INTENSIONS

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1. The Way Things Are

This note accompanies the introduction of Chapter 4 of the lecture notes. I shall provide some formal background and technology. Let a language $\mathfrak{L}$ be given that contains a few words such as **bachelor**, **married**, **renate**, **cardiate**, **clark kent** and **superman**. The facts as they are can be coded into a *structure*; it is a pair $\mathfrak{M} = \langle D, I \rangle$, where D is the domain, and I a function that interprets our symbols. As it happens, $I(\mathbf{bachelor}) \subseteq I(\mathbf{married})$ (since no bachelor is married), $I(\mathbf{renate}) = I(\mathbf{cardiate})$ (since renates turn out to be cardiates) and $I(\mathbf{clark\ kent}) = I(\mathbf{superman})$ (since Kent Clark is Superman). This can be rephrased differently. The first can be rendered

$$\mathfrak{M} \vDash (\forall x)(\mathbf{bachelor}\ x \rightarrow \neg\mathbf{married}\ x) \tag{1}$$

which says that whatever is a bachelor is not married. The second becomes

$$\mathfrak{M} \vDash (\forall x)(\mathbf{renate}\ x \leftrightarrow \mathbf{cardiate}\ x) \tag{2}$$

which says that whatever is a renate is a cardiate and conversely. (Recall that $\varphi \leftrightarrow \chi$ is the same as $\varphi \rightarrow \chi \& \chi \rightarrow \varphi$.) The second can be rephrased as

$$\mathfrak{M} \vDash \mathbf{clark\ kent} = \mathbf{superman} \tag{3}$$

I find it useful to rewrite this into

$$\mathfrak{M} \vDash (\forall x)(x = \mathbf{clark\ kent} \leftrightarrow x = \mathbf{superman}) \tag{4}$$

The phrase '$x =$ **clark kent**' will be rendered 'x Clark Kents'; therefore the above says that whatever Clark Kents also Supermans and conversely.

From this we can shown the following arguments to be valid.

$$\frac{\texttt{Pedro is a bachelor.}}{\therefore\texttt{Pedro is not married.}} \tag{5}$$

$$\frac{\texttt{Pedro owns a cardiate.}}{\therefore\texttt{Pedro owns a renate.}} \tag{6}$$

The proof can be done in essentially two ways: the first is to interpret these sentences in $\mathfrak{M}$. We perform this now. If the first sentence is false, nothing needs to be done. If it is true, the second will be. In the first example, $I(\mathbf{pedro})$ is an object, say o. By assumption, $o \in I(\mathbf{bachelor})$. Now, $o \in I(\mathbf{married})$ does not hold, since the two sets are disjoint. So, `Pedro is married.` is false, hence `Pedro is not married.` is true. In the second example, assume that the premiss is true. So o owns some o' which is a renate; that is to say, $\langle o, o' \rangle \in I(\mathbf{own})$ and $o' \in I(\mathbf{cardiate})$. Since $I(\mathbf{renate}) = I(\mathbf{renate})$ we also have $o' \in I(\mathbf{renate})$. It follows that `Pedro owns a renate.` is true.

The second method for showing the validity of the argument is to throw in the additional facts that the structure supports. Informally, this corresponds to the validity of the following arguments *under no condition on the model*:

(7)
$$\frac{\texttt{Pedro is a bachelor.} \quad \texttt{No bachelor is married.}}{\therefore \texttt{Pedro is not married.}}$$

(8)
$$\frac{\texttt{Pedro owns a cardiate.Every renate is a renate.}}{\therefore \texttt{Pedro owns a cardiate.}}$$

In these forms they are actually standard syllogisms! The fact is now that the arguments (7) and (8) do not require any condition to hold in the model. They are always true, by pure logic.

