The semantics of many, much, few, and little

Abstract

The words many, much, few and little (and their cross-linguistic counterparts) are quite unusual semantically. They have traditionally been characterized as quantifiers (like every) or adjectives (like tall); however, these analyses can only account for instances of these terms in which they encode information about an individual or a set of individuals, as they do when they occur pronominally (in e.g. much traffic). Recent degree-semantic analyses (Rett, 2007, 2008b, 2014; Solt, 2009, 2015) instead characterize the meaning of these words in terms of intervals, or sets of degrees; this accounts for their canonical uses as well as uses in which they don’t appear to be ranging over individuals (as in their differential use, e.g. much taller).

1 Introduction

The words many, much, few and little have received several very different semantic treatments in the literature. This arguably due to the fact that they are quite versatile, and appear to be quantifiers, or adjectives, or modifiers, depending on which of their uses are taken to be canonical. The goal of this paper is to provide a comprehensive view of the meaning and distribution of these words, and to review and evaluate semantic treatments of them.

2 An empirical overview

The semantic status of many, much, few and little is so indeterminate that there is no standard term for them and their cross-linguistic counterparts. I will settle on the relatively theory-neutral term “quantity words”.

2.1 The meaning(s) of quantity words

Quantity words are associated with a number of different semantic properties. This section reviews those properties which have received the most attention in the theoretical literature: the contribution of negative quantity words; the cardinal/proportional ambiguity; and the evaluative, ‘above-the-standard’ meaning of some constructions that contain quantity words.
2.1.1 Differences between quantity words

It’s generally assumed that the difference between many/few on the one hand and much/little on the other corresponds to the difference between count and mass nouns. While it is unclear what that distinction is (see Chierchia, 2010, for an overview), a common working hypothesis is that many and few are associated with discrete, countable entities whose quantities can be measured with whole numbers, while much and little are associated with non-discrete entities whose quantities must be measured with rational numbers. A consequence, further discussed in §2.2, is that the distribution of much and little is broader than that of many and few: the former, but not the latter, can measure the quantity of events and abstract intervals as well as individuals.\(^1\)

Many languages don’t lexicalize a count/mass distinction in their quantity words (e.g. Romanian, Rett 2007), but they do generally lexically distinguish between positive (e.g. many, much) and negative (e.g. few, little) quantity words. The polarity distinction has, consequently, received a fair amount of attention in the theoretical literature, which generally assumes that e.g. many and few are antonyms, just like big and little. Negative antonyms are widely viewed as marked relative to their positive counterparts; i.e., a negative antonym like little is the negated version of its positive counterpart much, but not vice-versa (Lehrer, 1985; Rett, 2015). Heim (2007) makes this point explicitly for little, arguing that it involves a semantic negation that can scope above or below a modal.

2.1.2 Cardinal and proportional readings

Many constructions containing prenominal quantity words can receive a cardinal or at least one proportional reading (Milsark, 1977; Westerstål, 1985; Partee, 1989; Büring, 1996; Herburger, 1997). This is demonstrated in (1), where \(S\) denotes the set of Scandinavians, \(W\) the set of Nobel Prize winners, and where \(d\) ranges over some contextually salient quantity threshold.

(1) Many Scandinavians have won the Nobel Prize in literature.
   a. \(|S \cap W| \geq d\) \hspace{1cm} \text{cardinal}
   b. \(|S \cap W| / |S| \geq d\) \hspace{1cm} \text{proportional}

\(^1\)However, the distribution of much in English is subject to additional restrictions relative to the other quantity words: it is only acceptable under negation in some (e.g. *John ate much rice) but not all contexts (e.g. Much attention was paid to this document). Israel (1996) identifies much as the paradigm case of a so-called ‘attenuating NPI’.
In (1-a), the number of Scandinavians who are Nobel Prize winners is understood to be a large number. In (1-b), the number of Scandinavians who are winners is understood to be large relative to the number of Scandinavians. Finally, in (1-c), the number of Scandinavians who are Nobel Prize winners is understood to be large relative to the number of Nobel Prize winners generally. As Partee observes, the proportional readings can be paraphrased with partitive sentences: Many of the Scandinavians are Nobel Prize winners (for the proportional reading) and Many of the Nobel Prize Winners are Scandinavians (for the reverse proportional reading).

Theoretical treatments of the semantics of quantity words, discussed further below, vary in their treatment of these readings. Earlier analyses (e.g. Partee, 1989) treated them as an ambiguity; Herburger (1997) argues that syntax and focus play a role in the availability of the three interpretations. Recent semantic analyses have tried to derive the cardinal and proportional interpretations from a single underspecified, context-sensitive lexical entry, both in quantificational accounts (Westerståhl, 1985; Lappin, 2000; Cohen, 2001; Romero, 2015) and non-quantificational accounts (Rett, 2008b; Solt, 2009). The availability of the ‘reverse proportional’ interpretation means that, if quantity words denote quantifiers, they violate conservativity (Fernando and Kamp, 1996); this has lead those working in a quantificational framework to conclude that quantity words require an intensional analysis (Greer, 2014).

2.1.3 Quantity words and evaluativity

Quantity words, like adjectives, participate in evaluative (or ‘norm-related’) constructions. A construction is evaluative iff it requires of some degree that it exceed a contextually-valued standard of evaluation (see Rett, 2015, for an overview of this phenomenon).

