Standard and Non-Standard Quantifiers in Natural Language

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Since the early 1980s we have been witnessing an explosive growth in our knowledge of natural language quantifiers, specifically of their denotations as opposed to their form, distribution, or compositional interpretation. Our concern here is to unify and extend these results, highlighting generalizations of linguistic interest. We present first an inventory of semantically defined classes of quantifiers expressible in English, and then a list of linguistic generalizations stated in terms of these classes and their defining concepts. We close with two types of quantification that lie outside these classes and which invite further research as less is known about them.

1. Semantic classes of quantifiers The best studied type of natural language quantifier is that expressed by the italicized expressions in the Ss in (1).

(1) a. *No teacher* laughed at that joke
 b. *All students* read the Times
 c. *Most poets* are vegetarians

Such Ss are interpreted as True (T) or False (F) in a situation (model). They may have different values, T or F, in different situations. One place predicates (P1s), such as *laughed at that joke*, etc. denote properties of individuals, which we represent extensionally as subsets of the *domain* E_s of individuals[1] in situation s (usually writing just E for E_s). P_E, the set of subsets of E, is the set of possible P1 denotations in s. So *Sue laughed* is true (in s) if and only if (iff) Sue is an element of the set LAUGH (We note denotations in upper case).

The italicized DPs (Determiner Phrases) in (1) denote functions from P_E into the set {T,F}. They are called *generalized quantifiers* (GQs) or *functions of type* <1> (over E).

So the truth value that (1a) has in a situation s is the one that NO TEACHER maps the LAUGH AT THAT JOKE property to in s, etc.

Def 1 Given a domain E, the set GQ_E of *generalized quantifiers* (*type <1> functions*) *over E* is$_{def}$ $[P_E \rightarrow \{T,F\}]$, the set of functions from P_E into $\{T,F\}$.

The DPs in (1) are built by combining the Dets *no*, *every*, and *most* with a Noun (N). Dets are naturally interpreted by functions of type <1,1> **–** maps from P_E into GQ_E, given that Ns like *student* also denote subsets of E. So NO maps the STUDENT set to the generalized quantifier NO(STUDENT). The Dets in (1) exemplify three natural classes of type <1,1> functions: **intersective**, **co-intersective** and **proportionality** ones, noted INT_E, CO-INT_E, and $PROP_E$ respectively.

1.1 Intersective Dets denote type <1,1> functions whose value at sets A,B just depends on $A \cap B$, the set of objects that lie in both A and B. We don't have to know about As who aren't Bs for example. Thus whether NO(STUDENT) maps LAUGH to T (True) is decided just by looking at STUDENT$\cap$LAUGH, the set of students who laughed. If that set is empty the S is true, otherwise it is false. We define:

Def 2 For D of type <1,1>, D is *intersective* iff for all sets A,B,X,Y

if $A \cap B = X \cap Y$ then DAB = DXY

Thus for D intersective, DAB does not change when we replace A by X and B by Y as long as A has the same intersection with B as X has with Y. NO, below, is intersective.

(2) For all sets A,B NO(A)(B) = T iff $A \cap B = \varnothing$ ($\varnothing$ is the empty set)

(3) **some intersective Dets in English** some, a/an, no, several, more than six, at least

six, exactly six, fewer than six, at most six, between six and ten, just finitely many,

infinitely many, about a hundred, a couple of dozen, practically no, nearly twenty, approximately twenty, not more than ten, at least two and not more than ten, either fewer than five or else more than twenty, that many, How many?, Which?, more male than female, just as many male as female, no...but John

Most but not all of the Dets in (3) are not merely intersective, they are *cardinal*, meaning that D(A)(B) is determined just by *how many* elements are in $A \cap B$, we don't have to know precisely which elements they are. Writing $|X|$ for the cardinality of the set X, the truth of *Between five and ten boys laughed* is decided just by giving $|BOY \cap LAUGH|$, the number of boys that laughed.

Def 3 A function D of type <1,1> is *cardinal* iff for all sets A,B,X,Y

$$\text{if } |A \cap B| = |X \cap Y| \text{ then } DAB = DXY$$

Being cardinal is a special case[2] of being intersective. From Def 3 we infer that vague Dets such as *about fifty*, *nearly a hundred*, etc. are cardinal: If the number of sparrows on my clothesline is the same as the number of coins in that fountain then *About fifty sparrows are on my clothesline* has the same truth value as *About fifty coins are in that fountain* (even if we disagree on how many coins are to count as "about fifty").

