half (the), the_{sg}, the_{pl}, a, some, zero, one, . . . , twenty, no, several, a few, a score of, a dozen, finitely many, infinitely many, many, few, this, these, them, my, his

We have been generous (here and elsewhere) in counting certain expressions in (3) as syntactically simple so as to not make our claim that Det is not L-free depend on an unjustified choice of what we regard as syntactically simple. For similar reasons we have included several dets which cannot (see K&S) be treated extensionally, such as *many, few*, and the demonstratives (*this, my*). We may however think of *occurrences* of these expressions as being interpreted the same as various properly extensional dets. For example, *many* on any given occurrence will denote something like *more than n*, where n is some number determined by context. Similarly, in a given context, an occurrence of *this* identifies an individual J and associates a property p with J if J has p (otherwise it associates the empty set Ø of properties with p).

Now it is easy to see that most of the dets in (3) always denote increasing functions, where a function f from properties to property sets is increasing if its value at any property is an increasing set. However, *no, neither, few* (extensionalized as above) and (*just*) *finitely many* denote decreasing and not increasing functions.

It would appear then that like NP, lexical dets denote monotonic functions. We believe this to be correct, as do Barwise & Cooper (1981). However some (e.g. Thijsse, 1983) consider that bare numerals such as *two* are to be interpreted in the sense of *exactly two* and not in these of *at least two*, in which case they denote functions which are not monotonic. We prefer the "at least" sense: it is unnatural to think that a question such as *Are there two free seats in the front row?* would be truly answered *No* in a situation in which there were three free seats there.

However, even accepting that bare numerals have an "exactly n" reading, it is not difficult to show that Det is still not L-free. A general observation which establishes this is that the non-increasing dets in (3) are all "logical". That is, they are always interpreted by functions which are *automorphism* (*permutation*) *invariant* (AI). Informally, to say that a function f from properties to sets of properties is AI is to say that in deciding whether to put a property q in the set it associates with a property p, f ignores which particular individuals with p have q. f may however be sensitive to how many p's have q and more generally to what proportion of the individuals with p have q. To appreciate the difference between AI and non-AI dets consider the interpretative differences between (4a, b, c) below:

(4) a. Some doctor is a vegetarian
b. Every doctor is a vegetarian
c. John's doctor is a vegetarian

Clearly the truth of (4a) and (4b) is determined once we have specified which individuals are doctors and which are vegetarians. But that information is insufficient to determine the truth of (4c). For that we must in addition specify which individual John is and which individuals he "has". Thus the function which interprets *John's* makes a real world commitment as to which individuals have which properties and which ones are related to which others. So *John's*, in distinction to *some* and *every*, is not AI. (A properly general, i.e. category independent definition of AI is given in the Appendix. See also Westerstahl (1984) for a slightly different treatment, one not incompatible with ours).

We may then consider that lexical dets are either increasing or AI. And to show that Det is not L-free it is sufficient to find complex dets which are neither. There are many such. One class is given by possessive dets such as *only John's, neither John's nor Mary's, no student's, exactly two students'*. Another class is given by comparative AP dets such as *more male than female, fewer male than female*, etc. Exception dets are another group: *every ... but John* (as in *Every student but John left early*), *no ... but John, every ... but John's*, etc.

We conclude then that Det_1 is not lexically free. Analogous arguments show that Det_2 is not L-free. The simplest det_2's are ones like *more ... than ...* and satisfy the very weak condition of being either AI or increasing. But complex det_2's such as *more of John's ... than of Mary's ...* (as in *More of John's dogs than of Mary's cats*) and *more male ... than female ...* (as in *More male dogs than female cats*) are neither AI nor even monotonic.

We consider finally the more complicated case of AP's. Extensionally AP's may be represented by functions from (extensional) properties to (extensional) properties. K&F observe that extensional AP's always denote *restricting* functions, i.e. functions f such that all f(p)'s are p's. E.g. all female artists are artists, all tall doctors are doctors, etc. Within this class there are two semantically distinct sorts of lexical AP's: *absolute* ones and *relative* ones. We consider first the absolute AP's (AP_{abs}). They are illustrated by lexical AP's such as *male* and *female* as well as by more complex AP's of the relative clause sort (*who Sue kissed*). To say that *female* is absolute is to say e.g. that the female artists are just the artists who are female *individuals*. *Relative* AP's such as *tall* do not license this inference: A tall artist need not be a tall individual (i.e. "absolutely" tall), he need only be tall relative to artists. Note that two absolute AP's are extensionally distinct just in case their values at the property of being an individual are different. It is in fact easily seen (K&F) that the set of absolute functions is isomorphic to the set of properties (the function h sending each absolute function f to the value of f at the individual property being the isomorphism).