Constructions containing unbound quantity words and relative adjectives are always evaluative, as demonstrated in (2). (2-a) is true only if the height of the students in question exceeds the contextual standard for tallness (or shortness). Similarly, (2-b) is true only if the quantity of students in question exceeds the contextual standard for many (or few).

(2)  
   a. Tall/Short students received new desks.  
   b. Many/Few students received new desks.

\[ |\frac{S \cap W}{W}| \geq d \quad \text{reverse proportional}\]

As Herburger (1997:67) notes, this reading is available when Scandinavians is focused.
However, not all of the constructions that quantity words and relative adjectives occur in are evaluative. This is demonstrated below for the equative construction.

(3) a. Mary is as tall as John... which is to say, she’s short.
    b. Mary is as short as John... #which is to say, she’s tall.

(4) a. Mary teaches as many students as John... which is to say, few students.
    b. Mary teaches as few students as John... #which is to say, many students.

Neither construction in (3-a) or (4-a) requires that the degree in question exceed a contextually-valued standard, as evidenced by the acceptability of the continuations. In contrast, the negative-antonym constructions in (3-b) and (4-b) are evaluative, judging by the unacceptability of the continuations. But not all constructions containing negative-antonym quantity words and relative adjectives are evaluative; comparatives formed from negative quantity words are not evaluative, as demonstrated in (5).

(5) Mary teaches fewer students than John... but of course, she still teaches many students.

These data have lead semanticists to draw the same conclusion about quantity words as they have about relative adjectives (Cresswell, 1976; Bierwisch, 1989; Rett, 2008a): an empirically comprehensive theory of evaluativity requires the assumption that evaluativity is not lexically encoded in the meaning of quantity words themselves, but rather that it arises for external reasons, only in certain configurations. This conclusion runs contrary to early, quantifier-based treatments of quantity words that encode evaluativity in the lexical entry of quantity words.

2.2 The distribution of quantity words

Quantity words have a very wide distribution across constructions and categories. The canonical use of quantity words is their prenominal or attributive use, as in (6).

(6) a. Many/few guests left.
    b. Much/little rice was consumed.

It is this use that makes it tempting to classify them as quantifiers, given the parallel data in (7-a), although (7-b) shows that adjectives can occur in
this position as well.

(7)  
a. All/some guests left.
b. Tall/short guests left.

Quantity words in this prenominal position can combine with demonstratives like that (exemplified in (8)) and wh-phrases like how and what (although the latter only in exclamatives, Rett 2009). Many, few and little can additionally be modified by the determiner the (9), and few and little can additionally be modified by the indefinite determiner a. Quantity words share this ability with adjectives, but not with canonical quantifiers, as demonstrated in (9).

(8)  
a. *John didn’t meet that all/some guests.
b. John is not that tall/short.
c. John didn’t each that much/many cookies.

(9)  
a. *The all/some guests left.
b. The tall/short guests left.
c. The many/few guests left.

Also like adjectives, but unlike quantifiers, quantity words can occur in predicative position, as shown in (10), although the mass quantity words much and little are generally reported to be less acceptable in this position.

(10)  
a. *John’s regrets are all/some.
b. John’s children are tall/short.
c. John’s regrets are many/few.

There are other canonically adjectival positions in which quantity words can occur: those described by Bresnan (1973) as involving degree quantifiers like the equative as, the excessive too, and so.

(11)  
a. John is as tall as Sue.
b. John has as many regrets as Sue.

(12)  
a. John is too irresponsible to be an assistant.
b. John has too much experience to be an assistant.

(13)  
a. John is so clumsy that he can’t play tennis.
b. John has so few suitcases that he had to borrow mine.

Two other constructions that fit in this paradigm are the comparative morpheme -er and superlative morpheme -est (Heim, 2000; Schwarzschild,
2008). While the forms of these constructions involving *few* are transparently constructed from quantity words, the other forms (*more, most and less, least*) very clearly involve suppletion (Bresnan, 1973; Bobaljik, 2012). It should be clear that standard quantifiers like *all* and *some* cannot occur in these positions.

From the discussion so far, quantity words pattern with quantifiers in only one configuration: prenominally. But they pattern with adjectives in several (prenominally; predicatively; with overt determiners; and with degree quantifiers). Quantity words are, however, acceptable in three additional configurations which admit neither quantifiers nor adjectives: the modification of verb phrases (VPs); preposition phrases (PPs); and comparatives, demonstrated in (14)–(16) below.

(14)  a. John doesn’t go to the movies much.
     b. John sleeps little.

(15)  a. The car drove much/little over/under the speed limit.
     b. The picture isn’t much above the mirror.

(16)  a. John is much/little taller than Sue.
     b. John drove much/little faster than Sue.
     c. The desk is much/little farther away than the couch.

These distributional patterns are summarized in Table 1.\(^3\)

In what follows, I will be making a single distinction within the contexts in which quantity words can occur. ‘Individual uses’ of quantity words are those in which they appear to be ranging over individuals (or sets of individuals). This category includes the prenominal and predicative contexts, as well as prenominal uses of quantity words in degree quantifier constructions. In these cases (e.g. *(the) many guests, The guests were many, as many friends as*) we have an intuition that the quantity word encodes information about the measure of an individual or a plurality of individuals.

