Gnosis

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Abstract

The transition from form to meaning is not neatly layered: there is no point where form ends and content sets in. Rather, there is a continuous semiosis that procedes from form to meaning. That semiosis cannot be a straight line. Very often we hit barriers in our mind, due to the inability to represent the exact content of the words just heard.

[Die Physik] weiß, daß gerade die Grundlage einer Wissenschaft von der Wirklichkeit nur als Hypothese ausgesprochen werden kann. Von dieser Auffassung her könnte man genau umgekehrt als Lorenzen die Evidenz der Logik preisgeben und dafür ihren ontologischen Charakter festhalten. So versteht Picht die Sätze der Logik als „erkannte Seinsgesetze", die aber das Seiende nicht so zeigen, wie es an sich selbst ist, sondern ihm eine bestimmte Perspektive auferlegen.

C. F. von Weizsäcker: *Zeit und Wissen*, p. 684

1 Introduction

This paper is about the problem of using one's knowledge of the meaning of something in understanding complex expressions. It contends that there are limits to what we can understand just by unpacking a definition and applying it. These limits come in part from a particular design of our reasoning system, in part they have to do with the problem of abstractness of content: logical notions do not have counterparts in ordinary experience except in the most trivial way. Logical languages allow to express facts about the world through concepts that seem to reach beyond it. One such notion is that of consequence and implication. There is, I contend, no physical correlate to them. They are purely conceptional, and must be treated as that. If that is right, then the meaning of these entities have more to do with our conceptualizations than with the facts themselves. This may provide a synthesis of Lorenzen against Georg Picht. At a lower level, logical notions do represent facts, but one level up it loses its counterpart in the world and trades on concepts instead.

More exactly, let the following formula be given:

(1) $$\varphi = \delta_0 \to (\delta_2 \to (\cdots \to (\delta_{n-1} \to \delta_n)\cdots))$$

Using the deduction theorem $\vdash \varphi$ can be reduced to

(2) $$\delta_0; \cdots ; \delta_{n-1} \vdash \delta_n$$

The arrows of the original formula have disappeared. If the δ_i are basic formulae, they point to certain propositions, and so (2) exhibits a regularity of the world, so to speak. By contrast, if δ_0, say, contains an occurrence of $\to$, there is no way to eliminate this occurrence through the use of the Deduction Theorem. Then (2) does not describe a regularity of the world because the arrow has no physical correlate. Its meaning is typically defined precisely with the DT as follows:

(3) $$T \vdash \varphi \to \psi \text{ :iff } T; \varphi \vdash \psi$$

However, the definition as given makes use of the judgement sign '$\vdash$', and thus cannot be inserted into (2). In other words: the definition is such that it places limits on reducibility which are not matched by syntactic restrictions. We have not excluded leftward nesting of arrow, but when it happens it creates a problem.

This tension exists also in human sentence processing and reasoning. We try to use a definition by simply unpacking it and see where we can take things. But we may get stuck. What comes to rescue is that we may actually introduce correlates to the linguistic objects. Functions and sets are notions that arise through reification of this sort, and can be used to overcome the restrictions in expressing our thoughts. It is helpful in this respect to look at computer languages. A function is generally defined by saying what its values are on the inputs. Languages with type systems may allow to use functions as inputs to functions. Once defined, a function may then also be applied to a function as long as it matches the type requirement of the input variable. The type of a function that uses inputs of type δ_i, $i < n$, is exactly as given in (1). Unless arguments are themselves understood to be functions (as the definition of the function may require), each of the type δ_i is actually primitive. The complex types not covered by (1) can come into existence only through reification of functions. Notice, however, that the function is a purely symbolic notion. The function does not exist, so I claim, in the same way as a chair exist. Its correlate is a particular program for the computer.

The main interest below is actually neither type theory nor meaning. It is the idea that the limitations of the sort explained above have consequences for semantics of natural languages. They suggest that there is something fundamentally different between logical and nonlogical notions in that logical notions may require reficiation of higher order concepts, but that this reification comes at high costs. In mathematics we get trained to reason effectively with notions that are conceptually very difficult: one example is the notion of a continuous function. It takes most people a very long time until they even understand what it says not to mention the problem of applying the notion in proofs.

2 Understanding What Is Said

As a particular example we take Peirce's Law:

(4) $((p \rightarrow q) \rightarrow p) \rightarrow p$

Other than being a formula of a syntactic kind, what does it actually *say*? When I try to apply the standard meaning of implication (which is something like (2)) I cannot not grasp what the entire formula is saying. There also is, I find, no adequate way to actually read the formula. The best I can come up with is

(5) `If p follows from the fact that p implies q then p.`

In logic classes, it would perhaps be read like this

(6) `p arrow q arrow p arrow p`

with appropriate intonation to indicate bracketing. In philosophical seminars it was read like this

(7) `p implication q implication p implication p`

Both impersonations make no attempt to help the listener. They present the formula merely as a formal object. Contrast this with the following law.

(8) $p \to (q \to (p \wedge q))$

I can paraphrase it as follows.

(9) `If p then if q then p and q.`

This one is much easier to understand and to phrase meaningfully. So, what is the problem? The problem is that there are laws of logic that can be understood with ease, and others are almost impenetrable. No matter how hard we try, we cannot make sense of them. If on the other hand we understand well what the primitives mean (here: implication), where does the difference come from? And what is it that we understand in one case but not in the other?