Given this correspondence between absolute AP's and properties it is perhaps unsurprising that syntactically they behave more like CN's than do properly relative AP's. E.g. they combine more freely with dets to form NP's, as in *The university hires more males than females* but **The university hires more talls than shorts*. There may then be syntactic grounds for distinguishing absolute AP's from others, in which case we may observe that the category AP_{abs} is lexically free. As the point has some independent interest in what follows let us establish that fact.

There are several ways of forming complex absolute AP's. For example, relative clauses (formed on extensional positions) are absolute. E.g. a student who Sue kissed is a student and an individual who Sue kissed. Similarly (though often awkward to form) boolean combinations of absolute AP's are absolute. E.g. a *non-male* student is a student who is a non-male individual. Third, comparatives of relative AP's with NP arguments are absolute: A student *taller than Bill*[3] is a student who is an individual taller than Bill. Equally, as Manfred Bierwisch (pc) pointed out to me, relative AP's combined with measure phrases such as *five feet tall* are absolute. A student five feet tall is a student who is an individual five feet tall, not just five feet tall relative to students but perhaps some other height relative to, say, vegetarians. Finally, and important in what follows, AP's formed from AP_1's plus NP's appear to be absolute. E.g. a student *fond of Mary* is a student who is an individual fond of Mary. A student *angry at every teacher* is a student who is an individual angry at every teacher. And since these complex AP's are absolute they do not denote functions inherently undenotable by lexical absolute AP's. I.e. it could be that for each p, the p's who Sue kissed are just the male p's.

We conclude then that if AP_{abs} is a grammatically definable subcategory it is lexically free. But the full category AP itself is not lexically free. To see this, consider that lexical relative AP's are not interpreted as arbitrary restricting functions. They must meet a variety of very strong conditions (see e.g. Kamp, 1975, and Bartsch, 1975 for much discussion). One such condition is exemplified in (5) below.

(5) *Continuity Condition:* if John and Bill are both doctors and also both lawyers and John is a tall doctor but Bill is not, then it is not the case that Bill is a tall lawyer and John not.

The antecedent of (5) guarantees that John is taller than Bill. If it were so that Bill was a tall lawyer and John not, that would imply in addition that Bill was taller than John, an obvious impossibility.

It appears then that lexical AP's must denote restricting functions which are either absolute or continuous as above. (In fact many additional conditions must be satisfied by the lexical relative AP's, but Continuity is sufficient for our purposes here).

Now it is not difficult (but it is slightly tedious) to show that boolean combinations of relative AP's need not satisfy Continuity. Consider e.g. *neither tall nor short*. Let p be a property possessed by just the individuals I_5, I_4, and I_3 where I_k is k feet tall. It is clearly possible that I_5 would be the only tall p, I_3 the only short one, and I_4 a neither tall nor short p. Now let q be the property possessed by just I_7, I_6, and the individuals with p, where I_7 is 7 feet tall, etc. It is surely plausible that the tall q's are just I_7 and I_6, the short q's are just I_4 and I_3, and the neither tall nor short q's are just I_5. But in such a case (more individuals can be added improving plausibility) Continuity fails. To see this, write f for the function denoted by *neither tall nor short*. Then we have that both I_5 and I_4 are p's and also q's. And I_4 is an f(p) and I_5 is not. But I_5 is an f(q) and I_4 is not, so *neither tall nor short* fails Continuity.

We infer then that AP is not lexically free. But note, as Dick de Jongh points out, that our argument relies crucially on the empirical claim that English possesses no lexical AP *blik* with the meaning of *neither tall nor short*. Might we not try such AP's as *average*, or *middling?* But clearly *an average man* is not synonymous with *a neither tall nor short man*. What is needed here is a complex AP such as *of average height*. It appears then to be a non-trivial fact concerning the expressive nature of English that it does not lexically codify the middle range of scales determined by relative AP's such as *tall* and *short*. So we are claiming that *blik* above with the meaning given is not a possible extension of English.

Finally, let us note, perhaps surprisingly, that AP_1 does appear to be lexically free. Extensionally such expressions (*fond (of)*, etc.) denote functions from individuals to AP denotations – in fact to absolute AP denotations. And there appear to be several ways of forming complex AP_1's. Some combine with intensifiers like *very*, as in *very fond (of)*. But obviously it could be that for each individual J, the property of being very fond of J was possessed by just those individuals who were proud of J, so *very fond (of)* does not extensionally denote a function in principle undenotable by a lexical AP_1 such as *proud (of)*. Similarly boolean combinations do not lead to extensionally new functions. It could be that, for each individual J, the individuals who are *fond but not envious of* J are just those who are *proud of* J. And finally, complex comparative forms such as *much taller than, twice as tall as* are not extensionally new. It could be that for each individual J, those who are twice as tall as J are just those who are fond of J.

