Presupposition Projection

Three Valued Logic

Three valued logic. Suppose there are three true values, 0, 1 and $*$. Let minimally the following connectives be defined.

(1)

$\downarrow$	1	0	$*$
1	1	$*$	$*$
0	0	$*$	$*$
$*$	$*$	$*$	$*$

$\neg$	
1	0
0	1
$*$	$*$

For conjunction, there are various choices.

(2)

$\overset{\shortmid}{\wedge}$	1	0	$*$
1	1	0	$*$
0	0	0	$*$
$*$	$*$	$*$	$*$

$\overset{\triangleright}{\wedge}$	1	0	$*$
1	1	0	$*$
0	0	0	0
$*$	$*$	$*$	$*$

(3)

$\overset{\triangleleft}{\wedge}$	1	0	$*$
1	1	0	$*$
0	0	0	$*$
$*$	$*$	0	$*$

$\overset{\diamond}{\wedge}$	1	0	$*$
1	1	0	$*$
0	0	0	0
$*$	$*$	0	$*$

The first is the **Bochvar**–and or **weak Kleene conjunction**; the last is the **string Kleene conjunction**. The second is the left–to–right dynamic version, the third the right–to–left dynamic version. The rationale in the dynamic versions is that we take advantage of 'a priori' knowledge. If p is false, $p \wedge q$ is false, no matter what q is (this is akin to the supervaluations by van Fraassen). But the rationale is only motivation. In a computer we find the left–to–right dynamic version implemented. This is also the case is disjunction, while we generally find that natural language disjunction is ambiguous between a left–to–right and a parallel version. Given a formula, and a mapping v from the variables to the truth values, v can naturally be extended to all formulae over these variables. We then define $\Gamma \vDash \varphi$ if for every valuation v, if $v(\gamma) = 1$ for all $\gamma \in \Gamma$, then $v(\varphi) = 1$. (Notice that Γ can be partially false and partially undefined in v. In this case, no condition on φ.) φ is **inconsistent** with Γ if there is no assignment v that makes Γ and φ true.

Definition 1 *$\varphi \gg \chi$ iff for every valuation v: if $v(\varphi) \in \{0,1\}$ then $v(\chi) = 1$. Alternatively, $\varphi \gg \chi$ iff $\varphi \vDash \chi$ and $\neg\varphi \vDash \chi$.*

Now, by definition $p \overset{\shortmid}{\wedge} (q \downarrow p)$ presupposes p (in symbols $p \overset{\shortmid}{\wedge} (q \downarrow p) \gg p$). However, $p \overset{\triangleright}{\wedge} (q \downarrow p) \not\gg p$! So, choosing a different control structure makes all the difference.

Context Selection

Below is an outline of the theory by Rob van der Sandt. However, rather than following his definitions strictly, I have tried to reread the definitions in the light of the preceding discussion. It includes a departure from his view that the underlying logic is two–valued. This creates a number of complications, but is actually necessary. Notice that the English connectives `and`, `or` and `if` $\cdots$ `then` now enjoy different readings, since their three–valued extensions are not unique. Depending on the choice, we get a different formula that translates our original sentence. The formula is three–valued, so it has presuppositions, and the presupposition relation on them is easily defined.

- The interpretation of a sentence offered in isolation is always restricted to the set of contexts in which they can be uttered acceptably and coherently.

- The elementary presuppositions of sentences are indicators for context selection. Their selecting role, is, however, subordinated to (a).

So, the idea is that presuppositions tell us what conditions a context has to meet if the sentence just uttered is successful. Now, it hardly is the case that discourse precedes in that orderly a fashion. Therefore, in order to make a sentence acceptable despite the fact that it does not meet the presuppositions people often 'accommodate' the context: they treat the presupposition as an implicit claim; they add it to the discourse as if having been made. So, upon hearing 'If John stays until late, his wife will be angry.' we will tacitly add the fact that John is married to the discourse so that the utterance now becomes acceptable. This update of the context however may be unsuccessful. If we know that John is unmarried, this update will make our beliefs inconsistent. In this case we are likely to protest and interrupt the speaker, making him aware of the fact that we disagree with his (implicit) claim. Thus, the felicity of an utterance is contingent on not contradicting previous discourse (or context). This is quite general. This is basically Grice's Maxims of Informativeness.

A sentence however does not have a unique interpretation. The interpretation depends on the context in various ways. Van der Sandt draws the following conclusions from the above statements.