In contrast are the ‘non-individual uses’ of quantity words, in which we have an intuition that the quantity word doesn’t encode information about an individual. When they function as VP modifiers, quantity words encode information about the measure of an event (Doetjes, 2007; Nakanishi, 2007), e.g. its duration or frequency. When they function as PP modi-

\(^3\)I use the term ‘degree quantifier’ to refer constructions headed by degree quantifiers like *so* or *too*, as in *too much rice* or *so many pizzas*. While these examples could be considered a type of prenominal use, it’s important to note that quantity words can also occur with these degree quantifiers when they modify events, e.g. *go to the movies too much*, which would best be characterized as non-individual uses, along the lines of (14).
fiers, quantity words encode information about the size of a vector (Zwarts, 1993). And when they modify comparatives (their ‘differential’ use), quantity words encode information about the size of a differential interval along some dimension of measurement (Schwarzschild, 2006a), e.g. height.

To summarize: quantity words are available in a number of different configurations. Their prenominal use, often treated as canonical, has lead some to analyze quantity words as quantifiers, and others to analyze them as adjectives, but they pattern like neither in their overall distribution. The rest of this article outlines semantic treatments of quantity words as quantifiers (§3) and as adjectives (§4), and argues that these distributional differences mean that neither sort of theory can accurately characterize the behavior of quantity words. The details are interesting, but the overall point should be clear, given Table 1: analyzing quantity words as quantifiers predicts that they have the same distribution as quantifiers do, which they don’t. And analyzing quantity words as adjectives predicts that they have the same distribution as adjectives do, which they don’t. In particular, they incorrectly predict that quantity words only have individual uses.

§5 presents recent analyses of quantity words in which they range over intervals, or sets of degrees. These accounts work well for the diverse data reviewed here because the framework of degree semantics allows for a variety of different semantic objects (individuals, events, etc.) to be associated with sets of degrees corresponding to their measure in a given context.
2.3 Crosslinguistic differences

Although most of the semantic literature on quantity words focuses on English, recent research has revealed some additional ways in which quantity words can vary across languages.

Quantity words vary in the extent to which they contribute to quantity *wh*-words (the equivalent of English *how many*): in Balkan languages, quantity words can optionally occur with quantity *wh*-words, and effect subtle semantic differences when present (Rett, 2007, 2008b).

(17) a. Căte *how many-f.pl* femei *women* cunoaste?
     how-many-f.PL women know-3.SG
b. Cît *how many* de multe *of many-f.pl* femei *women* cunoaste?
     how-many of many-f.PL women know-3.SG
     ‘How many women does he know?’

Some Slavic languages additionally seem to include two sets of quantity words, even setting aside distinctions of polarity and count/mass. Krasikova (2011) argues that the Russian adverb *mnogo* can only receive a cardinal interpretation, while the adjectival *mnogie* can only receive a proportional interpretation. Stateva and Stepanov (2016) present an in-depth look at the Slovenian quantity words *precej* and *veliko*. They argue that, contrary to Russian, the words do not differ in terms of the cardinal/proportional distinction, but rather in their distribution (see §2.2): both can combine with NPs or the superlative, but only *precej* can combine with adjectives, adverbs, or PPs.

Finally, there are cross-linguistic differences in the way quantity words can range over events. Doetjes (2007, 2008) observes that Standard French *beaucoup* can range over events in e.g. *J’ai beaucoup dormi* (‘I have slept a lot’), while Burnett (2012) observes that *beaucoup* can range over individual/event pairs in Québec French (Nakanishi 2007 makes similar observations for Japanese classifier constructions).

Despite these very interesting cross-linguistic differences, it is cross-linguistically quite common for quantity words to have uses as both quantifiers and modifiers of comparatives. This strong tendency provides compelling support for an analysis that does not characterize quantity words exclusively in terms of their interaction with individuals.
3 Quantificational accounts of quantity words

One interpretation of the word *quantifier* describes expressions that involve counting or measuring. This certainly characterizes quantity words. But there is a specific use of the term wherein it describes expressions that denote relations between sets of individuals, as it does in Generalized Quantifier Theory (GQT). In this section, I’ll argue that quantity words cannot accurately be analyzed as quantifiers in this more specific sense.

In GQT (Barwise and Cooper, 1981; Keenan and Stavi, 1986; Greer, 2014), quantifiers like *all* and *some* are analyzed as denoting relations between sets of individuals $A$ and $B$, as (18) shows.

\[(18)\]
\[
\begin{align*}
\text{a. } \llbracket \text{all} \rrbracket &= \lambda A \lambda B [A \subseteq B] \\
\text{b. } \llbracket \text{some} \rrbracket &= \lambda A \lambda B [A \cap B \neq \emptyset] \\
\text{c. } \llbracket \text{many} \rrbracket &= \lambda A \lambda B [|A \cap B| \geq d, d \text{ a large number}]
\end{align*}
\]

(18-a), for instance, predicts that a sentence *All ants boogied* is true iff the set of ants is a subset of the set of individuals that boogied. An equivalent way of writing these truth conditions is with an overt logical quantifier ranging over individuals, as in (19) for the quantifier *some*.