The explanation that I am pursuing is this: the minute we hear something we try to understand what it says. Understanding is an act, something we do. It takes time, and we may not be able to reserve enough time and resources to see the consequences that an utterance has. Also, certain acts involved in understanding are automatic and straightforward, others involve some sort of conscious bookkeeping, yet others involve reflection on acts of reasoning, and it is these that are problematic and come at a high cost. In what is to follow I shall expose a theory that explains understanding as a process that involves a stream of elementary acts, the most prominent of which are *conversion* and *reasoning*. Conversion is the process of trading some piece of form for its meaning (and back). It means unpacking the parcel into which the message has been wrapped to recover the content. Reasoning is the step that applies certain rules to arrive at conclusions that are strictly speaking not 'in the message'. The mental process of understanding can be reflected on; humans are able to retrace it if only partially. The reflection gives rise to concepts which denote the kinds of mental acts just talked about. In turn, since these concepts denote mental acts, they can be used to steer the process of understanding in the listener. This is what makes some ways of saying the same thing more digestible than others.

3 Enacting Meanings

There is an interpretation of the conditional attributed by Quine to Rhinelander, which goes as follows. Suppose I say

(10) `If Paul is a raven, he is black.`

Rather than reading this as an assertion about Paul, this interpretation says it is not an assertion at all until the premiss is satisfied. It turns into an assertion if Paul is a raven. And what it says is that Paul is black. This interpretation is akin to Ramsey's interpretation of conditionals: a conditional statement $p \to q$ means that q on condition that p. Ramsey also thought that conditional probability $p(A|B)$ is not a probability assigned to a pair of events, but to the event A alone, but its use is restricted to situations where B obtains. Similarly, a conditional obligation is an obligation that is enforced only when the condition is met; a conditional promise is void if the condition is not met. If I promise you a meal when you repair my bicycle, as long as you do not repair my bicycle there is no promise. There is nothing you can claim from me on the basis what I said.

Now, even if (10) may be read in the way just described: who is to prevent me from seeing it as a statement simpliciter? As the statement that either Paul is not a raven or he is black? How do we judge the matter? Similarly, in deontic logic it has been claimed that the obligation of a tautology is a harmless quirk: if something is true anyway you may at no cost be obliged to make it true.

My answer will be sibyllinic: I think that the implication forms a simple claim, but what it denotes (!) is a certain disposition that in turn results in a chain of mental acts which amount to the Rhinelander conditional. Let me explain. I consider the arrow $\to$ and the words `if....then` as linguistic objects (of different languages). If asked to defend the claim $p \to q$ we do the following. We ask the listener to suppose p. Then we proceed to a demonstration of q. (This is precisely the dialogical interpretation of implicationof Lorenzen.) Similarly, the way we see whether or not $p \to q$ holds, we first suppose p and see whether or not q. We symbolically describe the fact that after supposing p q follows by

(11) $p \vdash q$

The notation is an objectified version of a subjective disposition. In logical theory (11) means 'there is a proof of q from p'. Subjectively speaking it means 'if a consents to p then a will consent to q'. Such dispositions can be attributed also to animals. We say that (11) in turn is the *meaning* of $p \to q$. The equivalence between $p \vdash q$ and $\vdash p \to q$ is valid only in the objectified version; for the subjective version it requires being able to represent meanings because $\vdash p \to q$ is an attitude one holds towards a linguistic entity.

Now, if presented with $p \to q$, I wish to find out whether it holds. Even if the question is about objective validity, I have to do the reasoning myself, trusting that I can perform the actions in the right way. As we shall see below, the pure calculus actually is transsubjective because it requires the application of Modus

Ponens alone and requires neither complex reasoning nor empirical facts. Now, to check whether or not $p \to q$ holds, i.e. whether or not $\vdash p \to q$, I cannot simply undo it to become $p \vdash q$. The latter is a disposition that I must already have, and even if I did, I must be able to see that I do. For the fact that I posses the disposition of holding q true after holding p true is not something that I can actually grasp of myself (and in fact of anyone else). [1] The way to find out, then, is to *enact it*. I suppose p and the see for myself. The enaction is what makes me see whether or not (11) is true. The abyss here is real: you and I may have different ideas of what it means for one thing to follow from another. In that case it seems more straightforward to parametrize $p \vdash q$ for the agent that is performing the reasoning. This introduces a parameter that the phrase `if...then` does not display. Humans are aware of the problem and are (with limitations) able to perform acts of reasoning as if another agent. I have chosen to refer here to the 'mathematical mode', which is assumed to be independent of the impersonator, to avoid getting into deep water before having stated my point. [2]

To see the difference consider Pavlov's dog. Recall that Pavlov had trained his dog by always giving it food after he rang a bell. After doing that a number of times the dog had developed an expectation. When Pavlov rang the bell, the dog expected food. We might describe this as saying that the dog believes this: 'if the bell rings there will be food'. But as far as we know, dogs do not put their beliefs into words. The dog does not believe it as that. In particular, it has no notion of implication. Rather, what it has internalised is the connection between the bell ringing and there being food. In other words, the correct ascription to the dog is [3]

(12) `The bell rings.` $\vdash$ `There is food.`

The difference is that the latter does not lead to any expectations in case that the bell does not ring. This is a rule that only fires when the premiss is satisfied. No

[1] To see whether I possess the disposition $p \vdash q$ one would have to identify a specific pattern of my mind that corresponds to it. Without that $p \vdash q$ can be checked only by trying out. It is like reverse engineering: if you do not know whether the chip makes it function in a particular way (because the manufacturer refuses to tell you what hardware he uses) you run a few tests and then decide.