- ☞ When a sentence has an elementary presupposition and the text coming about as a result of the addition of the elementary presupposition to the context set is acceptable, then the sentence allows a presuppositional reading.

- ☞ When a sentence has an elementary presupposition and the addition of this elementary presupposition to the context set makes the corresponding text unacceptable, the presupposition is lost.

What is a context set? It is a set of sentences. Wat does it mean to 'lose a presupposition'? We take the interpretation of sentences to be coded in formulae of propositional or predicate logic, but three–valued logic for that matter. (We are committing ourselves to three–valued logic where Rob van der Sandt does not, for reasons tat I have explained earlier.) Pragmatically, in two valued logic, φ is defined to be acceptable in Γ, in symbols $A(\Gamma, \varphi)$, if neither $\Gamma \vDash \varphi$ nor $\Gamma \vDash \neg\varphi$. Notice that this means that it is neither the case that all assignment that make Γ true also make φ true, or that all assignments that make Γ make φ false. In three–valued logic, we need to say the following:

Definition 2 *φ is **acceptable** in Γ ($A(\Gamma, \varphi)$) iff both $\Gamma; \varphi$ and $\Gamma; \neg\varphi$ are satisfiable.*

(RvdS takes a context to be more than a set of propositions, but that offers no help here.)

The basic philosophy is this: sentences are uttered in a language L, which we take here to be English. These sentences are converted into formulae. This rendering depends however on the context. This rendering is also referred to as presupposition projection; I find this a misnomer. I call this **reallocation**. It basically reallocates the presupposition. Moreover, the process known as **accommodation** is an update on the context done on the basis of the presuppositions of the given sentence. Both accommodation and reallocation go hand in hand. If a presupposition is reallocated at highest level, the context is immediately accommodated. The intermediate reallocations, however, are less straightforward.

What is the strategy at play? Lexical entries trigger so–called **elementary presuppositions**. We can represent them as χ in the formulae $\varphi \downarrow \chi$. (If there are several, we may take the conjunction.) The explicit presence of χ triggers a check for the satisfaction of the felicity conditions. If the felicity conditions are consistent with the context, they are added. The accommodation can be mimicked by treating the utterance of $\varphi \downarrow \chi$ as short for $\chi \stackrel{\triangleright}{\wedge} (\varphi \downarrow \chi)$. So, (4) is considered to be short for (5).

(4) `John regrets having put money into his Swiss bank account.`

(5) `John has put money into his Swiss bank account and he regrets it.`

Notice that in assuming (4) to be a disguised version (5) means that we are assuming that it has a different logical form. This is unfortunate. One wishes to say, rather, that it triggers a pragmatic process of accommodation. But there are other cases in which the choice between reading a different form into it and assuming an update is not clear. Here is an exammple by Rob van der Sandt.

(6) `Either John has stopped smoking or he has just started doing so.`

(7) `John has stopped smoking.`

(8) `John has been smoking until now.`

(9) `John has started smoking.`

(10) `John has not been smoking until now.`

The first disjunct by itself, (7), presupposed (8); the second disjunct, (9), presupposed (10), which contradicts the first. Now, which of the presuppositions is accommodated? Since we cannot choose, we opt to accommodate neither. In this case, Rob van der Sandt says that the presuppositions are lost. What does that mean? The intuition is that it means the presuppositions now function as pure assertions, as if the sentence is saying

(11) `Either John has been smoking and stopped smoking or he he has not been smoking and just started`

The two `and` must here be interpreted by $\overset{\triangleright}{\wedge}$. Accommodation is a main level pehnomenon, an update on the context, so the present example is interesting inasmuch as it seems to suggest that we need a different process of 'lower level accommodation'. To see the need for this, let us take the following translation of the sentence as

(12) $(\sigma \downarrow \tau) \overset{\diamond}{\vee} (\neg\sigma \downarrow (\neg\tau))$

(13) $\sigma =$ `John has been smoking until now.`

(14) $\tau =$ `John is not smoking now.`

Let us map out the truth values of this:

(15)

$(\sigma$	$\downarrow$	$\tau)$	$\overset{\diamond}{\vee}$	$((\neg$	$\sigma)$	$\downarrow$	$(\neg$	$\tau))$
1	1	1	1	0	1	$*$	0	1
1	$*$	0	$*$	0	1	0	1	0
1	$*$	$*$	$*$	0	1	$*$	$*$	$*$
0	0	1	$*$	1	0	$*$	0	1
0	$*$	$*$	1	1	0	1	1	0
0	$*$	$*$	$*$	1	0	$*$	$*$	$*$
$*$	$*$	1	$*$	$*$	$*$	$*$	0	1
$*$	$*$	0	$*$	$*$	$*$	$*$	1	0
$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$