Interrogative Dets such as *How many?* and *Which?* in *Which students?* are usually not treated in studies of generalized quantifiers (Gutiérrez-Rexach 1997 is an exception). But our invariance condition approach to defining Dets includes them. Different approaches to questions differ with regard to what they denote, but they agree that their semantic interpretation is decided by their true answers. So we consider HOW MANY? to be cardinal (and thus intersective) on the grounds that *How many As are Bs?* and *How many Xs are Ys?* have the same true answers in any model in which the number of As that are Bs is the same as the number of Xs that are Ys..

Are there intersective Dets which are not are cardinal? The answer is not obvious. Note that *some*, even if defined as in (4b), is cardinal since even so (4c) is true.

(4) a. SOME(A)(B) = T iff $A \cap B \neq \emptyset$

b. SOME(A)(B) = T iff $\exists x (x \in A$ and $x \in B)$

c. (SOME)(A)(B) = T iff $|A \cap B| \geq 1$

However some Dets in (3) are not cardinal. *Which?* is not, accepting (5):

(5) Which As are Bs? = Which Xs are Ys? if $A \cap B = X \cap Y$

Someone who asks which students are vegetarians wants to know what the elements of STUDENT∩VEGETARIAN are, not merely how many of them there are. Similarly *no...but John*, interpreted as a Det as in (6b), is intersective but not cardinal.

(6) a. No student but John came to the lecture

b. (NO...BUT JOHN)(A)(B) = T iff $A \cap B = \{JOHN\}$

(6b) says that (6a) is true if and only if the set of students who came to the lecture is the unit set John, that is, John is the only student who came. Our analysis (Keenan & Stavi 1986), yields correct results, but since *no student but John* is syntactically complex we cannot rule out a syntactic analysis which does not treat *no...but John* as a Det.

Finally, given that *male* and *female* are intersective adjectives – interpreted by functions f from properties to properties satisfying $f(A) = A \cap f(E)$, so that the female lawyers are the lawyers who are female individuals, the reader can write out the truth conditions of (7a) as in (7b) and show that that Det is intersective but not cardinal:

(7) a. More male than female students were drafted

b. (MORE f THAN g)(A)(B) = T iff $|f(A) \cap B| > |g(A) \cap B|$

But again *more male than female students* is syntactically complex. The most convincing case for the existence of intersective non-cardinal Dets would come from syntactically simple Dets. Our only candidate is interrogative *Which?*

1.2 Co-intersective Dets denote functions whose value at a pair A,B of sets just depends on A – B, the set of objects in A not in B. To verify that ALL is co-intersective, verify that *All As are Bs* iff the set of As that are not Bs is empty, that is, $A - B = \emptyset$.

Def 4 For D of type <1,1>, D is *co-intersective* iff for all sets A,B,X,Y

if $A - B = X - Y$ then DAB = DXY

So *All poets daydream* is true iff the set of poets who do not daydream is empty. (8a), the usual definition, is equivalent to (8b), which shows it to be co-intersective.

(8) a. ALL(A)(B) = T iff $A \subseteq B$

b. ALL(A)(B) = T iff $A - B = \emptyset$

c. ALL(A)(B) = T iff $\forall x(Ax \rightarrow Bx)$

(9) **some co-intersective Dets in English** all, each, every, almost/nearly all, not all, all but

six, all but at most six, all but finitely many, every...but John, as in *Every student but John came to the lecture*.

Impressionistically the co-intersective Dets exhibit less internal structural diversity than the intersective ones. We find no unequivocal interrogative Dets that are co-intersective. Like the intersective Dets most co-intersective Dets are (*co-*)*cardinal*:

Def 5 D of type <1,1> is *co-cardinal* iff for all sets A,B,X,Y

$$\text{if } |A - B| = |X - Y| \text{ then } DAB = DXY$$

Of the Dets in (9) only *every...but John* is not co-cardinal. Dets of the form *all but n* which map A,B to T iff $|A - B| = n$ obviously are. (Note that ALL = ALL BUT 0)