\[(19)\]
\[
\llbracket \text{some} \rrbracket = \lambda A \lambda B \exists x [A(x) \land B(x)].
\]

In GQT, and in many logic traditions, quantity words like *many* are analyzed in the same fashion, although (as (18-c) shows) they require additional machinery. In particular, the GQT treatment of *many* incorporates the cardinality operator $|.|$ and an incorporation of evaluativity via the free variable $d$. (18-c) predicts that a sentence *Many ants boogied* is true whenever the cardinality of the intersection of the set of ants and the set of individuals that boogied is greater than or equal to some large number $d$, which can receive different values depending on the context. This appropriately characterizes constructions with prenominal quantity words as evaluative and thereby as context-sensitive; what counts as *many* or even as *many ants* can vary quite a bit from context to context.

Accounting for the semantics of negative quantity words has posed addi-
tional challenges for formal semanticists. van Benthem (1983) pointed out a particular pitfall for analyses that utilized an existential quantifier over individuals, widely referred to as ‘Van Benthem’s Problem’. While the informal characterization of positive quantity words in (20-a) works intuitively well, the informal characterization of negative quantity words in (20-b) is too weak. In a situation in which 100 students took semantics (and in which 40 students as a high number), (20-b) nevertheless predicts that the sentence *Few students took semantics* is true. This is because there is some plurality of semantics students – for instance, a plurality of two – whose quantity qualifies as low in the context.

(20) an informal semantics of quantity words (to be revised)

a. *many/much* NP VP: there is some plurality *x* in the extension of NP and VP such that the quantity of *x* is significantly high

b. *few/little* NP VP: there is some plurality *x* in the extension of NP and VP such that the quantity of *x* is significantly low

There have, however, been recent attempts to revive quantificational treatments of quantity words in light of this problem (Buccola, 2015; Romero, 2015).

Romero (1998) and then Hackl (2000) propose an updated analysis of quantity words as quantifiers, based on their observations about the behavior of quantity words in *how many* questions (Romero, 1998) and in comparatives like *more* (Hackl, 2000). Both (see also Romero, 2015) analyze quantity words as ‘parameterized determiners’: quantifiers with an additional degree argument *d*, as in (21), making them a hybrid between a quantifier and a gradable adjective in degree-semantic analyses (which will be discussed in more detail in §4).

(21) \[ \text{[many]} = \lambda d \lambda A \lambda B \exists x [A(x) \land B(x) \land |x| = d] \]

Although the parameterized determiner analysis was proposed in contrast to the GQT analysis, the two have in common that they analyze quantity words as encoding existential quantification (or its set-theoretic equivalent) over individuals. They therefore predict that quantity words are defined only when they range over individuals. Quite simply, these accounts incorrectly predict that quantity words are undefined in the non-individual uses described above (e.g. the comparative modifier or differential use).

At the very least, then, adopting a quantifier-based analysis of prenominal quantity words requires a separate treatment for their other uses, which suggests that it’s an accident prenominal quantity words are homophonous.
with e.g. differential ones. But there are still more strikes against quantifier-based approaches. They cannot account for predicative uses of quantity words, for example. And they incorrectly predict that prenominal quantity words cannot combine with overt determiners like the, which are also analyzed as ranging over individuals. (If many did in fact bind the individual argument of guests in The many guests left, we would predict that argument to be unavailable to the, resulting in vacuous quantification.)

There are still more problems with quantifier approaches to quantity words, as detailed in Rett (2008b) and Solt (2015). For instance, quantity words are optional in how many questions in several Balkan languages (Rett, 2007), but quantifier treatments predict that they’re necessary to bind the individual variable introduced by the nominal. And the analysis proposed in Hackl (2000) requires a null positive quantity word many in every case in which a noun must be associated with a degree corresponding to its measure. As a consequence, he predicts that even nominal comparatives formed with few (e.g. John has fewer shoes than Bill) contain a covert many in addition to an overt few.

These issues raise the question: if quantity words aren’t quantifiers, are they better analyzed as adjectives?

4 Adjectival accounts of quantity words

Table 1 suggests that quantity words and adjectives mean the same sort of thing, at least some of the time. This perspective has a modest history of support in linguistic semantics (Bresnan, 1973; Cresswell, 1976; Hoeksema, 1983; Grosu and Landman, 1998; Partee, 1989), based on the fact that quantity words can “...form comparatives and superlatives, they can combine with degree expressions like too or very,.... Many and few can also be used in predicative position (his sins were many, his virtues were few)” (Hoeksema, 1983, 65).

If quantity words are adjectives, they are gradable adjectives like tall (as opposed to non-gradable ones like polka-dotted). A gradable adjective is characterized by its ability to be modified by intensifiers like very and to occur with degree quantifiers like the comparative, crucially without semantic coercion to some related scale (as the non-gradable pregnant is often coerced to a temporal scale in e.g. very/more pregnant; Cruse, 1986). Quantity words pass both tests (as demonstrated by very few and fewer).