[2] To be fair, it must be said that $\vdash$ (11) should rather be written $\vdash_T$, since in logical theory there is not one universal deductive relation but a continuum of them. Thus, effectively, the lack of intersubjectivity of $\vdash$ is already present in logic.

[3] To be exact, the dog does not know about English, so I would have to replace the English strings by something else. However, this would result in overly pedantic notation and obscure matters at hand.

bell, no food, not even the expectation of a food. In particular, it is incorrect to ascribe to the dog

(13) `There is no food.` $\vdash$ `There is no bell ringing.`

This is because if there is no food, there is no expectation of bell ringing. Modus Tollens is not a mode of reasoning that dogs use...

Now, the statement

(14) `If the bell rings there is food.`

uses a linguistic object, here the words `if` and `then`. It puts to words what $\vdash$ says. Thus, once we have have grasped (12) we are able to understand (14) because all the difference is in the introduction of a linguistic object whose meaning is clear to us. In turn, the rule (12) is assessed through enaction; in this way (14) expresses a reflection on an internal process. For its meaning is (indirectly) the enaction scheme sanctioning (12). A human subjected to Pavlov's experiment would not only be able to put words to the thought 'the bell rings' and 'there is food'. He will inevitably at some point stop and think this way: whenever the first holds, then the second holds as well. In symbolic terms, he has realized that (12) (through using his own concepts and judgements as carriers of the thought). And he may phrase this as (14).[4] The difference between conditional speech acts and their enaction is very important and often overlooked. But the importance cannot be overestimated. The first consequence is that there are things that differ not in truth conditions but in what one may call 'packaging'. Suppose I say

(15) `If people continue to drive cars then oil prices will rise.`

Was I saying (15) as if to claim that either people will not continue to drive or else the oil price will rise? Or was I rather saying that the oil price will rise, but I added a condition that I really only say this if people continue to drive cars? I think I would have a hard time telling you. Similarly in logic. There is no real difference in the following two claims.

[4] To be able to reason this way, one will also have to be able to representat the fact that the bell is ringing out of context. This requires symbolic capacities that presumably only humans have. On the other hand, note that the fact that the dog actually comes to acquire (12) through learning means that facts are somehow represented and the dog acually remembers them. The connection between the ringing of the bell and food in the plate is reinforced in stages. A moth is unable to learn that way. It acts on light always in the same way.

❶ $p \vdash q$

❷ $\vdash p \to q$

In fact, as I have said above, I am claiming that ❶ is what ❷ acually means. So, when people hear me say (15) they are given a choice: to represent this in ‘logicalese’ either as (16) or as (17).

(16) $\vdash$ `People continue to drive cars` $\to$ `Oil prices will rise.`

(17) `People continue to drive cars.` $\vdash$ `Oil prices will rise.`

However, as matters stand, (17) is not a way that things can be represented in someone’s head. Rather, the only way it can be represented at all is through a *process*: by enacting it. (17) is only short for a conditional claim. Now suppose you want to *understand* what I am saying when I say (15). Then (16) is one step further from the goal than (17). Because understanding means unpacking the meaning of $\to$, which leads you straight into enacting (17).

The upshot is this: you may either refuse to look at the content of $\to$ (you listen, but you don’t understand) or you do look at it. In the latter case, you will have to enact it. You suppose that people will continue to drive cars and then see for yourself whether the oil price rises. The enacting can actually be directly invoked in the following way.

(18) `Suppose that people continue to drive cars. Then oil prices will rise.`

The meaning of the latter clearly is not a statement. The first half is an imperative (`suppose`) even thougb the sentence is not ended by an exclamation mark. It asks you to picture the situation in order to enact the rule.

4 The Calculus of Enacting Meanings

Here is how the whole thing works.

(19) `If p then if p implies q then q.`

To see whether this is correct, we enact it. First, we suppose `p`. We are then asking ourselves whether `if p implies q then q` holds. Again, we suppose that `p implies q` and aim to see whether `q`. In other words, we check whether the following holds.

(20) $p, p \to q \vdash q$

Now, we have supposed that `p` and also `p implies q`. This allows to enact the implication: we suppose (again) `p`. Now use `p` ⊢ `q` to obtain `q`. This concludes the proof.

A fair bit of reasoning is involved. Let's see what it takes to represent it in your (or my) head. I propose to write : p for the fact that p is assumed; p on the other hand is the claim that p is true. Then the steps are as follows, where each line pictures what we need to keep score of. When we take the next step, the previous line disappears.

1. .
2. $: p$.
3. $: p, : p \to q$.
4. $: p, : p \to q$.
5. $: p, : p \to q, p \to q$.
6. $: p, : p \to q, p \to q, p$.
7. $: p, : p \to q, p \to q, p, q$.
8. $: p, : p \to q, p, q$.
9. $: p, : p \to q, q$.
10. $: p, (p \to q) \to q$.
11. $p \to ((p \to q) \to q)$.