1.3 Proportional Dets are ones whose value at a pair A,B of properties just depends on the proportion of As that are Bs.

(10) **some proportional Dets in English** most, more than half, less than two thirds, exactly half, at most ten per cent, between a half and two thirds, between ten and twenty per cent, every third (as in *Every third child was inoculated*), (not) one...in ten (as in *not one student in ten was inoculated*), seven out of ten, all but a tenth

So the proportionality Dets include mundane fractional and percentage expressions. Their first, or Noun, argument is usually assumed to be finite. In general the functions they denote are not intersective or co-intersective, though a few extremal cases are: *Exactly zero per cent* = *no*, *a hundred percent* = *every*, *more than zero per cent* = *some*, *less than a hundred per cent* = *not every*. Formally, here is our definition with two examples:

Def 6 D is *proportional* iff for all A,B,X,Y if $|A\cap B|/|A| = |X\cap Y|/|X|$ then DAB = DXY

(11) MOST(A)(B) = T iff $A \neq \varnothing$ and $|A\cap B|/|A| > ½$

(AT LEAST SEVEN OUT OF TEN)(A)(B) = T iff $A \neq \varnothing$ and $|A\cap B|/|A| \geq 7/10$

Several proportionality Dets combine directly with Ns to form DPs. These include *most*, *every second*, *n out of m*, and *less than three...in ten*. But fractional and percentage Dets are usually constructed on a partitive pattern: *two thirds <u>of</u> the students*, *less than ten per cent <u>of</u> John's students*, etc. Here the Det is followed by *of* which governs a "definite DP". As Barwise and Cooper (1981) were the first to notice, such DPs identify a property p, and map to T just the supersets of p. We note such functions by F_p, the *principal filter generated by* p. So *the ten boys* denotes F_{BOY} if $|BOY| = 10$; otherwise it denotes **0**, that GQ which maps all sets to F. *of* in partitives functions to map such DPs to their generating property p, whence fractional and percentage Dets can still be treated as functions mapping properties to GQs.

Some Complex DPs For DPs of the form Det+N, nothing about our semantics of Dets changes when the N is syntactically complex, as when it is modified by adjectives and relative clauses. The interpretation of *every Greek student who Mary knows* is just the value of EVERY at a property, that denoted by *Greek student who Mary knows*. But there are DPs that are not of the form Det+N. We consider three cases:

(12) a. **$\mathbf{Det_2}$ + (N, N)**: *<u>more</u> students <u>than</u> teachers*; *<u>not</u> <u>as</u> <u>many</u> boys <u>as</u> girls*

b. **Proper Nouns**: *John*, *Mary*,...

c. **Boolean Compounds of DPs**: *at least two boys <u>and</u> <u>not</u> more than ten girls*; *<u>neither</u> every student <u>nor</u> every teacher*; *<u>either</u> John <u>or</u> some student*

$\mathbf{Det_2}$s: We treat expressions such as *more...than...*, *fewer...than...*, *exactly/twice as many...as...*, etc. as two place Determiners: they combine with two (possibly complex) Ns to form comparative DPs, which do exhibit typical DP distribution:

(13) Subjects: More students than teachers came to the party

Objects: John interviewed fewer men than women

Indirect Objects: I gave presents to more women than men

Objects of Prepositions: Mary has gone out with more students than teachers

Raising to Object: We believe more students than teachers to have signed the petition

Passivizes: More students than teachers were believed to have signed the petition

Existential There: There are exactly as many dogs as cats in the garden

Multiple Occurrences[3]: More students than teachers read more plays than poems

The pair of N arguments of Det_2s have several properties in common with the single N argument of one place Dets (Keenan 1987). For example, PP and adjectival modifiers of single Ns serve to restrict their denotations. In *Most students in this class are women* we only make a claim about students in this class, not students in general. Similarly *at the party* in (14a) is most naturally understood to restrict the denotation of both Ns, making its natural reading a paraphrase of (14b).

(14) a. More students than teachers at the party signed the petition

b. More students at the party than teachers at the party signed the petition

The Det_2s we find most natural are *cardinal*, meaning the DP they build decides to map a predicate property X to T or F based solely on the cardinality of the intersection of each N argument with X. Here is a paradigm example:

(15) $(\text{MORE A THAN B})(C) = T \text{ iff } |A\cap C| > |B\cap C|$

How many more...than...? is a cardinal interrogative Det_2. There seem to be no co-cardinal Det_2s.