We conclude then that AP_1 is lexically free and thus that Claim 1 is established.

3. EXPLAINING LEXICAL FREEDOM

We should like to know whether the fact that certain categories are L-free and others are not is merely an accidental fact about English or whether there is some principled basis for expecting just the distribution of L-

freedom given in Claim 1. More specifically, does the property of being L-free correlate with any other semantic property of the categories which might be used to predict, or at least lead us to expect, that the L-free categories are just those indicated?

We shall show here that L-freedom correlates with the "size" of a category, and moreover, that this property may plausibly be used as a basis for explaining the distribution of L-freedom across categories.

Regarding "size", observe that in a model with n individuals, there are 2^n extensionally distinct CN denotations, one for each set of individuals. Similarly there are just 2^n extensionally distinct P_1 denotations and 2^n AP_{abs} denotations. Further, the set of extensional P_2 denotations (as well as the CN_1 and AP_1 denotations) corresponds to the sets of ordered pairs of individuals, whence there are 2^{n^2} such. Similarly there are 2^{n^3} extensional P_3 denotations. These facts and others are summarized in the table below, where we write n for the cardinality of the set I of individuals of the model and, as before, $Den_I C$ is the set of possible denotations of expressions of category C in a model with universe I.

(6) *Lexically Free Categories C*

	P_0	NP_{prop}	P_1	CN	AP_{abs}	CN_1	AP_1	P_3
$\lvert Den_I C \rvert$	2	n	2^n	2^n	2^n	2^{n^2}	2^{n^2}	2^{n^3}

By contrast consider the comparable figures for categories which are not L-free:

(7) *Lexically Restricted Categories C*

	NP	AP	Det_1	Det_k
$\lvert Den_I C \rvert$	2^{2^n}	$2^{n \cdot 2^{n-1}}$	2^{3^n}	$2^{(2^{k+1}-1)^n}$

Justifying the figures in (7) is rather more difficult than for those in (6) and raises one question of some theoretical interest.

Consider the figure given under the heading NP. In a model with n individuals (n may be infinite) there are 2^n properties (sets of individuals) and thus 2^{2^n} sets of properties. But are all of these properties sets possible full NP denotations? That is, given any property set, can one guarantee that there is an English NP which may be interpreted as that set? The answer to this question (and the comparable ones for the other categories above) is far from obvious, and requires a serious study of the actual expressive power of English. Several such questions are investigated in K&M and several partial results are obtained, but equally several questions remain open. In the case at hand, NP, we may infer that the answer is yes when n is finite. If n is infinite it is unknown whether an arbitrary infinite property set is denotable by an English NP under some acceptable interpretation.

The positive answer for the finite case follows directly from a theorem in K&S which gives a positive answer for Det_1. Note first that Det_1's denote functions from properties to sets of properties. The total number of such functions is thus 2^{2^n} raised to the power 2^n, which computes out to 2^{4^n}. However K&S argue on empirical grounds that not all of these functions are needed as denotations for English Det_1's. Rather we only need functions f which satisfy the Conservativity condition given below:

(8) f is *conservative* iff for all properties p, q,
$p \in f(q)$ iff $(p \wedge q) \in f(q)$

In effect the condition says e.g. that *every doctor is a vegetarian* iff *every doctor is both a doctor and a vegetarian*, and moreover, the equivalence remains true no matter what English Det_1 is substituted for *every*. We may compute then (K&S, Thijsse 1983) that only 2^{3^n} functions satisfy the Conservativity condition. But is this condition sufficient? That is, is every conservative function a possible Det_1 denotation, or may we impose still further conditions? K&S show that for n finite no further conditions may be imposed. That is, given any conservative function f over a finite universe, K&S provide a way of constructing an English Det_1 which may be interpreted as f. This then justifies the figure given under Det_1 in (7), a point of some importance in what follows.

It is now easy to see that the figure in (7) for NP is accurate. For the nonce, write 1 for that property which all individuals have. So 1 is arguably the denotation of the CN *individual* (or at least the complex CN *object which is either animate or inanimate*). It is then easy to prove that for any set Q of properties, the function f_Q which sends 1 to Q and all other properties to $\emptyset$, the empty set, is conservative. It follows then that *d individuals* will denote Q, where d is the English Det_1 interpreted as f_Q whose existence is guaranteed by the theorem in K&S.

