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SEMANTIC ORDER
&
SEMANTIC ANSWERS TO SYNTACTIC QUESTIONS

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Let C be a grammatical category {S, NP, VP, ...} of natural language, and consider the collection of things we may reasonably think of expressions of category C as denoting (= being semantically interpreted as). Experience shows that in general this set is not just some random collection -- rather its elements are *ordered* in a certain, usually quite specific, way.

We shall be concerned in this article with several semantic generalizations about English (and hopefully natural language in general) which build on the notion of a semantic order. The generalizations all concern, at least as special cases, the interpretation of NPs or quantifiers. Most of them are available in the literature, though some more accessibly than others.

Our purpose here is expository: to make these generalizations accessible to the non-specialist and to exhibit the sense in which semantic analysis may contribute to the solution of problems which arise in a syntactic setting. In addition we unify the generalizations by presenting them all in an order theoretic perspective.

1. On the notion of order

As an illustrative example consider (tensed) VPs, such as those italicized in (1):

(1) a. John *laughed*
    b. John *laughed loudly*
    c. John *both laughed and cried*
    d. John *either laughed or cried*

The VPs of these Ss are semantically related in some obvious ways. Extending standard usage, the VP of (1b) "entails" that of (1a), meaning that whenever *laughed loudly* is true of an individual then so is *laughed.* One standard way to say this is as follows:

In a given a situation σ, a VP of the sort in (1) is true of some (possibly all) of the objects we might be talking about in σ. For p a VP expression write $\mathbf{p}_\sigma$ for the set of things p is true of in σ. Write E_σ, called the *universe* of σ, for the set of things we might be talking about in situation σ. (We usually omit the subscript σ since we are not comparing different situations).

And we define: a VP p *entails* a VP q iff for all situations σ, $\mathbf{p}_\sigma \subseteq \mathbf{q}_\sigma$. So to say that *laughed loudly* entails *laughed* is to say (omitting subscripts) that in all situations σ, **laughed loudly** $\subseteq$ **laughed**. That is, any object that *laughed loudly* is true of is an object that *laughed* is true of.

exercise Verify informally that *both laughed and cried* entails *laughed* and that *laughed* entails *either laughed or cried* #

Note that *laughed* does not entail *slipped,* since there are situations in which **laughed** $\not\subseteq$ **slipped**, that is, situations in which someone laughed who didn't slip. But there are also situations in which **laughed** $\subseteq$ **slipped**, that is, ones in which everyone who laughed did slip. Thus in a given situation some VP denotations stand in the subset relation and some do not.

Now the subset relation is a basic *order* relation: *transitive* (if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$), and *antisymmetric* ($A \subseteq B$ and $B \subseteq A \Rightarrow A = B$). Standardly,

Def 1 A binary relation R defined on a set E is called an *order* relation iff

i. R is transitive
(viz. for all $a,b,c \in E$, aRb & $bRc \Rightarrow aRc$) and

ii. R is antisymmetric
(viz. for all $a,b \in E$, aRb & $bRa \Rightarrow a = b$)

Note that antisymmetry rules out that distinct objects each bear the relation to the other, but it allows that a given object bear the relation to itself. Indeed the subset relation is *reflexive,* meaning that for each set A, $A \subseteq A$.

And in general the order relations we are linguistically motivated to consider below are reflexive. We often use $\leq$ as a symbol denoting a reflexive order relation. (Note e.g. that the natural $\leq$ relation among numbers in arithmetic is a reflexive order). Anticipating a more general use, we write $\leq_{VP}$ for the reflexive order relation on VP denotations defined above. (So $\leq_{VP}$ is just the subset relation as we presented it.)

A second basic order relation is the "implication" order $\leq_S$ defined on possible Sentence denotations:

Let x and y be possible S denotations (in a situation σ). Then x $\leq_S$ y iff an arbitrary sentence of the form *if p then q* is true when p denotes x and q denotes y. For example, thinking of Ss as denoting either **T** ("True") or **F** ("False"), we see that the $\leq_S$ relation is completely given by:

(2) $\mathbf{T} \leq_S \mathbf{T}$, $\mathbf{F} \leq_S \mathbf{T}$, and $\mathbf{F} \leq_S \mathbf{F}$.

The only case where a truth value x fails to bear the "implication" relation to a truth value y is when x = **T** and y = **F**. So to show that x $\leq_S$ y we must merely show that if x = **T** then y = **T** (since if x = **F** then x $\leq_S$ y no matter what truth value y is).