2. The Way Things Might Be

The interesting fact is that not everyone buys (6). Some person, say Arnold, may believe that Pedro owns a cardiate but fail to believe that he owns a renate. And this could even be so despite the fact that he knows what these words mean. He simply believes that Pedro only owns a rabbit, that this rabbit has a heart but no kidney. His belief is factually wrong, but that does not affect its nature as a belief. Beliefs do not have to true. On the other hand, if Arnold believes that Pedro is a bachelor and fails to believe that Pedro is unmarried then we will think that Arnold does not know what these words mean. Anyone who knows English must know that bachelors are unmarried. Its part of the word 'bachelor' means. Thus, (5) goes through, because we must assume that he knows English. (It is OK to assume that Arnold does not know any English; in that case we are attributing to believe what these sentences express. That will however strengthen my point. Because then he believes that Pedro satisfies the concept of bachelorhood (in whatever he would express that) and therefore also believe that Pedro satisfies the concept of being unmarried, since that is part of the concept of bachelorhood.)

In order to analyse the situation we shall say that (1) is a meaning postulate. Knowing it is part of knowing the language. (2) and (3), however, are not of that kind. Generally, language places no restriction on what names denote. We are free to assign them to any person or thing we please. Thus, knowing English does not imply that you know that Clark Kent is Superman. It also does not imply that you know that cardiates are renates. And this is because it is not part of the meaning. However, and this is the difficult part, despite not being part of the meaning it might well be a consequence of what things mean. It is a consequence of the fact that n is a natural number that it is the sum of four squares, but the latter is hardly part of the meaning. In order to separate the two we shall talk about alternative structures. Let us fix D for the discussion. Let us however imagine a function I' that assigns different individuals to our constants, different sets to the one place predicate letters and different relations to the binary predicate letters. To be exact, I' may differ from I in any given letter, and may agree in others. For example, if $I(\mathbf{pedro}) = o$, now we could have $I'(\mathbf{pedro}) = o' \neq o$.

Let's see a concrete example. Let $D = \{o, p, q\}$, let $I(\mathbf{pedro}) = o$, $I(\mathbf{own}) = \langle o, p\rangle$, and $I(\mathbf{cardiate}) = I(\mathbf{renate}) = \{p, q\}$. In this world, cardiates are renates and conversely. Let us define also $I'(\mathbf{pedro}) = o$, $I'(\mathbf{own}) = \langle o, p\rangle$, $I'(\mathbf{cardiate}) = \{p\}$ and $I'(\mathbf{renate}) = \{q\}$. Put $\mathfrak{M} = \langle D, I\rangle$, $\mathfrak{M}' = \langle D, I'\rangle$. In $\mathfrak{M}$, (2) holds, but in $\mathfrak{M}'$ it does not. $\mathfrak{M}$ is a candidate for the actual structure, $\mathfrak{M}'$ is not. However, $\mathfrak{M}'$ is well-formed; it corresponds to all laws of English, as far as we know them. $\mathfrak{M}'$ shows us a way things could have been.

The technical jargon is to call $\mathfrak{M}$ and $\mathfrak{M}'$ **worlds** (or also **possible worlds**). There are many possible worlds even over a three element domain. Given three elements, a name has three choices as to what it shall refer to. A one place predicate is mapped to a subset of D. There are $2^3 = 8$ such subsets (including trivial cases as the empty set or the entire set); and there are $2^{3\cdot 3} = 2^9 = 512$ ways to map a binary predicate letter to a relation. However, not all of these worlds exist. For every world must satisfy the law (1). This reduces the choices of worlds somewhat. A **possible world** over D is now a structure over D that satisfies all meaning postulates. We write w, w' and so on for worlds. Notice that worlds are structures over D. That is to say, $w' = \langle D, I'\rangle$ for some I'. One of these worlds is distinguished as the actual world. We write w^* for it. Since worlds are structures, it makes sense to write $w \vDash \varphi$ for a formula of our language; moreover, let σ be a function from our set of variables to D. Then also $\langle w, \sigma\rangle \vDash \varphi$ is completely defined: it means that φ is true in the model $\langle w, \sigma\rangle$.