The figure given for AP in (7) is empirically more problematic. That figure is the number of restricting functions from properties to properties, where, recall, such a function f is restricting iff for each property p, the individuals with f(p) are a subset of those with p. But as we have seen, lexical AP's are not freely interpreted in the set of restricting functions. Some, such as *male* and *female*, may only be interpreted by absolute functions. Others, the relative AP's like *tall*, must meet the Continuity condition (5), and in fact must meet somewhat stronger conditions. The issue then is whether we have sufficiently rich ways of forming complex AP's so as to allow us to denote any restricting function (at least over a finite universe). Under one plausible set of assumptions concerning these stronger conditions, which space prevents us from presenting here, we may prove that any restricting function may be built up as a finite boolean combination of ones meeting

the stronger conditions. To this extent then the figure given in (7) under AP is reasonable. But more work is needed to determine whether we might not be able to impose even stronger conditions yet on the interpretations of lexical AP's.

Finally, the general figure given for Det_k's in (7) is simply the number of k-place conservative functions given in K&M. We have no proof comparable to that in K&S for Det_1's that, even over a finite universe, any k-place conservative function can actually be expressed in English. However, for purposes of what follows, it is sufficient that the figure given for Det_k be at least as large as that given for Det_1, and a quick perusal of K&M shows that to be the case.

Consider now the significance of the figures in (6) compared with those in (7). It is obvious that the denotation sets for L-restricted categories grow much faster as a function of the size, n, of the universe than is the case for L-free categories. For example, choosing n infinite, the denotation set for any L-restricted category has size 2^{2^n}, which is strictly larger than that for any L-free category, whose size is always $\leq 2^n$. In fact, for all but the smallest universes ($n \leq 9$) the denotation set for any L-restricted category is strictly larger than that for any L-free category.

Let us define then a category C to be *small* if its denotation set always has size $\leq 2^{f(n)}$, where f(n) is a polynomial function in n. And call C *large* if its size is always $2^{g(n)}$, where g(n) is itself a non-trivial exponential function in n. We then have the following generalization:

GEN 1: Small categories are lexically free, large ones are not[4]

Note that GEN-1 is simply an empirical generalization about English. It has perhaps the status of a "law" in (roughly) the sense of Werner's Law or Grimm's Law – namely, an empirical regularity in the observed data.

It does however suggest an explanation for the distribution of lexical freedom among our categories: Let us think of the relative sizes of the denotation sets as a *measure* of the relative complexity in learning the meaning of, and interpreting on an occasion of use, an arbitrary expression in the category. Such a measure has some intuitive appeal. Correct application of an arbitrary expression from a small category requires being able to discriminate among many fewer states of affairs than for an arbitrary expression from a large category. E.g. assuming for the moment perfect knowledge of the world, imagine that we are faced with the use of a new vocabulary item, say, *blik* used as a P_1, as in *John bliks*. To know whether the statement is true or not we need merely be able to discriminate among 2^n possible states of affairs, where n is the number of individuals in our relevant universe of discourse. But if our interlocuter asserted *Blik students are vegetarians* we should have to be able to discriminate among 2^{3^n} possible states of affairs (were the category Det_1 is not lexically restricted).

Further, the learning and use problem can be expected to be greater for lexical items in a category then for syntactically complex ones, since the interpretation of complex expressions is (modulo idioms, etc.) determined as a function of the interpretations of their syntactic parts. But for lexical items there precisely are no syntactic parts, so the meanings of these elements must be learned directly. Thus the problem of determining the meaning distinctions among lexical expressions in a large category is reasonably considered more difficult than that for small categories. It is then cognitively advantageous in learning and using the meanings of expressions in large categories if the learners and users can assume that the lexical expressions in those categories do not denote freely in the set in which expressions in that category in general denote, but rather they may only denote in a very limited subset of that set.

While these considerations are admittedly speculative, they receive further support from the fact that in general it appears that the set of possible denotations we need for the set of lexical expressions of a large category is itself small.

Consider first Det_{lex}, the set of lexical one place dets. Several of these dets are deictic (*this, these, my, your*). That is, we understand that their reference is given by the context in which they are used. Reasonably then the use problem here is minimal. Equally "pronominal" dets such as *his, their* have their reference provided by the linguistic or non-linguistic context of use. Calling all of these elements deictic, we see that the learning and use problem for lexical dets largely reduces to that for the non-deictic items. Now most of those items are in fact logical constants. They are "logical" in the sense that they denote AI functions, and they are constant in that mostly there is only one function from properties to property sets which they denote (in each case). The learning and use problem then virtually reduces to that of learning the meanings of a handful of items – say 35 to be safe, and is not of the order expressed by 23^n, the size of Den_IDet. However, certain lexical dets such as *several* and *a few* appear not to be constant, though they still denote AI functions. But even if they denoted freely in the AI set (which they don't – e.g. *several* is increasing), we may compute (K&S, Thijsse, 1983), that there are only $2^{(n+1)(n+2)/2}$ such functions. Thus we may infer that the denotation set for Det_{lex} is by our definition small.