And as with VPs, we define: a sentence p *entails* a sentence q iff for all situations σ, $\mathbf{p} \leq_S \mathbf{q}$, where **p** and **q** are the respective denotations of p and q in σ.

One sees by inspection of (2) that $\leq_S$ as defined is reflexive (= for all x ε {**T**,**F**}, x $\leq_S$ x). Equally no two different truth values each stand in the $\leq_S$ relation to the other, so $\leq_S$ is antisymmetric. And transitivity is checked by cases in (3). To show that if x $\leq_S$ y and y $\leq_S$ z then x $\leq_S$ z, for x, y, z ε {**T**,**F**}, it is sufficient to consider the choices of values for x, y, z which make the "and" clause true.

(3)

x $\leq_S$	y and	y $\leq_S$	z	x $\leq_S$	z	??
T	**T**	**T**	**T**	**T**	**T**	yes
F	**T**	**T**	**T**	**F**	**T**	yes
F	**F**	**F**	**T**	**F**	**T**	yes
F	**F**	**F**	**F**	**F**	**F**	yes

Our concern now is with denotations of NPs, such as those italicized in (4). Such NPs combine with VPs to form Ss and may naturally be interpreted by functions which map VP denotations to S denotations (the truth value of such a sentence then being the value that the NP function assigns to the VP denotation).

(4) a. *John* is asleep
b. *Most students* can read
c. *More students than teachers* read the Times

Def 2 Now, if F and G are possible NP denotations (in a situation σ) we say that F $\leq_{NP}$ G iff for all possible VP denotations **p**, F(**p**) $\leq_S$ G(**p**). (That is, if F(**p**) = **T** then G(**p**) = **T**).

fact The $\leq_{NP}$ relation defined above is (for each σ) a reflexive order relation (see below).

And the entailment relation on NPs is defined as before: An NP A entails an NP B iff for all situations σ, **A** $\leq_{NP}$ **B**, where **A** and **B** are the respective denotations of A and of B in σ.

For example, *every student and some teacher* entails *every student* since for any VP q, if *every student and some teacher q* is true then, obviously, *every student q* is true. And more generally,

(5) For C ε {NP, VP, S} and for x,y expressions of category C,

i. *both x and y* entails *x* and
ii. *x* entails *either x or y* #

The $\leq_{NP}$ order builds on the $\leq_S$ order on the set, {**T**,**F**} in which NP functions take their values. This way of inheriting orders is fully general:

Def 3 Let A be any set and let $\leq_B$ be a reflexive order on a set B. Then we define a relation $\leq$ on [A $\rightarrow$ B], the set of functions from A into B, as follows:

For all f,g $\in$ [A $\rightarrow$ B],

$$f \leq g \text{ iff for all } b \in B,\ f(b) \leq_B g(b)$$

fact: $\leq$ as defined is a reflexive order.

2. Some semantic generalizations

2.1 Constraints on interpreting lexical items

We will establish here a very non-trivial semantic constraint on the interpretation of lexical NPs -- one that extends with some success to lexical expressions of other categories.

To set up the generalization let us consider first the following entailment paradigm (cf. Aristotle).

(6) a. All socialists are vegetarians
b. Some doctors are socialists
∴ Some doctors are vegetarians

We understand (6) to mean that the first two Ss jointly entail the third. That is, in any situation in which the first two are interpreted as true the third is also interpreted as true.

Query Which NPs X can replace *some doctors* everywhere in (6) preserving the entailment (changing plurals to singulars if necessary)?

NPs which satisfy the **Query** are called *increasing*. For example *Mary* is increasing: if all socialists are vegetarians and Mary is a socialist then, obviously, Mary is a vegetarian. Some other increasing NPs are given in (7), as the reader is invited to check:

(7) a. she, this cat, more than two cats, at least one cat, some cat, every cat, the (ten) cats, John's (ten) cats, most cats, several cats, more than half the cats, his cat, every student's bicycle
b. at least two of the ten cats, most of John's cats, at least two thirds of the students, more than five of John's cats
c. John and some student, at least two teachers and more than ten students, either a student or a teacher, most liberal and all conservative senators

To give a properly general account of these entailment facts consider the semantic representation of (6) below (given a situation σ):

(8) a. **socialist** $\subseteq$ **vegetarian**
b. **(all doctor)(socialist)** = **T**
∴ **(all doctor)(vegetarian)** = **T**