Boolean compounds are formed by combining DPs with (*both*)... *and*..., (*either*)...*or*..., *not*, *neither*...*nor*... and some uses of *but.* Now the set {T,F} in which Ss denote possesses a familiar boolean structure, one which supports two binary functions, $\wedge$ (meet) and $\vee$ (join), whose values are given by the standard truth tables for conjunction and disjunction. For example, for x and y in {T,F}, $(x \wedge y)$ is T iff x = T and y = T. {T,F} also supports a one place function $\neg$ (complement), where $\neg T = F$ and $\neg F = T$. Using this boolean structure we interpret a conjunction of Ss as the meet of the interpretations of the conjuncts, a disjunction as the join of the interpretations of the disjuncts, and *not* ϕ as the complement of the interpretation of ϕ. *Neither* ϕ *nor* ψ is interpreted as the complement of the join of the interpretations of ϕ and ψ.

Now the set of generalized quantifiers over E possesses a natural boolean structure which allows us to directly interpret their boolean compounds. For F,G generalized quantifiers their meet, join, and complement functions are given by

(16) a. $(F \wedge G)(A) \quad = \quad F(A) \wedge G(A)$

b. $(F \vee G)(A) \quad = \quad F(A) \vee G(A)$

c. $(\neg F)(A) \quad = \quad \neg(F(A))$.

And as with Ss, we interpret conjunctions of DPs as the meet of the interpretations of the conjuncts, and analogously for disjunctions and negations. Note that F(A) and G(A) above are truth values, and we already know what meets, joins and complements of truth values are. These definitions guarantee the logical equivalences below (expressions are *logically equivalent*, $\equiv$, iff they are always interpreted the same):

(17) Every student but not every teacher came to the party

$\equiv$ every student came to the party but (not(every teacher)) came to the party

$\equiv$ every student came to the party but it is not the case that every teacher came...

Definitions of the form (16) are said to be done *pointwise*: the value of $(F \wedge G)$ at a "point" A is obtained by applying each of F and G to that point, and taking the meet of the result. So the values of F and G at an argument must themselves lie in a boolean structure. Similarly disjunctions and negations of DPs are interpreted pointwise. And boolean compounds of Dets are also interpreted pointwise:

(18) a. (most but not all)(students) $\equiv$ (most students but (not all)(students))
$\equiv$ (most students but not (all students))

Proper Nouns such as *John* and *Mary* are traditionally treated as denoting elements of the domain of the model, which suffices for simple cases such as *John loves Mary*. But this approach is not adequate to treat boolean compounds such as *both John and Bill, neither Mary nor any student*, etc. To see but one of the absurdities derived from the assumption that boolean compounds of proper nouns denote objects in the domain, imagine a situation in which *John* and *Bill* denote different objects and interpret the proper noun *Sam* as the object denoted by *both John and Bill*. So *both John and Bill are Sam* is true in this model. But by (16) conjoined subjects distribute over the predicate (e.g. *both John and Bill smoke* iff *John smokes and Bill smokes*) we infer *John is Sam* and also *Bill is Sam*, whence John is Bill contradicting our assumption that they denote different objects.

Another problem here is that if proper nouns denote objects and quantified DPs denote GQs, then why should it make sense to form boolean compounds of proper nouns and quantified DPs, as in *John and some student arrived early*, etc?

A not uncommon response to such problems is to suggest that boolean compounds of proper nouns, and quantified DPs in general, denote *sets* of objects (which is what common nouns such as *student*, etc. denote). But this response is still woefully insufficient. Given a domain E, $|P_E|$ is $2^{|E|}$ and $|GQ_E|$ is 2 raised to the power $2^{|E|}$. So if there are just three objects in E, then there are $2^3 = 8$ subsets of E, and $2^8 = 256$ generalized quantifiers over E, and all 256 of these functions are denotable in English just with boolean compounds of proper nouns. (In general for any finite E, all GQs over E are denotable in English – Keenan & Moss 1985; Keenan & Stavi 1986).