Beginning with Cresswell (1976), degree semantics treats gradable adjec-
tives as differing from non-gradable ones in taking a degree argument.\textsuperscript{5} In modern degree semantic approaches, degrees are semantic primitives, ranging over points on scales that correspond to different dimensions of measurement, like height, happiness, etc. From this perspective, the difference between gradable and non-gradable adjectives is one of valency, similar to the difference between transitive and intransitive verbs, demonstrated in (22). (Furthermore, in degree semantics, intensifiers and degree quantifiers range over degrees, which explains why they can’t modify non-gradable adjectives.)

\begin{align*}
(22) & \quad \text{a. } [\text{polka-dotted}] = \lambda x [x \text{ is polka-dotted}] \\
& \quad \text{b. } [\text{tall}] = \lambda d \lambda x [x \text{ is } d\text{-tall}]
\end{align*}

According to (22-b), the gradable adjective tall relates an individual \( x \) to a degree \( d \) whenever \( x \) is tall to at least degree \( d \). As a result, each individual is potentially associated with a plurality of degrees of tallness, or a degree interval. In the case of an individual \( j \) whose maximum height is 6ft, this interval would have an open lower bound of zero and a closed upper bound of 6ft, i.e. \( \{\ldots,1\text{ft},\ldots,6\text{ft}\} \), or \( (0,6\text{ft}] \).

In adjectival accounts of quantity words, they, too, take an individual and a degree argument, as in (23).\textsuperscript{6}

\begin{align*}
(23) & \quad [\text{many}] = \lambda d \lambda x [x \text{ is } d\text{-many}]
\end{align*}

As in (22-b), (23) states that many relates a (plural) individual \( x \) to a degree \( d \) whenever the cardinality of \( x \) is at least \( d \). As a result, each plural individual is associated with a plurality of quantity degrees, i.e. an interval. In the case of a plural individual \( g \) with an atomic quantity of 6, this interval would be \( \{1,2,3,4,5,6\} \), or a discrete interval with closed lower bound of 1 and a closed upper bound of 6, i.e. \( \{1,\ldots,6\} \), or \( [1,6] \).

In these degree-semantic accounts, gradable adjectives can combine with an overt measure phrase, if present, which value their degree argument (e.g. *John is 6ft tall*).\textsuperscript{7} In the absence of an overt measure phrase or degree quantifier, the extra degree argument is generally assumed to be bound by a null operator (e.g. *pos*, Bartsch and Vennemann 1972; Cresswell 1976;

\textsuperscript{5}See Schwarzschild (2008); Morzycki (2015) for an introduction to degree semantics.

\textsuperscript{6}These are the definitions for adjectives in predicative position; those in attributive position are assumed to have a related, type-raised definition that includes a set of individuals \( P \) as an argument, e.g. \( [\text{tall}(P)(x)(d)] \) is true iff \( x \) is \( P \) and \( d\text{-tall} \).

\textsuperscript{7}Quantity words cannot occur with overt numerals or measure phrases (cf. *6 many pizzas*), but neither can the vast majority of gradable adjectives (cf. *$20 expensive*). The difference is generally treated as a lexical rather than semantic one (Schwarzschild, 2005).
Figure 1: Antonymic relations as scale reversals

von Stechow 1984; Kennedy 1999) or to undergo existential closure (adding the $\exists$) and receive evaluativity (adding the ‘and $d > s$’ via some other null modifier or via conversational implicature (Rett, 2008a,b, 2015), as depicted in (24) and (25).

(24) $[\text{John is tall}] = \exists d [\text{John is } d\text{-tall } \land d > s_{\text{tall}}]$

(25) $[\text{The guests are many}] = \exists d [\text{the-guests are } d\text{-many } \land d > s_{\text{many}}]$

The evaluative truth conditions in (24) predict that the sentence is true in a situation in which there is some degree to which John is tall that exceeds on the ‘tall’ scale some contextually salient standard of tallness $s_{\text{tall}}$. The evaluative truth conditions in (25) predict that the sentence is true in a situation in which there is some degree corresponding to the quantity of guests that exceeds on the ‘many’ scale some contextually salient standard of quantity $s_{\text{many}}$.

In these degree-semantic approaches, antonymy is generally treated as a reversal in scale ordering (Bartsch and Vennemann, 1972; Morzycki, 2015; Rett, 2015); this accounts for the mutual entailment of sentences like $A$ has more than $B$ and $B$ has less than $A$. Thus the negative-antonym version of (25) would require that there be some quantity of guests that exceeds the contextually salient ‘few’ standard on the ‘few’ scale. This is equivalent to the requirement that the quantity of guests be lower than $s_{\text{few}}$ on the ‘many’ scale. This is illustrated in Figure 1 below.

It should be clear from the discussion above that adjectival accounts, like quantificational accounts, define quantity words in terms of their individual argument. In (23) (and its attributive counterpart), the quantity word many takes an individual argument, and the semantic contribution of the quantity word is to associate this individual with a degree argument corresponding to its measure. In these adjectival accounts, quantity words are undefined if they do not take an individual argument (although see Wellwood, 2015, for
an account that re-envisions adjectival meanings in light of this constraint. They therefore cannot extend to the non-individual uses of quantity words, which do not encode information about an individual. In the comparative *John is much taller than Sue*, for instance, the quantity word *much* intuitively measures the difference between John’s height and Sue’s height, not John or Sue themselves.

