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ANSWER KEYS, PART 2.

Ex 2.1 Let 'O' stand for the relation of owning, 'C', 'D' and 'F' for the property of being a car, a donkey and a farmer, respectively. Let 'p' denote Pedro. Calculate the truth conditions of the formulae to the right to show that they codify the truth conditions of the corresponding sentence on the left:

① *Pedro owns something.* ∃ x p O x
This sentence is true...
...iff for any assignment σ it is true$_\sigma$
...iff for any σ there exists an object o such that "p O x" is true$_{\sigma[^x/_o]}$
...iff for any σ there exists an object o such that $\sigma[^x/_o](p)$ stands in the relation that "O" stands for to $\sigma[^x/_o](x)$.
...iff for any σ there exists an object o such that Pedro stands in the relation of owning to o
...iff Pedro owns something.

② *Someone owns everything.* ∃ x ∀ y x O y
...iff for any assignment σ it is true$_\sigma$
This sentence is true iff for any assignment σ it is true$_\sigma$
...iff for any σ there exists an object o such that "∀ y x O y" is true$_{\sigma[^x/_o]}$
...iff for any σ there exists an object o such that for any object o', "x O y" is true$_{\sigma[^x/_o][^y/_{o'}]}$
...iff for any σ there exists an object o such that for any object o', $\sigma[^x/_o][^y/_{o'}](x)$ stands in the relation that "O" stands for to $\sigma[^x/_o][^y/_{o'}](y)$.
...iff for any σ there exists an object o such that for any object o' o stands in the relation of owning to o'
...iff Pedro owns something.

Also, calculate the truth conditions of the following formula and render it into natural English.

③ ∀ x C x → ¬ D x.
This sentence is true...
...iff for any assignment σ it is true$_\sigma$
...iff for any σ and any object o "C x →¬ D x" is true$_{\sigma[^x/_o]}$
...iff for any σ and o "C x" is false$_{\sigma[^x/_o]}$ or "¬ D x" is true$_{\sigma[^x/_o]}$ (or both)
...iff for any σ and o, $\sigma[^x/_o](x)$ is NOT a member of the set that "C" denotes or "D x" is false$_{\sigma[^x/_o]}$ (or both)
...iff for any σ and o, o is not a car or $\sigma[^x/_o](x)$ is NOT a member of the set that "D" denotes (or both)
...iff for any σ and o, o is not a car or o is not a donkey (or both).
...iff for any object o, o is not a car or o is not a donkey (or both).
"Everything is either not a car or not a donkey," or
"If something is a car, then it's not a donkey."

Ex 2.2 (With the conventions of the previous exercise.) Give all possible formal renderings of the following sentences (even if you think that some of the scopings do not match your intuitions):

① *Every farmer owns a donkey.*
every x { **farmer** x } **a** y { **donkey** y } x **own** y
Scope of **every**: "every x { farmer x } a y { donkey y } x own y"
Scope of **a**: "a y { donkey y } x own y"
a x { **donkey** x } **every** y { **farmer** y } y **own** x
Scope of **every**: "every y { farmer y } y own x"
Scope of **a**: "a x { donkey x } every y { farmer y } y own x"

② *Some farmer does not own every car.*
¬ **some** x { **farmer** x } **every** y { **car** y } x **own** y
some x { **farmer** x } ¬ **every** y { **car** y } x **own** y
some x { **farmer** x } **every** y { **car** y } ¬ x **own** y
¬ **every** y { **car** y } **some** x { **farmer** x } x **own** y
every y { **car** y } ¬ **some** x { **farmer** x } x **own** y
every y { **car** y } **some** x { **farmer** x } ¬ x **own** y

③ *No farmer owns every car.*
no x { **farmer** x } **every** y { **car** y } x **own** y
every y { **car** y } **no** x { **farmer** x } x **own** y

"**no** x { **farmer** x } **every** y { **car** y } x **own** y" is the same as " ¬ **some** x { **farmer** x } **every** y { **car** y } x **own** y"

"**every** y { **car** y } **no** x { **farmer** x } x **own** y" is the same as "**every** y { **car** y } ¬ **some** x { **farmer** x } x **own** y"

Give the scopes for the various readings of the first sentence. There are readings of the second and the third sentence which are equivalent. Which ones are they?

Ex 3.3 (a) Let 'P' stand for the property of being a penguin, and 'F' for the property of being able to fly. We have seen that the sentence 'Every penguin flies.' can be rendered as $\mathbf{every}_{\mathrm{x}}$\{P x\} F x. Show that it can also be rendered as

$\forall$ x P x $\rightarrow$ F x

By our method of calculating truth conditions, this sentence is true if for any object *o* either *o* is not a penguin or *o* flies. This is, in fact, an appropriate rendering of the English sentence "Every penguin flies." Consider: if an object *o* is a penguin, then the first half of the disjunct (i.e., that *o* is not a penguin) is false. This means that if *o* is a penguin the disjunction is only true if the sencond half of the disjunction (*o* flies) is true. If *o* is not a penguin, then the first half of the disjunction is already true so the second half is irrelevant. Thus, the logical formula "$\forall$ x P x $\rightarrow$F x" is true if and only if every penguin flies; thus it is a valid rendering of "every penguin flies."

(b) Based on the previous, suggest a formula equivalent to $\mathbf{every}_{\mathrm{x}}\{\varphi\}\psi$ using $\forall$ in place of **every**.

$\forall$x$(\varphi \rightarrow \psi)$

Ex 4.4 Draw the phrase structure of the following formula:

$\neg\mathbf{exists}_{\mathrm{x}}$\{D x & S x\} P x