Here is an example illustrating this. Let $|E| = 3$ and baptize its elements *Manny*, *Moe*, and *Jack*. Consider now the 8 DPs of the form X, Y and Z, where X,Y, and Z run through the three proper nouns and their negations: *Manny*, *Moe and Jack*; *Manny and Moe but not Jack*, ..., *Manny but not Moe and not Jack*, *neither Manny nor Moe nor Jack*. No two of these DPs denote the same GQ, as each is true of just one property, a different one in each case: the first DP is true just of {Manny, Moe, Jack}, the second is true just of {Manny, Moe}, the next to last is true just of {Manny} and the last is true of ∅. So we have 8 logically distinct DPs. Now, for each subset K of these DPs form the disjunction of its elements. This disjunction is True of just the properties that one of its members is True of, so disjunctions of different subsets K are not True of exactly the same properties.[3] Thus distinct subsets of this 8 element set form logically distinct DPs, and there are 256 such subsets. And we have only used boolean compounds of proper nouns! The use of DPs of the form Det+N just makes it easier to denote all GQs.

Conclusion: to provide denotations for logically distinct DPs their denotations must correspond to the *sets* of sets of objects. And we want proper nouns to denote GQs, those we call *individuals* below, using ∈ for the relation *is a member of*:

Def 7 For each $b \in E$, I_b, the *individual* generated by b, is that GQ which maps a property A to True iff $b \in A$. A GQ F is an individual iff for some b, $F = I_b$.

2. Some Linguistic Generalizations

We begin with three generalizations where the denotations we propose enable us to define classes of expressions independently needed in generative grammar.

Gen 1 To within a good first approximation the DPs which occur naturally in Existential There (ET) sentences are (boolean compounds) of those built from intersective Dets (See Keenan 2003 and references cited there, in particular Reuland & ter Meulen 1987).

We give some positive instances in (19) and some negative ones in (20). We test acceptability in ET contexts using negative or interrogative Ss to avoid certain special uses of affirmative Ss (such as "list" contexts: *– How do I get to UCLA from here? Well, there's always the bus, but it doesn't run very often*; Rando and Napoli 1978; **There isn't always the bus...*, **Is there always the bus...?*).

(19) a. Aren't there more than ten students in the class?
b. There aren't fewer than ten students in the class
c. Were there no undergraduates at all enrolled in the course?
d. Was there really no one but the janitor in the building?
e. How many students are there who object to that?
f. Weren't there about fifty students at the party?
g. ?Just which students were there at the party anyway?[3]
h. Weren't there more students than teachers arrested at the demonstration? i. Aren't there as
j. Weren't there the same number of men as women on the jury?

(THE SAME NUMBER OF ... AS ...) in (19j) is two place intersective, in fact cardinal.

(20) a. *Aren't there most students in my logic class?
b. *Isn't there the student who objects to that?
c. *Isn't there every student enrolled in the course?
d. ??Aren't there seven out of ten students enrolled in the course?
e. *Was there neither student arrested at the demonstration?
f. *?Which of the two students was there at the demonstration?

Surprisingly, the class of DPs occurring naturally in ET contexts is closed under boolean compounds – that is, if X and Y occur there then so do their conjunction, disjunction and negations (if well formed). A comparable claim holds for Dets as well, (21c):

(21) a. There are at least two dogs and not more than ten cats in the yard

b. There were more cars than trucks but not more BMWs than SUVs in the lot

c. There were at least two but not more than ten cats in the yard

Gen 2 The DPs which occur naturally in the post *of* position in plural partitives, such as *two of the ten students*, *more than two of John's ten or more cats*, are those built from Dets that create principle filters F_p, with $|p| \geq 2$ (Barwise & Cooper, 1981).

A generalized quantifier F is a *principal filter* iff for some $A \subseteq E$, F maps to T just the supersets of A. For example (THE TEN OR MORE)(CATS) maps a property B to T iff $CAT \subseteq B$ and $|CAT| \geq 10$. So *the ten or more cats* denotes **0** if $|CAT| < 10$; otherwise it denotes F_{CAT}, the prinicipal filter generated by CAT.

Def 8 A non-trivial D of type <1,1> is *definite plural* iff for all properties A with $|A| \geq 2$ there is an $A' \subseteq A$ with $|A'| \geq 2$ such that $D(A') =$ **0** or $D(A') = F_{A'}$.

The subset condition above is for cases like (JOHN'S TEN OR MORE)(CATS) which, when non-zero, denotes F_B, for B = CAT WHICH JOHN HAS, a subset of CAT.

