Answers to Sipser Ch. 0 questions

0.1
a. odd, positive numbers
b. even numbers
c. even, positive numbers
d. positive kmultiples of 6
e. palindromic binary numbers {0, 1, 00, 11, 000, 010, 101, 0000, 0110, 1001, 1111, 00000, 00100, 01110, 10001, 10101, 11111, etc.}  
f. empty set

0.2
a. \( \{ n \mid n = 10^m \text{ for some } m \text{ in } \mathbb{N}, 0 = m = 2 \} \)
b. \( \{ n \mid n \text{ is in } \mathbb{N} \text{ and } n > 5 \} \)
c. \( \{ n \mid n \text{ is in } \mathbb{N} \text{ and } 0 < n < 5 \} \)
d. \( \{aba\} \)
e. \( \{e\} \)
f. \( \emptyset \)

0.3
a. no
b. yes
c. \( \{x, y, z\} = A \)
d. \( \{x, y\} = B \)
e. \( \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\} \)
f. \( \{\emptyset, \{x\}, \{y\}, \{x, y\}\} \)

0.4
\( ab \)
because every member of A (and there are \( a \) of them) appears in one pair for each member of B (and there are \( b \) of them)

0.5
\( 2^c \)
because to form a subset of \( C \), decide whether each element of \( C \) is included or not. It’s a series of \( c \) 2-way choices, so there are \( 2^c \) different ways to choose a subset.

0.6
a. 7
b. domain is \{1, 2, 3, 4, 5\} = \( X \); range is \{6, 7\}
c. 6
d. domain is \{(1,6), (1,7), (1,8), (1,9), (1,10), (2,6), (2,7), (2,8), (2,9), (2,10), (3,6), (3,7), (3,8), (3,9), (3,10), (4,6), (4,7), (4,8), (4,9), (4,10), (5,6), (5,7), (5,8), (5,9), (5,10)\} = \( X \times Y \); range is \{6, 7, 8, 9, 10\}
e. \( g(4, 7) = 8 \)

0.7

a. \( \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\} \)
   real example: less-than-a-year-older-than
b. \( \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\} \)
   real example: subset-of
c. This one depends on the definition of transitivity. Sipser’s definition on p. 9
doesn’t require that \( x, y, \) and \( z \) be distinct. Consider then any non-empty relation
doesn’t require that \( x, y, \) and \( z \) be distinct. Consider then any non-empty relation
that is symmetric and transitive. If \( aRb \), then \( bRa \) by symmetry, so \( aRa \) by
transitivity. The only way for the relation to be non-reflexive is to have some
element in the universe that doesn’t participate in the relation at all. The empty
relation \( \emptyset \) would qualify, as would
\( \{(1,2), (2,1), (2,3), (3,2), (1,3), (3,1), (1,1), (2,2), (3,3)\} \)
if the universe is \( \{1,2,3,4\} \).

But, if we redefine transitivity thus,
\( R \) is transitive iff for every distinct \( x, y, \) and \( z \), \( xRy \) and \( yRz \) implies \( xRz \)
then the following relation would qualify:
\( \{(1,2), (2,1), (2,3), (3,2), (1,3), (3,1)\} \)
real example: sibling-of

0.8

Degree of node 1 is 3.
Degree of node 3 is 2.
Path from node 3 to node 4 (not the only possibility) is in bold.

0.9  \( \{(1,2,3,4,5,6), \{(1,4), \{1,5\}, \{1,6\}, \{2,4\}, \{2,5\}, \{2,6\}, \{3,4\}, \{3,5\}, \{3,6\}\}\} \)

0.10  The proof assumes that \( a = b \). But then it divides both sides of an equation by
\( (a-b) = 0 \). The result of division by zero is undefined, so this step is illegal.

0.11  The induction step shows that all the horses in \( H_1 \) are the same color, and that all
the horses in \( H_2 \) are the same color, but does not show that the color of all the
horses in \( H_1 \) is the same as the color of all the horses in \( H_2 \). For example, if \( k = 1 \),
we take any set \( H \) of 2 horses (say, a brown one and a black one). If we remove
the brown one, the resulting set \( H_1 \) has only black horses, and if we instead
remove the black one, the resulting set \( H_2 \) has only brown horses.
0.12 I couldn’t solve this. I once found a proof on the web, but don’t seem to have brought it to MA with me and can’t find it again—it assumed some other theorem, though, so it can’t have been the intended answer.

I did find a good-looking hint at
http://www.math.niu.edu/~rusin/known-math/99/ramsey
but haven’t tried it out yet.

0.13 \[ Y = P \left(1 + \frac{I}{12}\right)^{(1/12)} \left(1 + \frac{I}{12}\right)^{t - 1} \]

\[ \frac{100000(1+ 0.08/12)^{360} (0.08/12)}{(1+0.08/12)^{360} -1} = 733.76 \]