The first lines consists in making assumptions. This is always admissible. The fourth consists in what I call **phatic enaction**: the assumption follows, it is now simply true. The next line enacts the meaning of →. Notice that when $p \to q$ is assumed, $p \vdash q$ is a something to which we explicitly subscribe. (But it need not be one of our ordinary dispositions. It is a momentary disposition on the basis of assuming $p \vdash q$. This fact is not represented, but see below.) We now enact the implication. We phatically enact p. Modus Ponens yields q. We may now forget that we had concluded q from p and $p \to q$ using MP. We are at Step 9. Now, we do the converse of enacting, we reflect: supposing $p \to q$ gave us q, so we have $p \to (q \to q)$. Once again we reflect, and we get the last line.

Notice that all the lines represent the content of our mind at any given stage; we do not, for the purpose of the proof remember more than is written on any of the lines. Thus here is a summary of the rules:

1. **Assumption** Add : φ.
2. **Reflection** Given : φ and ψ, add $\varphi \to \psi$.
3. **Phatic Enaction** Given : φ add φ.
4. **Firing** Given φ and $\varphi \to \psi$ add $\vdash \psi$.
5. **Forgetting** Erase φ.

A sequence that is obtained by following these rules is called a **mental proof**. I claim that any person with or without formal training uses these rules, at least implicitly in everyday reasoning. For they are needed to ensure successful enacting of conditional claims, promises, obligations etc. The rules are indeterministic. There always are many ways to proceed. If they derive a line containing just φ then φ is logically valid. These rules are sound and complete for intuitionistic logic of $\to$. It is known that Peirce's Law cannot be derived in intuitionistic logic, so no wonder it is hard to understand.

One may now wonder why $\vdash$ is now gone. Too see why let me contrast this with a different calculus, which I call the **external calculus**. This calculus does not describe the representations that are used for reasoning; instead it describes from an outside perspective what the reasoning is. It contains for justification of the steps ascriptions such as $\vdash p$ and $p \vdash q$. The previous calculus did not reveal a subtlety that the external calculus will show: the enacting of an implication consists in a conversion of $\vdash p \to q$ to $p \vdash q$ and the subsequent execution of this version of firing:

Firing. Given φ and $\varphi \vdash \psi$ add ψ.

The latter version of Firing has the advantage of not using any language at all: it is based solely on the notion of deduction. The rationale for doing it this way is as follows. $\vdash p \to q$ symbolizes the consent to $p \to q$ qua linguistic object. It means, however, that once consent to p is given, consent to q is a consequence. So, $p \vdash q$ is now true.

In the external calculus, the proof would look as follows.

1.
2. $: p$.
3. $: p, : p \to q$.
4. $: p, : p \to q$.
5. $: p, : p \to q, \vdash p \to q$.
6. $: p, : p \to q, p \vdash q$.
7. $: p, : p \to q, p \vdash q, \vdash p$.
8. $: p, : p \to q, p \vdash q, \vdash p, \vdash q$.
9. $: p, : p \to q, \vdash p, \vdash q$.
10. $: p, : p \to q, \vdash q$.
11. $: p, p \to q \vdash q$.
12. $: p, \vdash (p \to q) \to q$.
13. $p \vdash (p \to q) \to q$.
14. $\vdash p \to ((p \to q) \to q)$.

This calculus is different from the previous in using the expression $p \vdash q$. As I claimed above, this is not represented symbolically in my mind: it is a disposition I have not a representation thereof; the latter is written internally $p \to q$, and externally $\vdash p \to q$, showing that I have the disposition to assert $p \to q$. The calculus may be used only externally, to describe what is going on, not internally, to do it.

5 More On Reflection

A sign has at least two faces: a **signifiant** and a **signifié**, otherwise known as **exponent** and **meaning**, respectively. The exponent is a tag, and arbitrarily chosen entity that is coupled with the meaning in a sign. Only in the sign does the exponent get its meaning, and only in the sign is the meaning the meaning of the exponent. A language is constituted by a set of signs. Not knowing a language means not knowing what signs it actually has. The word `telephone` denotes some objects in this world, or maybe a concept. For a stranger to the language the word `telephone` is just a sound bit; he may understand that it has meaning but may fail to know which one. The situation is even more acute with a child. Children will inevitably have to be exposed to the words before they can see what they mean. They first have learn the fact that `telephone` is an actual word before they attempt to link it to any meaning. Second, in order to actually form the complete sign for the word they have to link the word to an appropriate concept. This concept has to be there. This situation is a known one: we often use words without knowing what they mean. What, for example, is `suprematism`? And who can faithfully declare not to use it before he knows exactly what it means. And, by the way, what does it exactly mean? Who is to tell?

Let's return to the stranger. Suppose now that we tell him what `telephone` means. If he is Finnish, we might say: it means `puhelin`. Now he knows, but only indirectly. He knows that the sign is composed of the word `telephone` paired with the meaning of `puhelin` (which is the Finnish translation of it). Now, it turns out that the formation and use of this sign in the head of the poor stranger is no simple thing, just like the disposition of Pavlov's dog. It cannot be created just by the will to have it. The mind wants to be trained through repetition. Moreover, 'natural use' of a language requires a level of control over the signs that few people can ever reach in their lifetime with a language they didn't grow up with. Thus, depending on his capacity to learn, for years to come our stranger will have to make a step that is normally hidden away from us, as it has become so automatic to almost not to be there: he will translate the soundbit into its meaning. I call this **conversion**.[5] Conversion takes away the word and returns the associated meaning.[6] There is a reverse process, equally automatic and equally difficult for

[5]Actually, very often people will convert not into the meaning but into their own language. So, our stranger will probably silently translate `telephone` into `puhelin` before finally being able to access the meaning of that word.