Generalizing from (8) we see that the function **all doctor** *preserves* the order relation on its arguments in the sense that if $\mathbf{p} \leq_{VP} \mathbf{q}$ then $(\mathbf{all\ doctor})(\mathbf{p}) \leq_S (\mathbf{all\ doctor})(\mathbf{q})$. That is, **all doctor** is *increasing* as defined below:

(9) Let A and B be ordered sets and F a function from A into B.

a. F is *increasing* (= *order preserving*) iff for all $a, a' \in A$,
if $a \leq a'$ then $F(a) \leq F(a')$

b. F is *decreasing* (= *order reversing*) iff for all $a, a' \in A$,
if $a \leq a'$ then $F(a') \leq F(a)$

c. F is *monotonic* iff F is increasing or F is decreasing

And the NPs in (7) are increasing in the sense that in all situations σ they denote increasing functions. And we now state:

Gen 1: Lexical NPs are monotonic -- in fact monotonic increasing with at most a few exceptions.

Here is a snapshot of the lexical NPs of English: they include one productive subclass, the *proper nouns*: *John, Mary, ..., Siddartha, Chou en Lai, ...* ('productive' here means that new members may be freely added without changing the language significantly). They also include listable sprinklings of (i) personal pronouns -- *he/him,..* and their plurals *they/them*; (ii) demonstratives -- *this/that* and *these/those*; (iii) possessive pronouns -- *his/hers .../theirs*; and (iv) a few "indefinite pronouns" as *all* in *A good time was had by all*, *some* in *Some like it hot*, and *many* and *few* in *Many are called but few are chosen.* Some linguists would include here *everyone, everybody, everywhere*; *someone, somebody, somewhere*; and *none, noone, nobody, nowhere,* though these expressions appear to have meaningful parts.

Excluding *none, noone, nobody,* and *nowhere,* which are properly decreasing, the lexical NPs noted above are increasing.

We shall discuss decreasing NPs in **Gen 2** below. Here let us just note that the NPs in (10) below are not monotonic.

(10) a. every student but not every teacher, every student but John, exactly five students, between five and ten cats, no student but John, John but neither Bill nor Sam, most of the students but less than half the teachers

b. either fewer than five students or else more than a hundred students, approximately a hundred students, more students than teachers, exactly as many students as teachers

Note that in any given situation the NPs in (10) will denote perfectly reasonable functions from possible VP denotations to possible S denotations, but those functions are not monotonic. Thus **Gen 1** is a strong semantic claim about natural language -- many functions that are denotable by NPs in English are not denotable by lexical NPs.

If we think of **Gen 1** as a constraint on the interpretation of human languages then it helps to explain how children learn languages quickly with limited exposure to limited data. They must learn the meanings of the expressions they use. And while learning the meanings of syntactically complex expressions is facilitated by knowning the meanings of their parts, learning the meanings of lexical items is not facilitated in this way. But it is facilitated if the child need consider only monotonic (increasing) denotations for his lexical NPs.

For further generalizations concerning constraints on denotations of lexical items see Keenan (1987). We turn now to our second generalization.

2.2 Negative polarity items

To characterize the set of expressions judged grammatical by native speakers of English, we must distinguish the grammatical expressions (11a) and (12a) from the ungrammatical (11b) and (12b).

(11) a. John hasn't ever been to Moscow
b.*John has ever been to Moscow

(12) a. John didn't see any birds on the walk
b.*John saw any birds on the walk

Npi's (negative polarity items) such as *ever* and *any* above, do not occur freely; classically [Klima 1964] they must be licensed by a "negative" expression, such as *n't* (= *not*). But observe:

(13) a. No student here has ever been to Moscow
b.*Some student here has ever been to Moscow

(14) a. Neither John nor Mary saw any birds on the walk
b.*Either John or Mary saw any birds on the walk

(15) a. None of John's students has ever been to Moscow
b.*One of John's students has ever been to Moscow

The *a*-expressions here are grammatical, the *b*-ones are not. But the pairs differ with respect to their initial NPs, not the presence vs. absence of *n't*.