As with the quantificational accounts, since adjectival accounts cannot extend to these non-individual uses of quantity words, they are forced to claim that it is an accident that the same word is used in English to measure individuals and non-individuals. Worse, they cannot predict that quantity words have this multiplicity of meaning across languages. What Table 1 suggests is that we need an account of quantity words in which their distribution is less semantically restricted.

## 5 Interval-based treatments of quantity words

If quantity words are neither quantifiers nor adjectives, what are they? Domain-general accounts of quantity words take the perspective that if the non-individual uses of quantity words can’t be properly treated in individual-based accounts, then the individual uses of quantity words should be treated in degree or scale-based accounts that are well-suited to treat the non-individual uses. In other words, instead of considering the prenominal or predicative instances of quantity words to be canonical (like the quantifier and adjectival accounts do, respectively), domain-general approaches treat the !differential instances as canonical.

### 5.1 Rett’s interval-based account

In a series of proposals, Rett (2007, 2008b, 2014) argued that quantity words should be recast in terms of intervals, or sets of degrees. Since there is independent evidence that all sorts of semantic domains – including individuals and events – need to be associated with degrees, the switch from individuals to intervals amounts to a domain-general account of quantity words.

The underlying intuition is this: in the differential comparative *John is much taller than Sue*, quantity words measure the size of the gap between John’s height and Sue’s height. This gap is modeled in degree semantics using a scale, i.e. a set of degrees representing points along a single dimension of measurement (Schwarzschild, 2006a), as illustrated in Figure 2 below.

In Rett’s account, quantity words are degree modifiers (type ⟨⟨d,t⟩, ⟨d,t⟩⟩), denoting relations between a set of degrees $D$ (i.e. an interval) and its size.
Figure 2: Scales and gaps in degree semantics

d, as shown in (26).\(^8\)

(26) \[ \text{[much]} = \lambda D \lambda d \text{[}d \text{is the size of } D\text{]} \]

Intervals are measured by subtracting the lower bound from the upper bound.\(^9\) In Figure 2, for example, the measure of the interval (5ft,6ft] – representing the gap between John and Sue’s heights – is 1ft.

A transparent instance of quantity words modifying intervals is their use as modifiers of comparatives, as in *John is much taller than Sue is*. A typical assumption in the degree-semantic literature is that a clausal comparative sentence like (27-a) has the logical form in (27-b), and that the comparative morpheme is akin to a degree quantifier, as in (28).\(^10\) In this formulation, the variable \(d\) ranges over differential degrees: those that are in the interval associated with the matrix subject but not the interval associated with the embedded subject (i.e. the gap between the measures of the two subjects).

(27) a. John is taller than Sue is.
    b. \(-er([\text{OP}_{d}\text{ Sue is tall to } d'])([\text{OP}_{d}\text{ John is tall to } d'])\)

\(^8\)The motivation for this characterization, as stated in Rett 2008b, is that quantity words are like other modifiers in their ability to be reduplicated, as in *much, much later*.

\(^9\)In mathematical measure theory, the size of an interval with endpoints \(a\) and \(b\) (where \(a < b\)) is \(b-a\). This measure is defined over closed, open, and partially closed intervals alike (Gordon, 1994). See Rett 2008b for discussion. When the sets are downward monotonic – i.e. when the inclusion of \(d\) in the set entails the inclusion of \(d-1\) – the measure of an interval is equivalent to its maximal degree, which yields some interesting results for natural language semantics (Rett, 2007).

\(^10\)This is a version of the ‘A-not-A’ analysis, which has its origins in Ross (1969); Seuren (1973); McConnell-Ginet (1973); Kamp (1975); Hoeksema (1983); Seuren (1984) and is reviewed in Schwarzschild (2008). It assumes that each clausal argument of the comparative denotes a set of degrees (Bresnan, 1973), and it analyzes the comparative morpheme as requiring that there be a degree on the interval \(D'\) associated with the matrix subject’s measure that is not on the interval \(D\) associated with the embedded subject’s measure.
In the case of unmodified comparatives, as in (27), the differential degree argument $d$ is assumed to be existentially bound, resulting in the informal truth conditions 'There is some degree $d$ such that John is tall to degree $d$ but Sue is not.'

It is this degree argument $d$ that quantity words like much modify in interval-based approaches: in a sentence like (29), much measures the size of the gap between John and Sue’s height. Rett assumes that, in the absence of any additional modification (see footnote 8), the degree argument of the quantity word is existentially bound. She also assumes that the construction is made evaluative via the usual mechanism for bestowing evaluativity on unbound adjectives and modifiers (see §2.1.3). This is depicted in (29-c), where the size $d'$ of the gap between John’s and Sue’s heights is required to exceed the contextually-valued standard for tallness gaps, $s_{t-interval}$.

Informally, (29-c) predicts that the differential comparative is true iff the size $d'$ of the gap between John’s height and Sue’s height (the interval (5ft,6ft]) is considered big in the context of utterance.

This interval-based account treats the use of quantity words as comparative modifiers as primary. To extend it to cases in which quantity words measure other sorts of things (i.e. its individual and event uses), Rett exploits existing mechanisms in degree semantic theories that use null operators or type-shifters to associate an individual with a degree corresponding to some salient measure of that individual.