Equally it is not hard to see that the set in which lexical NP's denote is small. Again excluding deictic items (*this, that, I, we mine*) and deictic/anaphoric items (*he, they*) we observe that lexical NP's are either proper nouns, and so denote individuals, or else belong to a small finite set of essentially logical constants (*everyone, noone, someone,* etc.). Basically then the measure of the learning and use problem for lexical NP's is given by n + c, n the number of individuals and c the number of lexical logical constants. Note that even if further logical elements are found among NP_{lex} and they could denote freely in the AI property sets, there are only 2^{n+1} such, so even in this case we may infer from our definition that $Den_I NP_{lex}$ is small. Thus we suggest:

GEN 2: For C large, the set in which non-deictic lexical elements of C denote is small

Note that Det_k's for k > 1 further support GEN-2. At best there are a handful of logical constants (*more ... than ...*) among the (non-deictic) lexical Det_2's. So the magnitude of the learning problem here is given by a constant c, c the number of lexical det_2's. And even if further work revealed the existence of logical Det_k's freely interpreted in the AI set, the number of such would still be technically small: 2 raised to the power $\binom{n+2^{k+1}-2}{n}$. (Thanks to Johan van Benthem for determining this figure for us.)

There is one open problem concerning GEN-2 however, namely the size of $Den_I AP_{lex}$. To be sure the absolute AP's denote in a set of size 2^n so their denotation set is small. But how many additional functions are needed to provide denotations for lexical AP's such as *tall?* At the moment we do not know.

4. A GENERALIZATION OF LEXICAL FREEDOM

The distinction we have drawn between L-free and L-restricted categories is rather naturally seen as a special case of a more general distinction (pointed out to us by Peter van Emde Boas). The basic idea behind lexical freedom is that complex expressions in a category may extensionally denote things which the simplest expressions of that category cannot denote. Generalizing, given a category C, may we continually denote new things as the complexity of expressions in C is increased, or rather is there a complexity bound beyond which new expressions will not let us denote anything that could not already be denoted?

To formulate the question more precisely, let us represent the complexity of an expression by its *length*, as measured by the number of lexical items in it (which is reasonable, given the essentially context free grammar for our expressions in K&F). Now, for C a category and k a positive integer, write C_k for the set of expressions in C of length ≤ k. And recall from Def 1 that for a universe I of individuals, $Den_I C_k$ is the set of possible interpretations of elements of C_k relative to the universe I. Then,

Def 2: k is an *extensional bound* for a category C iff for all sets I of individuals and for all k′ > k, $Den_I C_{k'} = Den_I C_k$

If C has no extensional bound it will be called *unbounded*, otherwise it is *bounded*, and the least such bound will be called *the bound for* C.

Note that any L-free category is bounded; it's bound is the length of the syntactically simplest expressions in C (e.g. 1, if C has proper lexical expressions). But if C is not L-free it does not follow that it is unbounded. So our precise question becomes: Which L-restricted categories are unbounded? Part of the answer is given by:

Thm 1: Det_1 is unbounded

The theorem follows as an easy corollary to the following two theorems from K&S:

(9) *The Finite Effability Theorem*
For each conservative function f over a finite universe of individuals there is an expression d in Det_1 which denotes f under some acceptable interpretation of L.

(10) *The Finite Ineffability Theorem*
For any finite subset D of Det_1 there is a finite universe I of individuals such that some conservative functions over I are undenotable by any d in D.

(Note that in (9) we construct the determiner expression d once f is given; in (10) we fix the determiner expressions D in advance.)

Thm 1 follows from (9) and (10) with the additional assumptions that Det_{1ex} is finite, that there are only finitely many ways of forming complex Dets, each one of which only introduces finitely many new expressions. These latter assumptions are satisfied by any reasonable grammar of English Dets. It then follows that for each positive integer k, $(\mathrm{Det})_k$, the set of Det_1's of length $\leq$ k, is finite. So let k be arbitrary. Then by the Ineffability Theorem, there is a finite universe I for which there are conservative functions undenotable by any element of $(\mathrm{Det})_k$. But by the Effability Theorem any of those functions is denotable, and thus denotable by a Det_1 of length greater than k. Thus $Den_I(\mathrm{Det})_k$ is a proper subset of some $Den_I(\mathrm{Det})_{k'}$, for some k′ greater then k, proving the theorem.