The linguistic problem: define the class of NPs which license the npi's, and state what, if anything, those NPs have in common with *n't/not.*

A syntactic attempt to kill both birds with one stone is to say that just as *n't* is a "reduced" form of *not* so *neither...nor...* is a reduced form of [*not* (*either...or...*)], *none* a reduction of *not one,* and *no* a reduction of *not a.* The presence of *n-* in the reduced forms is thus explained as a remnant of the original *not.* So on this view the licensing NPs above "really" have a *not* in their representation, and that is what such NPs have in common with *n't.* Moreover NPs built from *not* do license npi's:

(16) Not a single student here has ever been to Moscow
Not more than five students here have ever been to Moscow

However, as Ladusaw [1983] has taught us, this solution is insufficiently general: The initial NPs in the *a-* sentences below license npi's; those in the *b-* sentences do not. But neither present reduced forms of *not.*

(17) a. Fewer than five students here have ever been to Moscow
b. *More than five students here have ever been to Moscow
a. At most four students here have ever been to Moscow
b. *At least four students here have ever been to Moscow
a. Less than half the students here have ever been to Moscow
b. *More than half the students here have ever been to Moscow

An hypothesis which does yield correct results is a semantic one discovered by Ladusaw (1983), building on the earlier work of Fauconnier (1979). (See also Zwarts (1981)).

Gen 2 *The Ladusaw-Fauconnier Generalization (LFG)*

Negative polarity items occur within an argument of a monotonic decreasing function

To check that an NP is decreasing verify that (18) is valid when substituted for X.

(18) All linguists can dance
X can dance
∴ X is a linguist (are linguists)

This test shows that the NPs in (13) - (17) which license npi's are decreasing whereas those that do not are not.

Further the LFG yields correct results on structures like (19) and (20) below, not considered by Ladusaw or Fauconnier.

(19) No player's agent should ever act without his consent
*Every player's agent should ever act without his consent
Neither John's nor Mary's doctor has ever been to Moscow

(20) None of the teachers and not more than three of the students have ever been to Moscow

(19) draws on the fact that possessive NPs, ones of the form [X's N] such as *John's doctor*, inherit their monotonicity from that of the possessor X. Viz., X'*s doctor* is increasing (decreasing) if X is. (20) testifies that conjunctions (and disjunctions) of decreasing NPs are decreasing.

Finally we may observe from a linguist's perspective that the LFG is quite general. Denotation sets for most categories of expression in English are ordered (Keenan & Faltz, 1985) and thus most expressions of functional types are classifiable as increasing, decreasing or non-monotonic. We may expect then to find npi licensers in many categories, and we do.

A crucial case of course is that of ordinary negation *not* (*n't*). In general it denotes a complement operation in the set in which its argument denotes. E.g. at the VP level *didn't laugh* denotes E - **laugh**, the set of objects under discussion that are not in the **laugh** set. So **not (n't)** maps each subset **p** of E to E - **p**. And one shows easily that if $\mathbf{p} \subseteq \mathbf{q}$ then $E - \mathbf{q} \subseteq E - \mathbf{p}$. Which is just to say that the denotation of *not* (*n't*) is decreasing.

Thus the LFG finds a non-trivial and independently verifiable property which NPs like *no student* have in common with simple negation.

For further refinement see Nam (1992) and Zwarts (1990).

2.3 Partitives and definite NPs

We consider partitive NPs like *at least two of the students, all but one of John's children* and *most of those questions*. They appear to be of the form [DET_1 of NP], and more generally [DET_K (of NP)^K], like *more of the students than of the teachers*.

The linguistic issue: For which choices of DET_1 and NP is the sequence $\langle DET_1$ of NP$\rangle$ a grammatical NP? Some partial answers:

(21) a. *[at least two of X]* is a grammatical NP when X = the boys; the ten or more boys; these boys; these ten boys; John's cats; John's ten or more cats; my cats; the child's toys; that child's best friend's toys

b. *at least two of X* is ungrammatical when X = each boy; all boys; no boys; the boy; some boys; most boys; exactly ten boys; ten boys; no children's toys; most of the houses; at least nine students, more students than teachers, five of the students

Whether an NP of the form DET_1 + N occurs grammatically in the partitive context [*two of* __] depends significantly on *its* choice of DET_1. DET_1s acceptable here were first characterized semantically in Barwise and Cooper [1981]. We build on their analysis below.

Note that we might naively refer to NPs which occur naturally in these partitive contexts as "definite plural". So what is at issue is how to characterize that notion. We propose a semantic characterization. One problem that must be correctly handled here is the following: by various criteria NPs like those in (22) are definite plural, but, as indicated, they are at best problematic in (+count) partitive contexts, (23).

(22) a. the student and the teacher

b. this student and that student

c. John and Bill

(23) a. *?all/both of the student and the teacher

b. *most of this student and that student

c. *?none of John and Bill

We respond to this problem below by characterizing the NPs which occur in (+ count) partitive contexts in terms of the Det_1s used to build them, rather than directly in terms of denotational properties of the NP itself.