[6]There is a very short span when we are able to repeat verbatim; after that we are only able to repeat what has been said without knowing exactly how it has been said. Therefore I say that

non-native speakers: rendering the concept into words. This will also be called conversion.

Thus, on the face of it, understanding a message would simply constitute a sequence of conversions, from exponent to meaning. But this is not the case. There are several reasons why this is not everything that is going on. I shall concentrate here on one: in some cases we simply cannot convert an exponent to its meaning since there is no way to fit the meaning into the internal representation. Let me explain. I may learn the meaning of $p \to q$ by being told that it means the same as $p \vdash q$. (More exactly, I may told something that is tantamount to saying what $p \vdash q$ says.) The latter, however, is not a representation. It describes, a disposition to answer q when p. If you like, it is a program inside of me that is activated by p and returns q. If that is so, what is the meaning of $p \to (q \to r)$? Well, it is a program that is activated by p and returns ... a program that is activated by q and returns r. Now I ask: in what ways is the second program there? The first is clear: it may be described by a disposition to act on an input. The second however cannot be likewise resolved into a disposition to output r when q is given. If we did this, we would end up responding with r on input q, and that is not what it should do. Rather, the whole actually denotes a program that returns r when q and p are given. Thus, the best we can do is this:

(21) $p, q \vdash r$

This however is not the same disposition as

(22) $p \vdash (q \to r)$

But even though (22) has more ‘texture’ than (21), only (21) is a meaning that the mind can implement within itself. To elaborate this point a little more: (22) is a disposition that I can create or learn, namely, to respond to input p with a linguistic object, here $q \to r$. This object is formal, and the way to see what it means is to enact it. I assume q and respond with r; alternatively, if q is already there, then the whole will simply be enacted to r. I cannot turn the object $q \to r$ into a simple disposition of mine under the present circumstances. If I were to do this, I would have to do it under the assumption that p, and that had to enter somewhere.

There is a natural reaction to this that runs as follows. If converting $\vdash p \to q$ to $p \vdash q$ gives the wrong result then we should not use $\to$ in that way. In other words,

conversion *takes away* the word; once you have the letter you do not care about the envelope.

the fact that the algorithm gives the wrong result indicates that the use of $\rightarrow$ has to be regimented in the same way as $\vdash$. I think there is some truth to the matter. As I explained right at the start, an informed reading of logical postulates that involve stacked implications will naturally end up choosing alternative words (like `given`, `follows from`) or change the actual phrasing of the formula. I might read

(23) $p \rightarrow ((p \rightarrow q) \rightarrow q)$

as

(24) `If p and q follows from p then q.`

This is a reading that spells out the content of the formula rather than reading it aloud. For notice that it replaces the first implication by conjunction, as if the formula had been

(25) $(p \wedge (p \rightarrow q)) \rightarrow q$

This has I suggest severe consequences beyond merely being awkward. The choice of axioms in logic is more often than not thought to be a matter of technical convenience. This attitude pays no attention to the idea that axioms are things that we ought to be able understand immediately, and without hesitation. (Formalists will disagree here.) If that were the case, we should actually resist using too many arrows.

There are other cases like this. Consider the sentence

(26) `It is true that it is raining.`

What does it take to understand this? My proposal is to say that 'φ is true' is nothing but '$\vdash \varphi$'. Thus, the formula describes a judgement (or a disposition, whichever). Notice, however, that then we cannot represent the idea that (26) is true. This would require writing

(27) $\vdash\vdash$ `It is raining.`

However, $\vdash$ is external language and hence I cannot use it. What it technically says is 'there is a disposition to consent to there being a disposition to consent to that it is raining' or something of that sort. It is a second order notion that I claim does not exist. And this is because the disposition of myself to consent to the truth of p is not what I can apprehend. Recall that apprehension requires a formula. Facts are not apprehended. You don't judge seeing something; you only judge seeing something as something, in other words, when you categorise the experience using concepts. Thus all I can do is consent to p and see myself doing that. The disposition has to be enacted to become visible. Once I have observed myself giving consent to p I can express that in the thought: 'p is true', or formally $\mathsf{T}(p)$, with T the exponent of my own truth predicate. Thus, the rule of judgement for $\mathsf{T}(p)$ is this: judge it true if φ. Note that the consent to $\mathsf{T}(p)$ is now given; I can now apprehend $\mathsf{T}(\mathsf{T}(p))$. Since $\mathsf{T}(p)$ is there (in my memory), I judge $\mathsf{T}(\mathsf{T}(p))$ to be true. What is crucial is to understand that 'is true' is a concept and T is a sign. Every layer of T buries the content of the thought one level down; my original consent to p is history. Just now I have given my consent to $\mathsf{T}(\mathsf{T}(p))$. The fact that this was because I gave consent to p is something I may recall from memory. It is not something that I at this very moment can apprehend and judge. I can however always return and rethink my judgement, but no two judgements can be done at the same time.

At this point we may understand why paradoxes raise smiles rather than eyebrows in everyday life. Suppose I meet the following inscription in a classroom.

(28) `This sentence is false.`

Then I might say: so this is allegedly true, let's see. I convert `is false` to the disposition to reject the content, which is that very sentence. So I reject it. I enter again, converting the meaning of `is false` into the disposition to accept, and so on. I may continue like a moth spiralling into the light, or else recall that I had reached that point before. I smile and leave. I refuse to do any more work on that.