Even if both σ and τ are bivalent (cannot be $*$), the sentence is not free of presupposition. It's presupposition can be phrased as: 'John changed his smoking habit.' Interestingly, if the presupposition is true, the sentence becomes trivial! Thus, in the present translation the sentence is unacceptable. Thus, something of the translation offered below is actually needed:

(16) $(\tau \overset{\triangleright}{\wedge} (\sigma \downarrow \tau)) \overset{\diamond}{\vee} (\sigma \overset{\triangleright}{\wedge} ((\neg\sigma) \downarrow (\neg\tau))$

The truth table for this is

	$(\tau$	$\overset{\triangleright}{\wedge}$	$(\sigma$	$\downarrow$	$\tau))$	$\overset{\diamond}{\vee}$	$((\neg$	$\tau)$	$\overset{\triangleright}{\wedge}$	$((\neg$	$\sigma)))$	$\downarrow$	$(\neg$	$\tau))$
	1	1	1	1	1	1	0	1	0	0	1	$\ast$	0	1
	0	0	1	$\ast$	0	0	1	0	0	0	1	0	1	0
	$\ast$	$\ast$	1	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	0	1	$\ast$	$\ast$	$\ast$
(17)	1	0	0	0	1	0	0	1	0	1	0	$\ast$	0	1
	0	0	0	$\ast$	$\ast$	1	1	0	1	1	0	1	1	0
	$\ast$	$\ast$	0	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	1	0	$\ast$	$\ast$	$\ast$
	1	$\ast$	$\ast$	$\ast$	1	$\ast$	0	1	0	$\ast$	$\ast$	$\ast$	0	1
	0	0	$\ast$	$\ast$	0	$\ast$	1	0	$\ast$	$\ast$	$\ast$	$\ast$	1	0
	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	$\ast$

The revised interpretation has made the sentence bivalent for the cases that σ and τ are not $\ast$. This means that the presuppositions of σ and τ are inherited (which we expect).

This case is interesting in many ways. It shows on the one hand that accommodation as a process is not enough, on the other that the interpretation of a formula can be more complex that seen at first sight. Pragmatically, the second interpretation, being nontrivial, is stronger, since it can be false in certain circumstances. However, we should resists the conclusion that all that happens is that the presuppositions 'are lost'. This is misleading, partly for the reason that the presuppositions can be implicit in the lexical item (think of partial predicates). The partially can never be dropped, as if it were possible to remove the failure in dividing by zero or make the expression `the present of France` a denoting expression. What we can do, however, interpret the sentence as a pragmatically stronger statement, as indicated. The presuppositions of the new sentence is that σ and τ are bivalent, or formally: $(\sigma \vee \neg\sigma)$ and $(\tau \vee \neg\tau)$ (NB it does not matter which of the $\vee$ we take!).

Rob van der Sandt has later revised his position. He promoted the idea that presuppositions simply are anaphors. A presupposition is bound in its position if the local context implies it. The local context is something that must be explicitly defined. If a presupposition is bound in its position, it will not trigger any reanalysis. If it is not bound, it can move outside of its position. Where it can go, however, depends among other by the reanalysis that is triggered overall. If it renders the formula pragmatically unacceptable, the attempt to move the presuppositions up. For example, in a disjunction of the form we have discussed above, each disjunct contributes a presupposition, but the two are contradictory. In this case, the presuppositions are barred from moving higher. The interesting facet of this proposal is that it ties the accommodation of discourse markers practiced in DRT to the projection (or 'movement') algorithm. In DRT, you place a discourse marker for every pronoun or name that you meet. Some of the discourse markers are placed higher than others. Names generally introduce a discourse marker which is accommodated at the top level. The reason why this does not happen for a reflexive is that it must be bound, and that this binding preempts the marker from being moved up. Unfortunately, this view has its drawbacks. Markers are objects, they cannot be equated with DRSs. Presuppositions are DRSs, and binding of DRSs is a different notion. It amounts to the nonexistence of runtime errors for that presupposition. If accommodation or reanalysis is triggered by the need to avoid runtime errors, the fact that bound presuppositions stay put is easily explained. What we have not explained is the habit of presuppositions to move up as high as possible.

An example Analyzed

Presupposition projection depends also on a number of other factors; a sentence can have multiple readings (de re/de dicto; different qauntifiers scopes), and pronouns and anaphors can have different antecedents.