Now observe that Det_1s like *most, every, more than ten, at least and not more than ten, ...* combine with common nouns to form NPs. Semantically then we may interpret them by functions mapping common noun denotations to NP denotations. We shall take common noun denotations to be sets of objects (e.g. in a situation σ with universe E, the students in E are the objects in the set denoted by *student.* Here are some illustrative examples in an obvious notation:

(24) **some**(p)(q) = **T** iff $p \cap q \neq \varnothing$
every(p)(q) = **T** iff $p \subseteq q$
(**the ten**)(p)(q) = **T** iff $|p| = 10$ and $p \subseteq q$
most(p)(q) = **T** iff $|p \cap q| > |p - q|$

So e.g. *most p's are q's* is true iff the number of p's who are q's is greater than the number of p's who are not q's.

To avoid certain trivial cases in our characterization of "definite plurals" we note the (largely obvious) definitions of trivial functions: An NP function F is *non-trivial* in σ iff there are subsets **q**,**q'** of E such that F(**q**) = **T** and F(**q'**) = **F**. A Det_1 denotation **g** is *non-trivial* in σ iff for some $\mathbf{p} \subseteq E$, **g(p)** is non-trivial. And a Det_1 expression g is non-trivial iff for some situation σ, the denotation **g** of g is non-trivial in σ.

Lastly, an NP function F is said to be a *principal filter* iff for some $\mathbf{s} \subseteq E$, F(**q**) = **T** iff $\mathbf{s} \subseteq \mathbf{q}$. In such a case F is said to be *generated* by **s**.

For example in a situation with many cats the NP *the cats* denotes the filter generated by **cat**. So does the NP *the two or more cats*. If John has exactly two cats then *John's two cats* is the filter generated by **cat which John has**. We now propose an answer to our query:

Def 4 A Det_1 expression g is *semantically definite* iff g is non-trivial and for each situation σ and each $\mathbf{p} \subseteq E$ such that **g(p)** is non-trivial, **g(p)** is the filter generated by some non-empty $\mathbf{s} \subseteq \mathbf{p}$. If **s** always has at least two elements **g** is called *definite plural*.

(25) *Some semantically definite plural Det_1s*

the ten, ten two or more, the_{pl}, John's ten, John's two or more, $\text{John's}_{\text{pl}}$, these, these ten, these ten or more, John and Bill's ten, ...

We might note here that *every* is not semantically definite, and that the Det_1's *the one, John's one* are semantically definite but not definite plural.

Gen 3 An NP is grammatical in plural partitive contexts iff it is of the form [d N] where d is semantically definite plural or it is a conjunction or disjunction of such NPs.

We note that NPs in (22) such as *this student and that teacher* are excluded by this definition.

2.4. Existential NPs

Consider *Existential There* (ET) Ss like those in (26):

(26) There wasn't more than one student at the party
There are more dogs than cats in the garden
There was noone but John in the building at the time

Such Ss are typically used to affirm, deny or query the existence of objects (e.g. students) with a specified property (e.g. being at the party). NPs like *more than one student* which naturally occur in such Ss will be called *existential NPs*. So NPs italicized in (27) are not existential, as the Ss are either ungrammatical or assigned an unusual interpretation.

(27) *There wasn't *John* at the party
*There were *most students* on the lawn
*?There wasn't *every student* in the garden

The linguistic problem: define the set of existential NPs in English. Barwise & Cooper (op cit) again were the first to propose a semantic solution to this problem, and as in the previous case, located the solution in the nature of the DET_1s rather than the NPs themselves. The solution presented below is original here but draws on theirs and on Keenan [1987b]. See Reuland and ter Meulen [1987] for extensive discussion of the empirically problematic issues here.

We construct the existential NPs from ones built from *intersective* Determiners. To say that **more than ten** is intersective is to say that we can decide whether **more than ten** p's are q's just by checking $p \cap q$, the p's who are q's. We need not for example concern ourselves with p's which fail to be q's, as we must when checking whether **all** p's are q's.

Equally to say that a two place determiner such as *more...than...* is intersective is to say that we can decide whether more students than teachers are vegetarians is true just by checking the students who are vegetarians and the teachers who are vegetarians. We need know nothing about students or teachers who fail to be vegetarians. Formally,

Def 5 A function **g** mapping k-tuples of sets to possible NP denotations is *intersective* iff for all k-tuples $(p_1,\ldots,p_k)$ and all sets q,q' if $p_i \cap q = p_i \cap q'$, all

$1 \leq i \leq k$, then $F(p_1,...,p_k)(q) = F(p_1,...,p_k)(q')$.