The logician in me might protest thinking: how can the same thing be both true and false? And how come you didn't see it coming? Here I wish to answer only the second complaint: because understanding is an act that unfolds in time. It is an act that we may also refuse to perform or put to its proper (?) conclusion. Normally, facts radiate to some degree. Our mind produces conclusions in an instant. The word `Berlin` invokes images in me that the word and its meaning do not support; they are real for me, I have lived long enough to make that automatic. But the radiation only goes a certain way; I do not immediately start to picture everything I know about it; only a little bit. And the same for the sentence above. The words it has in it typically do not radiate very much. Since we have no intuitions about the sentence at all, so we go the pedestrian's way, converting the words into representations, until we either wake up to the fact that we have been fooled, or give up without result at some point.

6 A Formal Model

I will present a formal account of the various notions presented so far. This will be a model of what is going on in the mind of a person. To start, there are **acts** and **dispositions**. Dispositions are timeless; acts happen in time. If there is a disposition to perform an act *a*, *a* may be performed on different occasions. Dispositions may change, but this is a long term process and the within the scope of this paper. We are studying here the way dispositions are enacted. In fact, the topic is much more restricted than that: we are interested in the enactment of meanings.

We fix a language L of propositions or formulae, with certain syntactic rules and so on. L may use various natural languages, but most importantly it uses certain internal signs, such as $\rightarrow$, $\wedge$ and so on. Their meaning is private while the meaning of natural language signs is not. The is no problem in mixing signs of various languages. Thus, if `baker` is a word of English, it is alright to write `baker`(x) to say that it applies to x. It is equally alright to say $\mathsf{baker}'(x)$, where baker' is one's own concept of `baker`. This mixing of languages allows to replace parts of one language by parts of another bit by bit. It also allows to leave certain words unresolved. If you have never heard some word you do not simply reject the sentence that contains it but rather work around it as much as you can. It also allows not to double a public concept by a private one. You may or may not form your own concept of a baker. If you don't, you simply work with `baker` instead.

A **judgement** is a pair $\succ\varphi$, where $\succ$ is **phematic sign** and φ a member of L. Notice that a judgement applies to φ verbatim. That means that it is directed towards the exponent, not the meaning. That may be hard to take at first, but it explains why we in fact issue inconsistent judgements. At present the only phematic sign is $\vdash$, and it denotes **acceptance**. A judgement has a preparatory phase of **apprehension**. This is the moment when the formula is brought into focus for judgement. Since judgement is an act we should rather write $t : \succ\varphi$ to say that the judgement occurred at time t; we may consider adding more details to the conditions under which the judgement occurred. In fact, when stored in memory, such things will often be added. We typically remember how we came to a particular conclusion. Often enough, however, we do not remember how we arrived at a conclusion, we only recall the conclusion itself. Apprehension need not yield a judgement; there may be formulae whose truth value we do not know. But nevertheless, in order to find out, we have to apprehend them first. The result of apprehension is what we call **phematic act**. There are several types of phematic acts, judgemental (accept, reject) or non-judgemental (suppose, unsuppose).

A **conditional judgement** is a disposition of the form $\Delta \succ \varphi$ where Δ is a set of formulae and $\succ\varphi$ is a judgement. We can define it as follows. Time is discrete, steps being measured in terms of steps of the internal calculus. The idea of the following definition is this: we have the person apprehend each premiss; if he consents to all of them we will judge the conclusion to be $\succ$. To make sure that meanwhile a does not retract anything said before, or change his mind otherwise, we enforce the apprehension of the formulae in strict succession.

Definition 1 *Let $\Delta = \{\delta_i : i < n\}$. A person P satisfies $\Delta \succ \varphi$ iff for all series of successive time points $t_0, t_1, \cdots, t_{n-1}$ if P consents to δ_i at i if apprehending it, then upon apprehending φ at t_n P will reach the judgement $\succ\varphi$.*

A **theory** is a set of conditional judgements. Our theory of the world is thus described by a set of conditional judgements. There is no condition on this set, it may even be redundant. There is a notation finesse that needs to be explained. The notation $\succ\varphi$ denotes an **act**; act happen in time, and they may repeat. The conditional judgement, however, is not an act, it is a disposition for an act. There is a conditional judgement of the form $\varnothing \succ \varphi$. This is a disposition to judge φ as $\succ$ where it to be apprehended. The conditional judgement with empty premiss will not automatically fire to give $\succ\varphi$. Otherwise, our mind will be constantly overwhelmed by requests to apprehend the same facts over and over. But technically, it could do that; at present, since I am not talking about how to choose formulae for apprehension, the calculus that I am showing does not prevent any such situation to happen.

A **slate** is a sequence of formulae, possibly prefixed by :. Slates are internal representations. The internal calculus is a calculus for modifying slates, while the external calculus is a more encompassing view from the outside.

Slates are short term devices to keep track of one's actions. There is also **memory**. It allows to store events that happened, and I place to restrictions on that. Any slate can also be stored in memory, for example. Memory serves therefore as a way to be able to reflect on one's own behaviour. My disposition to consent to q when consenting to p before that is not explicitly represented, and I might not find out until I observe myself many times over. However, I need to recall that this is what happened, so memory is necessary. (It is the same problem with discovering someone else's dispositions, by the way.) **Reflection** is the process of discovering a rule.

7 More on the Internal Calculus

So far we have only dealt with implication and consent. The calculus can be enriched to contain conjunction. One way to do this is via conversion rules. We shall use natural deduction style format now. The following describe the enaction of conjunction.

