Appendix B: Recoding Harmony as a single value in two-candidate systems

Appendix to "Deriving the Wug-shaped curve:
A criterion for assessing formal theories of linguistic variation"

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This brief appendix explains the arithmetic behind a practice used consistently in the main text for displaying harmony: if there are just two viable candidates, we can reallocate all Harmony to just one of them and give the other a constant Harmony of zero — and still derive the correct output probabilities.

For simplicity, let us work with a concrete example, the phonetic perception experiment of Kluender et al. (1988) covered in xxx §**Error! Reference source not found.**. The two viable candidates in this case are the percepts [b] and [p]. We calculate the probability of [b] as in the following table.

$\Pr(\mathrm{b}) = \dfrac{\exp(-\Sigma_i\, w_i \mathrm{f}_i\,(\mathrm{b}))}{Z}$, where $Z = \Sigma_j\ \exp(-\Sigma_i\, w_i \mathrm{f}_i\,(x_j))$	The MaxEnt formula, from xxx (**Error! Reference source not found.**) in the main text.
$= \dfrac{\exp(H_b)}{Z}$, where $Z = \Sigma_j \exp(-\Sigma_i\, H_j))$	Replace the calculation of Harmony with a single cover symbol, *H*.
$= \dfrac{\exp(H_b)}{\exp(H_b) + \exp(H_p)}$	Approximate Z; there are just two viable candidates [b] and [p]; and all others contribute essentially zero eHarmony values to the total.
$= \dfrac{1}{1 + \exp(H_p)/\exp(H_b)}$	Divide top and bottom by $\exp(H_b)$.
$= \dfrac{1}{1 + \exp(H_p - H_b)}$	Division is same as subtracting exponents.

Given the perceptual constraints we adopted in xxx §**Error! Reference source not found.** of the main, the harmony difference $H_p - H_b$ will be a simple function of the closure duration of any particular input (i.e., value of closure duration in msec.); and, as the derivation shows, this number suffices for computing P(b). P(p) is also obtained, since it is (infinitesimally close to) 1 − P(b). Thus, by putting $H_p - H_b$ on the x axis of our graphs, we can obtain a simple two-dimensional figure that gives the analytic basis of the MaxEnt sigmoid.