Monotonicity and negative polarity items (npi's). Overt negation (Klima 1964) such as *n't* in (22b) licenses the presence of npi's such as *ever* and *any*:

(22)	a. *John has ever been to Pinsk	*He saw any birds on the walk
	b. John hasn't ever been to Pinsk	He didn't see any birds on the walk
(23)	a. *Some boy here has ever been to Peru	*Some boy saw any birds on the walk
	b. No boy here has ever been to Peru	No boy saw any birds on the walk

But, (23), some subject DPs (Ladusaw 1983) also license npi's. *Query*: Which DPs license npi's and what do they have in common with overt negation? An answer:

Gen 3 1. The DPs which license npi's denote decreasing functions,

2. S or Predicate level negation also denotes a decreasing function.

We first define *decreasing*. A set with a boolean structure, usually called a *boolean algebra* or *boolean lattice*, is characterized by its partial order relation, generically noted $\leq$ and definable in terms of meet ($\wedge$) as in Def 9.1 below[3]:

Def 9 1. For $x,y \in B$, B a boolean algebra, we define $x \leq y$ iff $(x \wedge y) = x$

2. If B and C are domains of boolean algebras F is a map from B into C,

   a. F is *increasing* iff whenever $x \leq y$ in B then $F(x) \leq F(y)$ in C, and

   b. F is *decreasing* iff whenever $x \leq y$ in B then $F(y) \leq F(x)$ in C

Looking now at GQs, maps from P_E into {T,F}, we consider the boolean relation $\leq$ on each of these sets. First, for $x,y \in \{T,F\}$ we compute that $x \wedge y = x$ iff $x = F$ or $x = y = T$. The only case ruled out is $x = T$ and $y = F$. Thus the $\leq$ relation is just the truth table for conditionals *if P then Q*, True iff the truth value of P is $\leq$ the truth value of Q. For P_E, the $\wedge$, $\vee$, and $\neg$ operations are $\cap$, $\cup$ and complement relative to E: $\neg X =_{df} E - X$. Def 9 tells us that $X \leq Y$ iff $X \cap Y = X$. But $X \cap Y = X$ iff $X \subseteq Y$, so $\leq$ is $\subseteq$ in P_E.

(24) For F a generalized quantifier,

F is increasing iff for all $X,Y \subseteq E$, if $X \subseteq Y$ then if $F(X) = T$ then $F(Y) = T$

F is decreasing iff for all $X,Y \subseteq E$, if $X \subseteq Y$ then if $F(Y) = T$ then $F(X) = T$

And a DP is *increasing* (*decreasing*) iff it always denotes an increasing (decreasing) function. To verify that a DP K is increasing check that the leftmost argument below is valid (that is, the conclusion follows from the premisses). To verify that K is decreasing check that the rightmost one is valid.

(25) a. All Xs are Ys
b. K is an X
Therefore, K is a Y

a. All Xs are Ys
b. K is a Y
Therefore K is a X

Proper nouns are increasing since, for example, if all poets are vegetarians and John is a poet it follows that John is a vegetarian. More generally

some increasing DPs

1. Lexical DPs (Pronouns, Proper Nouns): he, she, John, Mary, ...
2. DPs of the form Det + N for Det = every, some, most, at least n, the n, more than n, at least seven out ten, at least half
3. If X and Y are increasing DPs then so is (*both*) *X and Y* and (*either*) *X or Y*. In contrast *neither X nor Y* is decreasing, as is *not X* (if well formed)
4. If X is increasing then so are possessive DPs of the form X's N.

some decreasing DPs

1. Ones of the form Det + N, where Det = no, fewer than n, at most n, less than half
2. Conjunctions and disjunctions of decreasing DPs are decreasing; when X and Y are decreasing *not X* and *neither X nor Y* are increasing (if grammatical)
3. If X is decreasing so are possessive DPs of the form X's N

Here are some further minimal pairs of Ss which contrast an increasing DP with a corresponding decreasing one – and only the latter licenses npi's:

(26) a. *Either John or Mary has ever been to Pinsk
Neither John nor Mary has ever been to Pinsk

b. *More than half the children here saw any birds on the walk
   At most half the children here saw any birds on the walk

c. *Some student's doctor reads any Russian journals
   No student's doctor reads any Russian journals

DPs such as *exactly five boys*, *between five and ten boys*, *more boys than girls* are neither increasing nor decreasing. We turn now to some semantics internal generalizations.