(18) `If Nixon appoints J. Edgar Hoover to the cabinet then he will regret having appointed a homosexual.`

(19) `J. Edgar Hoover is a homosexual.`

(20) `Nixon will have appointed a homosexual.`

Take 1. Suppose we have no knowledge about J. Edgar Hoover (context: $\varnothing$). Then (18) seems to presuppose (20). Suppose, however, the context is $\{(19)\}$, then (18) does not presuppose (20).

Take 2. On closer inspection, there are several problems with the previous analysis. First, 'regret' is a verb of propositional attitude. Even if J. Edgar Hoover is homosexual, Nixon might not know about it. This opens the possibility of two readings of 'homosexual': in (18) we can think of `a homosexual` as a phrase describing J. Edgar Hoover from speaker's point of view (de re attribution), or it could be a description that Nixon could have used himself to refer to Hoover. Neither reading blocks `a homosexual` to refer back to Hoover. But suppose that Nixon does not know that Hoover is homosexual, yet he knows about another member of his cabinet, say Mr. X, that he is homosexual, then (18) opens itself to a third reading: if Nixon appoints Hoover, then he will regret having appointed Mr. X. That is not implausible. (Maybe having two homosexuals in a cabinet creates complications that Nixon will later regret thinking it was the appointment of Mr. X that caused the trouble.) The regret of having appointed remains a presupposition of (18) even if (19) is in the context set. In a way the problems arise because the sentences are not clear enough on the reading that is intended. What van der Sandt's theory would claim, though, is that having (19) in the context set will make it likely that the phrase `a homosexual` in (18) is used to refer to Hoover.

Take 3. Notice that the fact that Edgar Hoover is homosexual is not enough for Nixon to know that this is the case. So, for Nixon to regret it, he must be aware of it. (19) is strictly speaking not enough to ensure that. We need to add

(21) `Nixon knows that J. Edgar Hoover is a homosexual.`

Further, what is the content of his regret? It could be: (a) having appointed J. Edgar Hoover, (b) having appointed Mr. X (if that was the case), and (c) having appointed someone who is homosexual. For simplicity, assume that Nixon remembers correctly his actions. To regret having appointed Hoover presupposes that he did. Likewise, to regret having appointed Mr. X presupposes that he did, and to regret having appointed a homosexual presupposes that he did.

Notice further, as we pointed out above, (a) could be rendered as

(22) `Nixon regrets having appointed a homosexual.`

if it is common knowledge that Hoover is a homosexual. In this case, it seems to presuppose that Nixon appointed Hoover as well as that Nixon appointed a homosexual. This analysis is interesting. On the face of it, we expect the phrase 'to regret P' to mean that P is the case and that one has negative feelings towards P. Now it means that one thing is true and another is what the content of one's feelings is. Maybe this is not desirable. So let us read this as: Nixon has negative feelings towards having appointed Hoover, presupposing Nixon has appointed Hoover, where the latter is disguised as

(23) `Nixon has appointed a homosexual.`

Modulo (19) (which is in the context), this is implied by

(24) `Nixon has appointed J. Edgar Hoover.`

If indeed (22) is used to convey that Nixon appointed Hoover, there is nothing to worry about. Moreover, (18) does not presuppose (20).

We can play a similar game with pronouns.

(25) `If Nixon appoints J. Edgar Hoover to the cabinet then he will regret having appointed him.`

(26) `Nixon will have appointed Edgar Hoover.`

If `him` is used to refer to Hoover, then the presupposition by the second sentence, (26) is cancelled by the antecedent. In the zero context this is a likely scenario: in the absence of other candidates to look out for, the pronoun is linked to Hoover. But if the discourse contains another person, say Mr. X, it is possible to make it refer to that other person. In this case, (25) presupposes that Nixon appointed Mr. X. We can to the same effect point at Mr. X and utter

(27) `If Nixon appoints J. Edgar Hoover to the cabinet then he will regret having appointed HIM.`

This presupposes that Nixon appointed whoever we point to (Mr. X).

In sum, pronoun reference determines what presuppositions survive, since the fill the message with content that is different for different indexations. The indexations in turn depend on the context. Certain indexations are more likely than others. Likelihood is determined also with respect to the presuppositions the sentence eventually carries when an indexation has been decided upon. There is then a pragmatic loop: (a) decide indexation, (b) calculate presuppositions, (c) determine pragmatic acceptability. If uncaceptable, go back to (a) again.