We might note that the basic reason that the Ineffability Theorem for Det_1 holds is that the set of possible Det_1 denotations increases so much more rapidly than the denotation sets for the lexical items occurring in a Det_1. E.g. let $d = (d_1, \ldots, d_k)$ be a single Det_1 of length k. An upper bound on the set of possible denotations for d is given by the cross product for those of the d_i occurring in d. And as K&S show, this product, call it d(n), is sufficiently small compared to 23^n, the number of possible Det_1 denotations in a world of n individuals, that the limit of $d(n)/23^n$ goes to zero as n increases. And this implies that for sufficiently large finite n, d(n) is smaller (in fact as much smaller as we like) than 23^n. Whence there are possible Det_1 denotations which are not possible denotations for our fixed d. The proof remains essentially unchanged when d is replaced by a finite set D of Det_1's.

Further, though we lack a complete formal proof, it seems likely that NP ($=\mathrm{Det}_0$) is also unbounded. As a first step towards showing that NP is unbounded, we note that we have a Finite Effability Theorem for NP. In fact we have a stronger result:

Thm 4: For every finite universe I of individuals there is an interpretation m of L such that every element of Den_INP is denotable under m. That is, there is a subset D of NP such that m[D] = Den_INP.

The essential step in Thm 4 relies on the lemma below, also used in the proof of Finite Effability for Det_1.

Lemma There is a fixed subset D of NP such that for any countable set K of individuals, m[D] = K, for some interpretation m of L.

We may choose D in the lemma to be $\{np_k : k \geq 0\}$, where np_0 = *John* and np_{j+1} = *the oldest friend of* np_j, all j. It is clearly possible, given a denumerable sequence $J_1, J_2, \ldots$ of individuals, that J_2 is the oldest friend of J_1, and that in general J_{j+1} is the oldest friend of J_j.

As a consequence of the lemma, all of any finite number of properties are denotable (under a fixed interpretation). To denote a property q for example, construct the CN *individual who is either* np_i *or* np_j *or* . . . , where the finite disjunction of np's is taken over those in D which denote just the individuals with q. Call this CN cn_q. Equally we may denote q′ (the property of being a non-q) by the CN *individual who is not* [np_i or np_j or . . .], where again the np_i's are just those denoting individuals with q. Call this CN *non*-cn_q. Then the unit set {q} is the denotation of *every* cn_q *and no non*-cn_q. Call this NP NP_q. Then any finite set Q of properties is denoted by the finite disjunction of the NP_q's for q in Q. And this gives us the Finite Effability Theorem for NP (as well as CN).

The basic step remaining in the argument that NP is extensionally unbounded is to show that for any fixed d in NP, the number of possible ways of interpreting d increases less rapidly than 22^n, the total number of possible NP denotations. If we can establish this point, then given any d in NP, we can choose n large enough (but still finite) that some property sets are undenotable by d under any acceptable interpretation. And what holds of a single such d will extend as per the proof in K&S to any finite subset

D of NP, yielding a Finite Ineffability Theorem for NP. Then the proof that NP is unbounded follows exactly that for Det_1.

The basic argument that finite ineffability holds for NP proceeds as follows: We argue that for any finite set I of individuals and for any given NP d, Den_Id, the set of possible interpretations for d, is small, that is, it is of size $2^{f(n)}$, where n is the cardinality of I and f(n) is a mere polynomial function in n. (In fact a polynomial of degree $\leq$ 3.) So let I and d be arbitrary as above. Then d is representable as a concatenation of k lexical items ($d_1, \ldots, d_k$), and an upper bound on $|Den_I d|$ is given by the product of the $|Den_I d_i|$ and is small if each $|Den_I d_i|$ is small, since a product of numbers of the form $2^{f(n)}$ is itself of the form $2^{g(n)}$, for g(n) a polynomial in n.

Now suppose first that none of the d_i's is of category AP. Then for each d_i, either d_i lies in a small category, in which case $|Den_I d_i|$ is small, or d_i is a lexical NP or Det_k, in which case again, as previously shown, $|Den_I d_i|$ is small, or else d_i is a logical constant (e.g. *and*, etc.) in which case $|Den_I d_i| = 1$ and so contributes nothing to Den_Id. Thus $|Den_I d|$ is small when d contains no AP's.