In standard degree-semantic accounts, numerals and measure phrases denote degrees. Without further innovation, this would result in a type clash in phrases in which numerals modify nouns, like five guests, as nouns traditionally denote sets of individuals. A standard solution, beginning with Cresswell (1976), is to allow nouns to be optionally associated with a degree...
denotation (via a null operator or a type-shifting mechanism, as in (30), Schwarzschild 2002, 2005; Nakanishi 2007).

\[ \text{[M-Op]} = \lambda P \lambda d \lambda x [P(x) \land \text{the salient measure of } x \text{ is } d] \]

‘M-Op’ relates individuals \( x \) in the extension of some predicate \( P \) to their degree of measurement along some salient dimension (allowing for sentences like \textit{The board is 3 feet} to measure e.g. length or width, depending).\(^{13} \) In the case of \textit{five guests}, or any numeral phrase, the only available dimension of measurement is quantity. A sample derivation is in (31); as shown in (31-b), the individual variable is bound via existential closure in the absence of any overt binding (see Rett, 2014, for details).

(31) Five guests arrived.
   a. \( \text{[M-Op guests arrived]} = \lambda d \lambda x [\text{guests}(x) \land \text{arrived}(x) \land \text{the quantity of } x \text{ is } d] \)
   b. \( \text{[five M-Op guests arrived]} = \exists x [\text{guests}(x) \land \text{arrived}(x) \land \text{the quantity of } x \text{ is 5}] \)

This ability of different types of semantic objects to optionally associate with degrees allows interval-based accounts to generalize from the comparative modifier use of quantity words to their other uses. (32) illustrates the extension of this interval-based account of quantity words to their canonical prenominal use. In it, the individual argument undergoes existential closure, creating in (32-a) the characteristic function of the set of quantities of guests who arrived. As in (32-b), the quantity word measures this set; since the quantity word is not bound or modified, its degree variable is assumed to undergo existential closure.

(32) Many guests arrived.
   a. \( \text{[M-Op guests arrived]} \) (\textit{existential closure over } \( x \))
      \( = \lambda d \exists x [\text{guests}(x) \land \text{arrived}(x) \land \text{the quantity of } x \text{ is } d] \)
   b. \( \text{[many M-Op guests arrived]} \) (\textit{existential closure over } \( d' \))
      \( = \exists d' [d' \text{ is the size of } \{d : \exists x [\text{guests}(x) \land \text{arrived}(x) \land \text{the quantity of } x \text{ is } d] \} \land d' > s_{q-interval}] \)

\(^{13}\)Rett assumes that M-Op has a predicative formulation, related to the attributive version in (30) via the same type-shifter that associates predicative and attributive gradable adjectives. See Schwarzschild (2006b) and Rett (2014) for arguments that, while mass quantity words like \textit{much} can be associated with a variety of different dimensions of measurement, they can only be associated with dimensions that are monotonic on the relevant part-whole structure of an entity.
Informally, (32) predicts that the sentence is true if the size of quantities corresponding to the plurality of guests who arrived exceeds the contextually valued standard for intervals corresponding to quantities (of guests who arrived). Recall that the size of a downward-monotonic set of degrees corresponds to its maximal degree. In a context in which there are 5 guests who arrived, the set of degrees \(d\) will be \(\{1,2,3,4,5\}\), and the measure of that set will be 5.

As discussed in §2.1.3, the evaluativity represented by the final requirement in (32) (that \(d' > s_{q-interval}\)) must be contributed by something external to the quantity word itself. Because of the context-sensitivity of the dimension of measurement built into M-Op – as well as the context-sensitivity of the standard of comparison involved in evaluativity – quantity word constructions are underspecified with respect to their dimension of measurement, or what sort of quantities they’re measuring. Assuming that cardinal, proportional, and reverse proportional quantities count as different dimensions of measurement (as do height, length, breadth and depth, under typical assumptions), this context-sensitivity allows for just the sort of contextual variation between cardinal, proportional, and reverse proportional interpretations of quantity words reported in §2.1.2 (Rett, 2008b).

This interval-based account has been extended to other quantity-word constructions discussed in §2: following standard degree-semantic theories, negative-polar quantity words like few are just like their positive counterparts in that they relate an individual to a quantity, albeit on reverse scales (i.e. requiring that \(d\) exceed on the ‘few’ scale the contextually-valued standard for few). This successfully avoids van Benthem’s Problem, as argued in Rett 2008b (see also Buccola 2015).

Rett’s proposal involves other details that can’t be addressed here: for example, she argues that sentences like The many guests arrive involve type-shifted instances of determiners in which they range over degrees (Rett 2008b; see also Kotek 2011). And while M-Op is defined in (30) over individuals, she argues that the use of quantity words as VP modifiers requires that it range over eventualities, as well, as in Nakanishi 2007 and Doetjes 2007 (Rett, 2008b, 2014). Finally, although the interval-based definition in (26) allows quantity words to apply to gradable adjectives in principle, there is reason to think that constructions like *much tall are prohibited across languages for morphological reasons (Doetjes, 1997).
5.2 Solt’s interval-based account

In a series of proposals, Solt (2009, 2015) modifies this interval-based approach to characterize quantity words as gradable predicates of intervals, type \( \langle d, \langle d, t \rangle, t \rangle \).

\[(33) \quad \text{[much]} = \lambda d \lambda D[D(d)]\]

Like Rett’s proposal, Solt’s analyzes quantity words as relations between a degree and an interval, although she characterizes them as taking their arguments in the opposite order. This allows her to better highlight the intuitive parallel between quantity words and gradable adjectives, as discussed in §4; like gradable adjectives, quantity words take degrees as their first argument, but the two differ in the nature of their second argument (an individual for adjectives, an interval for quantity words).