Definition 1. *A sentence is **true** iff it is true$_\sigma$ in w^* for every valuations σ into w^*.*

The idea is now this. Let us attribute to Arnold a set of worlds. These are called his **belief worlds**. And we say that Arnold believes φ if φ is true in every one of his belief worlds. With this definition, let us enter into (6) for Arnold's beliefs:

(9)
$$\frac{\texttt{Arnold believes that Pedro owns a cardiate.}}{\therefore\texttt{Arnold believes that Pedro owns a renate.}}$$

Let us choose an LF first.

(10)
$$\frac{\textbf{arnold believe}\hat{\ }[\textbf{some } x\{\textbf{cardiate } x\}\textbf{pedro own } x]}{\therefore \textbf{arnold believe}\hat{\ }[\textbf{some } x\{\textbf{renate } x\}\textbf{pedro own } x]}$$

The existentials are not raised out of the clause; this is just one choice we can make.

I shall show that this argument does not go through any more. For that, we use the worlds w and w' constructed above. $w^* := w$; and moreover, the belief worlds of Arnold are $\{w'\}$. Now, the premiss is true iff it is true in w^*. It is of the form `Arnold believes that __`, which means that we have to see whether the underlined sentence is true in all of his belief worlds. There is only one belief world, w'. So we need to check whether

(11) $$w' \vDash \textbf{some } x\{\textbf{cardiate } x\}\textbf{pedro own } x$$

This is the case; p is such an object. However,

(12) $$w' \nvDash \textbf{some } x\{\textbf{renate } x\}\textbf{pedro own } x$$

In fact, even more holds, namely

(13) $$w' \vDash \neg\textbf{some } x\{\textbf{renate } x\}\textbf{pedro own } x$$

So, Arnold believes that Pedro does not own a renate.

This is successful. It also generates the correct analysis for the first argument. This is so since we have declared that belief worlds must satisfy the meaning postulates, in particular (1). Thus, since it is a meaning postulate that no bachelors are married, Arnold believes that this is so, under the present analysis.

3. Meanings

So far we have said that I is the function that returns what the symbols stand for—in other words, their meaning. But the meaning of the symbols must contain everything we need to determine what a given sentence means. So we revise our initial proposal in the following way. We say that the meaning of a word, say **bachelor**, which we denote by [**bachelor**], is a function which returns for every given world $w = \langle D, I\rangle$ the value I(**bachelor**). So, we may write

$$[\textbf{bachelor}](w) = I(\textbf{bachelor}) \tag{14}$$

The meaning of the word `bachelor` is something that tells us for all possible ways thing could have been what things are bachelors and which ones are not.

Similarly, [**own**] returns for each world $w = \langle D, I\rangle$ a binary relation I(**own**). We can use the bracket '$[\varphi]$' to generate meanings for complex expressions φ from simple ones. There are two ways of doing so. The first, and most simple one, is as follows. Pick a world w, and compute the value of the expression φ in w. It is true or false. Do this for all worlds. For example, the expression $\textbf{some}x\{\textbf{renate}x\}\textbf{pedro own}x$ has the value true in w and false in w'. This can be established in the way given above. The second way is more roundabout: it is done by induction on φ. We shall not explain how it can be done, as this is outside the scope of these notes.

Definition 2. *Let φ be a formula without free variables, and $\langle W, w^*\rangle$ the set of worlds over D with actual world w^*. The **intension** of φ is the function $[\varphi]$. The **extension at** w is $[\varphi](w)$, which is nothing but the truth value of φ at w. φ is **true** at w if $[\varphi](w) =$ true. It is **necessarily true** or **analytic** if $[\varphi](w) =$ true for all $w \in W$.*

If φ is not a sentence but a formula with variables, we first have to select an assignment σ. The intension is a function that depends on the assignment σ and the world w. The extension is the function $[\varphi](w)$, which still depends on the assignment. φ is true at w if $[\varphi](w)(\sigma) =$ true for every assignment. φ is necessarily true if $[\varphi](w)(\sigma) =$ true for every world and every assignment.