Gen 4 NPs which occur naturally in ET contexts are ones built from intersective Dets or they are boolean compounds (in *and, or, not, neither...nor...*) of such NPs.

Since (**more than ten**)(p)(q) = 1 iff $|p \cap q| > 10$ we see that it is intersective and thus *more than ten cats* is, correctly predicted to occur naturally in ET contexts. Some further examples of intersective DET_1s:

(28) some, a, exactly ten, fewer than ten, not more than ten, no, between five and ten, at most ten, at least two but not more than ten, just finitely many, uncountably many, no...but John, more male than female

One checks that NPs built from these Dets occur naturally in existential Ss, as do their boolean compounds. Equally one proves easily that boolean compounds (in *and, or,* and *not*) of intersective Dets are intersective. So we predict that NPs such as those in (29) occur naturally in ET contexts, a correct prediction.

(29) at least two and not more than ten cats, either exactly two or exactly four cats

By contrast the Det_1s displayed below are not intersective and do not naturally occur in existential contexts:

(30) most, all, all but two, every...but John, two out of three, less than half the, at most twenty per cent of the, the ten, John's ten, these, my

Equally one checks that cardinal comparative DETs like *more...than...*, *fewer...than...*, *exactly as many...as...*, *more than twice as many...as...*, as in (1c) are intersective functions of type $\langle\langle 1,1\rangle,1\rangle$. E.g. whether *fewer students than teachers are vegetarians* is true is determined by the sets **student** $\cap$ **vegetarian** and **teacher** $\cap$ **vegetarian**. Thus we correctly predict that *There are fewer students than teachers in the garden* is natural.

A closing remark: Our purpose here has been to present generalizations which rely on the underlying order in denotation sets. It may not be obvious here however just how the notion of an intersective Det is built specifically on the underlying order. That is because we took the denotations of common nouns and VPs as sets because of the familiarity of this notion. But we could essentially without change have taken Ns and VPs to denote functions from E into {**T**,**F**}. But then would could not have literally referred to "intersections" of such functions.

So the crucial point here is to recognize that intersection in set theory is characterizable purely order theoretic terms. Specifically $\mathbf{p} \cap \mathbf{q}$ is the *greatest lower bound* of $\{\mathbf{p},\mathbf{q}\}$, where that notion is defined for ordered sets in general as follows:

Def 6 Where $\leq$ is an order relation on a set A, and $K \subseteq A$,

a. an element $\alpha \in A$ is a *lower bound* (lb) for K iff for all $k \in K$, $\alpha \leq k$.

b. α is a *greatest lower bound* (glb) for K iff

(1) α is a lb for K, and

(2) for all lbs β for K, $\beta \leq \alpha$.

c. If a subset $\{x,y\}$ of B has a glb, it is noted $(x \wedge y)$.

fact The orders we have considered, e.g. $\leq_C$ for $C \in \{S, NP, VP\}$ are ones in which for all x,y in the set, $\{x,y\}$ has a glb.

Then the properly general definition of (one place for simplicity) intersective Det_1 functions would be:

(31) $\mathbf{g}$ is intersective iff for all $\mathbf{p}$, $\mathbf{p'}$, $\mathbf{q}$, $\mathbf{q'}$

if $\mathbf{p} \wedge \mathbf{q} = \mathbf{p'} \wedge \mathbf{q'}$ then $\mathbf{g(p)(q)} = \mathbf{g(p')(q')}$

In this way we see that the notion of intersectivity is built on the underlying order.

Indeed the **fact** above enables us to appreciate a final linguistic generalization, much elaborated in Keenan & Faltz (1985):

Gen 5 For x,y expressions of category C, the expression *x and y* is interpreted as the greatest lower bound of the denotations of *x* and of *y*. In other words, the common meaning that *and* in all its uses is as a greatest lower bound former. (Similarly *or* is a least upper bound former).

Using **Gen 5** then we account for the common meaning of *and* in (many of) its diverse occurrences without having to say that the non-sentence level occurrences are derived from coordinate Ss by some kind of reduction rules, ones that are of necessity non-paraphrastic given that e.g. *Exactly two students both came early and left late* is not a paraphrase of *Exactly two students came early and exactly two students left late.*

In conclusion: We have exhibited several semantic generalizations about English which are defined in terms of the underlying semantic order on the denotations of expressions of a fixed category. Several of these generalizations provide a reasonable (but never perfect) answer to queries that were first raised in a purely syntactic context.

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