The two approaches share a number of the same background assumptions: Solt employs existential closure over individual arguments in the absence of overt binding or modification and assumes that evaluativity is contributed to quantity constructions via the same mechanism typically assumed for gradable adjective constructions (hers involves universal quantification over degrees). Also in the same vein, she follows Schwarzschild in assuming that the relevant dimension of measurement is contextually valued (and constrained in terms of monotonicity), and she characterizes negative-polar quantity words as converses of their positive-antonym counterparts, accounting for (among other things) split-scope readings observed with few and little (Heim, 2007). Finally, she too employs a null measurement operator to associate entities with degree arguments, as in (34), though hers differs from Rett in two crucial respects.

\[(34) \quad \text{[Meas]} = \lambda x \lambda d[\text{the salient measure of } x \text{ is greater than or equal to } d]\]

While Rett assumes an attributive and predicate version of this null measurement operator, Solt assumes only the version in (34) and introduces an additional compositional rule to allow Meas to compose in the prenominal position. But the main difference is that Rett’s M-Op equates plural individuals with their measure, while Solt’s Meas relates a plural individual to some degree on a scale.

In Solt’s formulation, quantity words relate a degree and an interval in terms of set inclusion, rather than in characterizing the former as a measure of the latter. As a result, Solt’s account doesn’t involve the higher-order
measurement that Rett's does. Rett (2007) argues that this higher-order measurement is required for Romanian, but it is clearly not at play in English; to address this, Rett (2014) amends her original proposal to make this higher-order measurement optional for languages like English.

Below are illustrations of Solt’s treatment of prenominal and differential quantity words, respectively. In (35), the degree argument of Meas is temporarily suppressed to allow composition with the VP; the result is truth conditions that require that the quantity of guests who arrive exceeds all degrees in N, the interval representing ‘neutral’ quantities of guests who arrived.

(35) Many guests arrived.
   a. [Meas guests arrived] = \lambda d \lambda x [\text{guests}(x) \land \text{arrived}(x) \land \text{the quantity of } x \geq d]
   b. [Many Meas guests arrived] (existential closure over x)
      = \forall d \in N [\exists x [\text{guests}(x) \land \text{arrived}(x) \land \text{the quantity of } x \geq d]]

(36) Many fewer than 100 guests arrived.
   a. [Meas guests arrived] = \lambda d \exists x [\text{guests}(x) \land \text{arrived}(x) \land \text{the quantity of } x \geq d]
   b. [fewer than 100 guests arrived] = \lambda d \neg \exists x [\text{guests}(x) \land \text{arrived}(x) \land d \leq (100 \text{ minus the quantity of } x)]
   c. [many fewer than 100 guests arrived] = \forall d \in N [\neg \exists x [\text{guests}(x) \land \text{arrived}(x) \land d \leq (100 \text{ minus the quantity of } x)]]

Informally, the truth conditions in (36) hold in any situation in which no quantity of guests who arrived exceeded 100, and the difference between the quantity of guests who arrived and 100 is significant (i.e. outside of the salient neutral zone N).

Both Rett (2014) and Solt (2015) additionally extend their accounts to a syntactic phenomenon in English called ‘much-support’ (Corver, 1997), in which much cannot typically modify adjectives (e.g. *John is too much tall) but is required to modify proadjectives (e.g. *John is intelligent and Sue is too *(much) so). And they both extend their accounts to the VP modifier uses of quantity words, in which (following work in Nakanishi, 2007; Doetjes, 2007) they appear to range over events. Recent work by Wellwood and colleagues (Wellwood et al., 2012; Wellwood, 2014, 2015) more explicitly treats the VP modification of quantity words in a similar domain-general approach.
Summary

In the foundational traditions of first-order logic and Generalized Quantifier Theory, quantity words are analyzed as quantifiers, relating two sets of individuals. A long-standing competitor to this perspective characterizes quantity words as gradable adjectives, i.e. relations between degrees and individuals. However, as demonstrated in Table 1, both treatments of quantity words are unsatisfying for the same reason: they incorrectly predict that quantity words are only able to range over individuals, when in fact, they are able to range over events (as VP modifiers); vectors (as PP modifiers); and intervals (as comparative modifiers).

In a series of recent proposals, degree semanticists have argued that all of these uses of quantity words have in common that they range over intervals, or sets of degrees. This is transparently true when quantity words modify comparatives, as in John is much taller than Sue; but, as Rett (2007, 2008b, 2014) and Solt (2009, 2015) have argued, the characterization of quantity words as ranging over intervals also naturally extends to the other uses of quantity words. In addition to these distributional motivations for a domain-general characterization of quantity words, there are a number of empirical motivations for the switch, including (but not limited to) some phenomena discussed here: the distribution of evaluativity across quantity word constructions (Rett, 2015) and the antonymic relationship between positive quantity words like many and negative ones like few.

References