4. Quantifier Raising and Proper Names

The issue now arises for proper names that they can be interpreted in different ways. Even though Clark Kent is Superman, that need not be the case in Arnold's belief worlds. However, it turns out also that the sentence

(15) `Arnold believes that Kent Clark is Superman.`

can be interpreted in four different way, depending on whether the name `Clark Kent` is taken to denote the individual that Arnold believes it denotes or whether it is taken to denote what he actually denotes. The LFs brings this distinction out as follows.

(16) $\mathbf{arnold\ believe}{}^\frown[\mathbf{some}\ x\{\mathbf{clark\ kent}\ = x\}\mathbf{some}\ y$
$\{\mathbf{superman} = y\}\ x = y]$

(17) $\mathbf{some}\ x\{\mathbf{clark\ kent} = x\}\mathbf{arnold\ believe}{}^\frown[\mathbf{some}\ y$
$\{\mathbf{superman}\ x\}\ x = y]$

(18) $\mathbf{some}\ x\{\mathbf{superman} = x\}\mathbf{arnold\ believe}{}^\frown[\mathbf{some}\ y$
$\{\mathbf{clark\ kent}\ x\}\ x = y]$

(19) $\mathbf{some}\ x\{\mathbf{superman} = x\}\mathbf{some}\ x\{\mathbf{clark\ kent}\ y\}$
$\mathbf{arnold\ believe}{}^\frown[x = y]$

Notice that we have translated the notation $\mathbf{superman}_x\varphi$ of the manuscript into $\mathbf{some}\ x\{\mathbf{superman} = x\}\ \varphi$. This is harmless, and it allows us to treat our names like quantified expressions.

Let us try to paraphrase these LFs. (16) goes as follows:

> Arnold believes that there is an individual that Clark Kents and there is some individual that Supermans, and the two are the same.

In this rendering, both name attributions are de dicto: the pick out individuals according to Arnold's views. Similarly, (17) can be paraphrased by

> There is an individual that Clark Kents and Arnold believes of it that there is an individual that Supermans and is identical to it.

The is a de re belief about Clark Kent. Arnold has a belief about him, but he might not be aware of the fact that he is actually called Clark Kent. The third LF, (18), is paraphrased as

> There is an individual that Supermans and Arnold believes of it that there is an individual that Clark Kents and is identical to it.

This is a de re belief about Clark Kent. Arnold has a belief about Superman without knowing that he is called by that name. Finally, the last one, (19), is

> There is an individual that Supermans and an individual that Clark Kents and Arnold believes of them that they are the same.

Notice in passing that for Arnold to have a de re belief about Superman is to have a de re belief about Clark Kent: they are the same person. What he fails to know is whether or not that person is called by any of those names. We shall show that these are all factually distinct readings. Let us create four worlds, w_1, w_2, w_3 and w_4. w_1 is the actual world. The domain is $\{o, p\}$.

$$(20) \qquad [\mathbf{superman}] = \begin{cases} w_1 \mapsto o \\ w_2 \mapsto p \\ w_3 \mapsto o \\ w_4 \mapsto p \end{cases} \qquad [\mathbf{clark\ kent}] = \begin{cases} w_1 \mapsto o \\ w_2 \mapsto o \\ w_3 \mapsto p \\ w_4 \mapsto p \end{cases}$$

Suppose that the belief worlds of Arnold are $\{w_1\}$. Then he blieves everything that is the case. In this situation, all four sentences are true. Assume now that his belief worlds are only $\{w_2\}$. The first sentence is now false: in the only belief of Arnolds, there is no individual that Clark Kents and Supermans. The second sentence is also false: o is such that it Clark Kents in w_1, and of this individual it is not true in w_2 that it Supermans (because for Arnold that individual is p). The third sentence is true. The individual o is such that in w_2 it Clark Kents. So, Arnold believes de re about o that it is called Clark Kent. And o is Superman in w_1. The fourth is true. If actually w_3 is the only belief world of Arnold, then the second reading would be true and the third false. Finally, let us assume that w_4 is the only belief world of Arnold. Then the first sentence is true: p is the thing of which Arnold believes it Clark Kents and Supermans, though in reality both claims are false. The second and third sentence both come out false; in either case the belief is about o, but in Arnold's belief worlds it is neither called Superman nor Clark Kent. The fourth LF is once again true.