(29) $$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

These rules say the following: if you have φ and ψ you may write down $\varphi \wedge \psi$. Conversely, if you have $\varphi \wedge \psi$ you may write down φ, or you may write down ψ. Notice however that rules of enaction do not require deletion of the material, while rules of conversion do. Therefore, rules of conversion must be exact; so there must be something that we trade for the connective $\wedge$. It could be as simple as the following: it is the set of φ and ψ. This would require that the set is manipulated in the required way.

As for negation, we now need to introduce a new kind of judgement: rejection, denoted by $\dashv$. Write $\dashv p$ to say that p is rejected. In the internal calculus there is nothing that corresponds to it, just like conjunction. However, there is a conversion: if you reject φ, you may enter $\neg\varphi$, and vice versa.

8 Thema and Rhema

We shall apply the theory to provide a 'semantics' for topic and focus. There have been numerous attempts in the literature to provide a semantics for topic and focus. What we shall propose here is that the correct semantics is not truth conditional but can be expressed in terms of actions only. I am convinced that there is no semantics in term of truth conditions that can be given for topic and focus. The general line is this. Reasoning is a semiotic process; it evolves in time. It is a sequence of noetic acts, which get reflected in the topic focus articulation. In turn, the topic focus articulation is unpacked into a sequence of acts, though they may be different from the acts of the speaker. Even if different articulations may be truth conditionally identical their content is different because they may actually map into an inappropriate sequence of actions.

The idea that there are mental acts has a correlate in language: the mental acts correspond to **phatic acts**. A phatic act is not to be confused with a speech act, as will become clear. Rather, phatic acts are in correspondence with the steps of the mental process that we have looked at. There is an act of supposing. And there is an act of claiming. The words of a sentence are equally divided into theme and rheme, where theme is that part of the sentence that expresses the content of the supposition and the rheme is that part that expresses the content of the noetic act. There is a third part, the **pheme**, which expresses the nature of the judgement. To give an analogy, a conditional judgement has the following form:

(30)	δ_1;	$\delta_2; \cdots$;	δ_n	$\succ$	φ
	Theme 1	Theme 2	Theme *n*	Pheme	Theme

A sentence presents an enactment of a conditional judgement in the form:

(31) $: \delta_1 \quad : \delta_2 \ \cdots \ : \delta_n \quad \succ \varphi$

Let me give an example.

(32) `Tullius is Cicero.`

This sentence may express various noetic sequences.

❶ I picture the person named ‘Tullius’; and I picture the person named ‘Cicero’. I consent to the fact that they are the same.

(33) $: \texttt{Tullius}(x) \quad : \texttt{Cicero}(y) \quad \vdash x = y$

❷ I picture the person named ‘Tullius’. I consent to the fact that he is Cicero.

(34) $: \texttt{Tullius}(x) \quad \vdash \texttt{Cicero}(x)$

❸ I picture the person named ‘Cicero’. I consent to the fact that he is Tullius.

(35) $: \texttt{Cicero}(x) \quad \vdash \texttt{Tullius}(x)$

❹ I consent to the fact that ‘Cicero’ is the same as ‘Tullius’.

(36) $\vdash \texttt{Tullius}(x) \leftrightarrow \texttt{Cicero}(x)$

Not all of these noetic sequences are equally likely to be rendered by (32). There are alternatives to the sentence (italics represent emphasis):

(37) *`Tullius`* `is Cicero.`

(38) `Tullius` *`is`* `Cicero.`

(39) `Cicero is Tullius.`

It seems to me that (37) fits best with ❸, that (38) fits best with ❶, (39) with ❸. For ❹ the neutral intonation on (32) seems to be most appropriate.

This idea has several consequences. For example, if someone else is going to describe my belief state, he may have to choose among these options. For notice that in belief contexts the equivalence between these renderings breaks down. Thus, while the truth conditions of (32) — (39) may be the same, the corresponding embeddings in propositional attitudes are not.

(40) `Marcus believes that Tullius is Cicero.`

(41) `Marcus believes that` *`Tullius`* `is Cicero.`

(42) `Marcus believes that Tullius` *`is`* `Cicero.`

(43) `Marcus believes that Cicero is Tullius.`

In order to see this we need to explore what these sentences actually correspond to. Return to the sequence of noetic acts above. Suppose what counts as the content of my belief really is only the apprehended fact, not the suppositions. The suppositions are just ways to enter the objects into the scene. In that case the belief reports will have the following representation.

(44) $: \mathtt{Tullius}(x) \quad : \mathtt{Cicero}(y) \quad \vdash B_M(x = y)$

(45) $: \mathtt{Tullius}(x) \quad \vdash B_M\mathtt{Cicero}(x)$

(46) $: \mathtt{Cicero}(x) \quad \vdash B_M\mathtt{Tullius}(x)$

(47) $\vdash B_M(\mathtt{Cicero}(x) \leftrightarrow \mathtt{Tullius}(x))$

The first is now the de re identity belief: of the people that are called Tullius and Cicero, I regard them as the same (though you may not). The second and the third are de re attributions, and the fourth is completely de dicto. Notice that we could imagine a host of other representations, like this one:

(48) $: B_M\mathtt{Cicero}(x) \quad : B_M\mathtt{Cicero}(y) \quad \vdash B_M(x = y)$

This says (if representing something you say to a third person): think of the object that Marcus calls Cicero, and think of the object that he calls Tullius. I claim that these two Marcus believes to be the same. There are explicit ways of saying this:

(49) `Marcus believes that the person Marcus calls Tullius is the same person he calls Cicero.`

Additionally, you might believe of the two people that I call ‘Tullius’ and ‘Cicero’, respectively, that they are the same. But of none of that is what a simple belief report says. The underlying principle is that a simple belief report reports a belief state. It does not report any mental acts. The mental act that is packaged into the sentence is therefore on your side, not on mine.