This argument does not go through however if some d_i is an AP, since we do not know how many functions are possible lexical AP denotations. However by investigation (see the list of Dets in K&S and K&M) of the ways in which AP's may occur in NP's we may conclude that, surprisingly perhaps, AP's contribute nothing to the total number of ways of (extensionally) interpreting an NP.

To see this point, consider first an occurrence of an AP as a modifier, as in *every tall student*. Though there are an unknown number of ways of interpreting *tall, tall student* occurs here as a CN and there are at most 2^n ways of interpreting a CN, whether lexical or complex. Since there is only one way of interpreting *every*, there are at most 2^n ways of interpreting *every tall student*, just the same as the number of ways of interpreting *every student*. Thus we may say that the AP *tall* does not occur essentially in *every tall student*. That is, it contributes nothing to the set of possible ways of interpreting *every tall student* over and above what is inherent in *every student*.

Moreover, though the point is not completely obvious, we claim that all occurrences of AP's within NP's are inessential in this sense.

Obviously enough, AP's occurring within other modifiers, such as relative clauses (*student who is tall, student who is a tall carpenter*) contribute nothing to the total, since the relative clause is itself a modifier and thus the entire CN's only admit of at most 2^n possible interpretations.

Following however the generous syntax in K&S, we do find apparently "free" occurrences of AP's within Dets, as exhibited in (11) below:

(11) a. Only the liberal but not the conservative (delegates voted for Smith)
b. Neither the tall nor the short (students were chosen)
c. More tall than short (students passed the exam)
d. (John's biggest) cows
e. (More of the liberal than of the conservative) (delegates ...)

Here K&S consider that the AP has combined with the Det to form a complex Det. These occurrences of AP's do contribute (an unknown) degree of freedom with regard to the range of possible interpretations for the Det's they form, but still contribute nothing to the range of possible full NP interpretations, as can be seen by inspection of cases. Thus the NP's in (11) exhibit the same range of interpretations as their counterparts in (12) where the AP's do occur as modifiers.

(12) a. Only the liberal delegates but not the conservative delegates ...
b. More tall students than short students ...
c. The (biggest (cow which John has))

Clearly for example, Den_I(12a) is small, being $2^n.2^n = 2^{2n}$, since the only freedom of interpretation is given by the two CN's *liberal delegates* and *conservative delegates*, 2^n in each case, everything else being a logical constant. Analogous claims hold for (12c) and (12d).

We conclude then that the range of possible denotations for any NP over a finite universe is small. And if D is a finite set of such NP's its set of possible denotations is bounded by the product of its elements and is thus also small. We may infer then that for n sufficiently large, 22^n, the size of the set of denotable property sets, is larger than the set of possible denotations for any antecedently selected (finite) set D of NP's. We conclude then that, like Det_1, NP is an extensionally unbounded category. As we form increasingly complex NP's we may increasingly denote new sets of properties.

We are not however in a position to extend our argument and draw similar conclusions for the remaining large categories, Det_2 and AP. As regards Det_2 we lack a Finite Effability Theorem, and the one given for Det_1 in K&S does not immediately extend. And as regards AP, we also lack a Finite Effability Theorem, but we are optimistic that further research will yield one. $Den_I AP$ is defined as the boolean closure of $Den_I AP_{lex}$. Consequently for I finite, if every element of $Den_I AP_{lex}$ can be denoted, then any element of $Den_I AP$ can be denoted simply by forming finite boolean combinations of elements of AP_{lex}. But whether every element of $Den_I AP_{lex}$ can be denoted is an open question. We have after all only a fixed (finite) number of lexical AP's and these are not independently interpretable, e.g. we cannot interpret *short* and *tall* independently. So chosing n greater than the number of lexical AP's would more than guarantee that the number of restricting functions would exceed the number of lexical AP's. However, we may well be able to form sufficiently many complex AP's which are not absolute. E.g. *very tall, very very tall,* . . . , *somewhat tall*, etc. seem not to be absolute. Whether these processes are sufficiently productive and permit sufficient freedom as regards their interpretation to guarantee that all of finitely many elements of $Den_I AP_{lex}$ can be denoted (under a fixed interpretation) awaits further research.