The way things are, there is no way to make the fourth reading false. If Superman and Clark Kent pick out the same individual then it is impossible that Arnold believes that they are distinct. For notice that both attributions are now de re and pick out just one and the same individual without saying that Arnold believes it is called either Superman or Clark Kent. We are using here our ascriptions but we manage to pick out one and the same individual.

Now, what would happen if Arnold's belief worlds would be more than just a single world, say $\{w_2, w_3\}$? Then what he believes only what is true in both of them.

Let us finally note that every person has its particular belief worlds. Which set that is determines which facts that person holds to be true.

5. Nonextensional Contexts

We have started by noting that certain occurrences of words do not allow for substitution of identicals. More precisely, an occurrence of $\vec{x}$ is extensional if replacing it with an expression $\vec{y}$ does not change the truth value provided that $\vec{x}$ and $\vec{y}$ have the same extension. Talk of occurrences of words in a sentence is ambiguous, however. Look again at (15):

(21) `Arnold believes that Kent Clark is Superman.`

We have given it four different LFs. It turns out that the expression `Kent Clark` occurs extensionally in (17) and (19), but not in (16) and (18); and that `Superman` occurs extensionally in (18) and (19) but not in (16) and (17). It is therefore inappropriate make the assignment on the basis of the surface structure; we should rather do it on the LF (in the syntactic sense). So, let us suppose there are 4 different LFs:

(22) $\texttt{Arnold believes}\ [\texttt{Clark Kent}_1[\texttt{Superman}_2\ [t_1\ \texttt{is}\ t_2]]]$

(23) $\texttt{Clark Kent}_1[\texttt{Arnold believes}\ [\texttt{Superman}_2\ [t_1\ \texttt{is}\ t_2]]]$

(24) $\texttt{Superman}_2[\texttt{Arnold believes}\ [\texttt{Clark Kent}_1\ [t_1\ \texttt{is}\ t_2]]]$

(25) $\texttt{Clark Kent}_1[\texttt{Superman}_2[\texttt{Arnold believes}\ [t_1\ \texttt{is}\ t_2]]]$

We can now say in (22) – (25) the DP `Superman` occurs extensionally if and only if it is not c-commanded by `believe`; the same holds for the DP `Clark Kent`. An identical statement is that the occurrence is extensional if and only if it is not in the scope of **believes**.

The reason for this is hopefully now clear. The expression in the scope of `believe` will be the argument of **believe**. Whether or not an agent believes a proposition depends not only on its truth value at w^*, that is, its extension.

There are many words other than `believe` that create intensional contexts. These are

① Propositional attitude verbs: `think`, `doubt`, `judge`.
② Evaluatives: `good`, `perfect`, `immoral`.
③ Attitudinals: `alleged`.

Surprisingly, though, the analysis has other merits, too. For facts can change through time as well. Thus, strictly speaking any tense operator (`will`, past tense) also creates a form of nonextensionality. People are reluctant to call it that, because we think of the ‘world’ as the entire collection of worlds-at-each-moment. But let us take things at face value. If Pedro owns a donkey and it is grey, then the following is true:

(26) $$\frac{\texttt{Pedro owns a donkey.}}{\therefore \texttt{Pedro owns a grey animal.}}$$

But it is not necessarily true. More exactly, we do not have

(27) $$\frac{\texttt{Pedro always owns a donkey.}}{\therefore \texttt{Pedro always owns a grey animal.}}$$

This is simply because even if it were always guaranteed that Pedro owns a donkey, it may not be a case that he owns a grey animal. (Thinkof Pedro owning two donkeys now, just one of them grey. Later he sells the grey donkey, but is never getting another animal. Suppose finally that the other donkey he always keeps. Then the first sentence is true but the second false.)

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