If this is true, then also negative belief reports act that way.

(50) $: \texttt{Tullius}(x) \quad : \texttt{Cicero}(y) \quad \vdash \neg B_M(x = y)$

(51) $: \texttt{Tullius}(x) \quad \vdash \neg B_M \texttt{Cicero}(x)$

(52) $: \texttt{Cicero}(x) \quad \vdash \neg B_M \texttt{Tullius}(x)$

(53) $\vdash \neg B_M(\texttt{Cicero}(x) \leftrightarrow \texttt{Tullius}(x))$

(54) `Marcus does not believe that Tullius is Cicero.`

(55) `Marcus does not believe that` *`Tullius`* `is Cicero.`

(56) `Marcus does not believe that Tullius` *`is`* `Cicero.`

(57) `Marcus does not believe that Cicero is Tullius.`

I guess these claims are correct.

Let me also point out another fact. Suppose I have never heard the names ‘Tullius’ and ‘Cicero’ but have heard about a famous orator in Rome and you call him ‘Tullius’, and that I also think he was a politician, and you furthermore think that the politician was called ‘Cicero’, and furthermore you think they are different. Then you are right to claim that I believe Tullius to be Cicero, and that is what is rendered by (53). You may also (falsely) claim that I do not believe them to be the same, and that would be given by (53). But it would be false to ascribe any of the other beliefs to me. I do not believe of Tullius that he is Cicero, for example. Nor do I actually disbelieve of Tullius that he is Cicero. The latter calls for an explanation. The names `Tullius` and `Cicero` are linguistic entities. If I have never heard these names I will not be said to have the belief that a person is called ‘Tullius’. The difference here is between the name and the concept that personifies. I may believe that someone was a famous orator in Rome, but I do not entertain the belief that he has the name ‘Tullius’ nor do I entertain the belief that he does not have the name ‘Tullius’. For the latter belief to be one of my beliefs it is necessary that I have heard of the name in the first place. But it is not sufficient. For the latter I must concretely formulate the disbelief of the proposition ‘the famous orator is called Tullius’. The latter is an object of my internal language towards which I have a relation of disbelief. I take the matter really concretely.

I distinguish between not believing and disbelieving. Not believing is something like: not disposed to accept a proposition when apprehending it. Disbelief is however not a disposition. It is a result. I consider disbelief to be the result of a noetic act. Only when I have apprehended φ at some point and rejected it then I can be said to disbelieve that thought. In ordinary language the two often get confused; but the difference is felt. Suppose you are discussing some problem, say P=NP. Even if you do not think that I believe that it is true you would not conclude that I disbelieve it either.

Often there is also the idea that in order to believe something you need to have a notion of it. But what is that concept 'having a notion of'? I say simply: it is to have some predicate in your mental language that denotes that concept. Suppose I say that you have no notion of a solvable group. Then that means: there is no predicate in your own language that corresponds to it. In particular, you have not heard about the concept. Of course, you had all the tools at your disposal to define such a notion but you didn't. It never entered your mind. If so, there is no sense in using the concept inside a belief context of yours. You just don't have the notion. On the other hand, you may have just formulated the very concept of a solvable group but have called it something else; say, you call that being a group of `finite center rank`. If presented with some group, say A_5, you may now believe that it is of finite center rank (falsely). You read in a book that it is not solvable. No contradiction for you. Your belief is directed towards a proposition that involves the concept phrased as `finite center rank`. Simplifying somewhat I may say that the phrase actually *is* the concept. So, your beliefs are:

(58) $\texttt{finite center rank}(A_5); \neg\texttt{solvable}(A_5)$

Your belief involves some strings and these strings may have different concepts behind them for all you know.

The logical distinctions I am using here have long been noted; it has also been noted that emphasis can change the meaning and the topic focus articulation. What was missing was an account of how it is that the topic focus articulation bears on the question of de dicto ambiguities. What was missing so far was a theory that could explain how the sentences (32) — (39), which are truth conditionally equivalent, suddenly part company when inside a propositional attitude. Attempts have been made, for example the structured meaning approach, but they leave mixed feelings behind. What has not often been noted is that the phenomenon is not restricted to propositional attitudes alone. Even negation is sensitive to the topic focus articulation.

(59) `It is not the case that Tullius is Cicero.`

(60) `It is not the case that` *Tullius* `is Cicero.`

(61) `It is not the case that Tullius` *is* `Cicero.`

(62) `It is not the case that Cicero is Tullius.`

Consider the second sentence. It says of the individual named Cicero that he is different from Tullius. It seems to say (for many) that there is someone else who is. In the present case this is trivially given: it is Cicero. The aboutness is here cashed out as a supposition that some object has a property. The sentence is about Cicero: it starts with the assumption that x is named Cicero (you may also think of it as an assignment of x to Cicero, it does not matter). It then forms the claim that x is not called Tullius. In the same vein the third sentence is about both Tullius and Cicero and it says that they are different.