NOTES

1. For simplicity of presentation we shall treat P_1's and P_n's in general as interpreted by total functions rather than ones that are only defined on proper subsets of the set of individuals. We do not think that generalizing predicate interpretations in this way will affect our results.
2. For convenience we present the set I as a primitive of the model. On most approaches to formal semantics I would not be a primitive. E.g. on most approaches we would take a non-empty set E of entities as primitive. Then for each b in E we define I_b, or the *individual generated* by b, to be the set of subsets of E which have b as an element (that is, the set of principal ultrafilters over E). On the approach taken in Keenan & Faltz (1985) we take a complete atomic algebra P as primitive (It is the set of possible CN denotations) and define the individuals as the filters generated by the atoms of P. These two approaches yield isomorphic results and differences among them are irrelevant for our purposes here.
3. Tom Wasow (pc) points out that a student *taller than Bill* and *a taller student than Bill* are not logically equivalent. The latter requires that Bill be a student and the former does not. We may interpret *taller . . . than Bill* as a function whose value at a property p is the same as that which interprets *taller than Bill* at p provided p is a property of Bill. Otherwise its value at p is the zero property (that one which no individual has). Interpreted thus *taller . . . than Bill* would be a kind of "restricted" case of an absolute AP though not strictly absolute. Adding such functions to our set of possible AP denotations will not, we feel, alter the claims made about AP's in later sections. We do not however add them, as their distribution is so restricted. E.g. it is unnatural to say *every taller student than Bill*, etc. We prefer an analysis then on which *a taller . . . than Bill* is thought of as a complex Det_1, but do not pursue this analysis here.
4. An alternative generalization here, more appealing perhaps to logicians, is:

GEN 1′: First order categories are lexically free, higher order ones are not.

Certainly the denotation sets for L-free categories may be represented, up to isomorphism, as relations (of various degrees) on the set I of individuals of the model. Denotation sets for L-restricted categories on the other hand are only representable as relations on the power set of I. Taking this characterization of first vs. higher order, GEN 1′ seems supported. Moreover, Johan van Benthem points out that, under a suitable characterization of "order", GEN 1 and GEN 1′ are essentially equivalent.

APPENDIX

A. *The general definition of automorphism invariance (AI)*

Let h be a permulation of the universe I. (That is, h is a one to one function from I onto itself.) Extend h to {T, F} by setting h(T) = T and h(F) = F. Call such a function a basic automorphism. Now extend h to an automorphism on all denotation sets as follows:

(i) if h has been extended to a set A, then for each subset B of A, set $h(b) = \{h(b): b \in B\}$

(ii) if h has been extended to each of $A_1, A_2, \ldots, A_k$, extend h to their cross product by setting $h(a_1, \ldots, a_k) = (h(a_1), \ldots, h(a_k))$, all a_i in A_i.

As any denotation set is defined by taking power sets, subsets, or cross products beginning from I and {T, F}, h as extended is easily seen to be an automorphism of any denotation set (when restricted to that set). Note we use h abusively for the basic automorphism as well as its extension.

Now, an element f in any denotation set is *automorphism invariant* iff for all basic automorphism h, h(f) = f.

It is easily proven for the class of models we consider (K&S) that any expression e which meets the condition that for all models (I, m) and (I, m′) m(e) = m′(e) always denotes an AI element of its denotation set. The converse may fail, and does. E.g. *several* may denote differently in different models with the same universe, but it always denotes an AI element of its denotation set.

REFERENCES

Bartsch, R.: (1975), 'Subcategorization of Adnominal and Adverbial Modifiers' in *Formal Semantics for Natural Language*, E. Keenan (ed) CUP.

Barwise, J., & Cooper, R.: (1981), 'Generalized Quantifiers and Natural Language' *Linguistics and Philosophy* 4.1 pp 159–219.

Kamp, J.A.W.: (1975), 'Two Theories about Adjectives' in *Formal Semantics for Natural Language*. Reidel, Synthese Language Library. Dordrecht, Holland, 1985.

Keenan, E., & Moss, L.: (1984), 'Generalized Quantifiers and the Expressive Power of Natural Language' in *Generalized Quantifiers* J. van Benthem & A. ter Meulen (eds). Foris Publications, Dordrecht, Holland.

Keenan, E., & Stavi, J.: (1981): 'A Semantic Characterization of Natural Language Determiners' to appear in *Linguistics and Philosophy*.

Montague, R.: (1970), 'English as a Formal Language' reprinted in *Formal Philosophy* R. Thomason (ed) 1974. Yale University Press, New Haven, original.

Montague, R.: (1973), 'The Proper Treatment of Quantification in Ordinary English' in *Approaches to Natural Language* J. Hintikka, J. Moravcsik, P. Suppes (eds). Reidel, Dordrecht, Holland.

Thijsse, E.: (1983), 'Laws of Language: Universal Properties of Determiners as Generalized Quantifiers' Doctoraalsciptie Rijksuniversiteit Groningen, Holland.

Westerstahl, D.: (1984), 'Logical Constants in Quantifier Languages' to appear in *Linguistics and Philosophy*.