Gen 4 An advantage of interpreting expressions in sets with boolean structure is that we have a uniform way of interpreting *and*, *or*, *not* and *neither...nor...*: *and* always denotes a meet operation in the set in which its arguments denote (different for different categories of arguments), *or* a join operation, *not* a complement operation[3] and *neither...nor...* a complement of a join.

Gen 4 suggests a deeper, but more speculative claim: the meaning of the boolean connectives (*and*, *or*, ...) is not category specific: *X and Y, X or Y*, etc. make sense for virtually all X,Y of the same category. So their meaning is not tied to that of any particular type of denotation. This suggests that these connectives express ways humans conceptualize things as opposed to properties of the things themselves. So *the boolean operators represent properties of mind*, not properties of objects, which is what Boole (1854) thought in titling his book ...*The Laws of Thought*.

Gen 5 A semantic universal? The Dets considered above all satisfy a very non-trivial condition on their denotations: they are **conservative**.

Def 10 A function D of type <1,1> is *conservative* (*Cons*)[3], only iff for all A,B,B'

$$\text{if } A \cap B = A \cap B' \text{ then } DAB = DAB'$$

Theorem D of type <1,1> is conservative iff for all A,B $D(A)(B) = D(A)(A \cap B)$

So the value a Det denotation assigns to a pair A,B of properties may depend on A and on

$A \cap B$ (from which we compute $A - B$), but it cannot depend on $B - A$. If B and B' have the same intersection with A then we must get the same value. All the intersective, co-intersective, proportional and definite Dets discussed above are Cons. Since *all* is Cons (27a) holds. And **Gen 5** suggests that (27b) holds for all choices of Det:

(27) a. All cats are grey $\equiv$ All cats are cats that are grey

b. Det cats are grey $\equiv$ Det cats are cats that are grey

Despite the apparent triviality of these equivalences not all type <1,1> functions are Cons:

(28) For $|E| \geq 2$, define H of type <1,1> by: $H(A)(B) = T$ iff $|A| = |B|$

For $a \neq b \in E$, $H(\{a\})(\{b\}) = T$ since $|\{a\}| = |\{b\}|$. But $H(\{a\})(\varnothing) = F$ since $|\{a\}| = 1 \neq 0 = |\varnothing|$. And since $\{a\} \cap \{b\} = \varnothing = \{a\} \cap \varnothing$, H fails to be conservative. And surprisingly, conservativity turns out to be a very strong constraint. The number of type <1,1> functions over an E of cardinality n is 2 raised to the power 4^n. The number of these functions that are conservative is just 2 raised to the power 3^n (Keenan & Stavi 1986). So in a model with just two entities ($|E| = 2$) there are $2^{16} = 65{,}536$ logically possible denotations for Dets, but only $2^9 = 512$ are conservative! (And, footnote 2, only 16 are intersective). Here is similar but independent semantics internal generalization, where we write d_E for a possible denotation of a Det d in a model with domain E:

Gen 6 Natural language Dets are Domain Independent (van Benthem 1984)

Def 11 A one place Det d is *Domain Independent* iff for all E, E' and all sets A,B which are subsets of E and also of E', $d_E(A)(B) = d_{E'}(A)(B)$

To see the force of **Gen 6** consider a hypothetical Det *blik* whose denotation satisfies $blik_E(A)(B) = T$ iff $|E - A| = 4$. Thus *Blik cats are grey* iff there are exactly four non-cats. *Blik* is not Domain Independent since $blik_E(\varnothing)(\varnothing)$ is True if $|E| = 4$ and False otherwise. **Gen 6** says that *blik* is not a possible natural language Det. But for each E, $blik_E$ is conservative. Leading up now to **Gen 7** we need:

Def 12 A non-empty subset K of type <1,1> functions is *closed* under the boolean operations in (16) iff whenever F and G are in K then so are $F \wedge G$, $F \vee G$ and $\neg F$. In such a case K is said to be a *subalgebra* of Type <1,1>.

Gen 7 The following classes of Dets defined above are closed under the pointwise boolean operations and are thus subalgebras of Type <1,1>: $CARD_E$, INT_E, $CO\text{-}CARD_E$, $CO\text{-}INT_E$, $PROP_E$, and $CONS_E$.

Thus *at least two and not more than ten* denotes an intersective function since it is a boolean compound of Dets with that property (*at least two*, *more than ten*). **Gen 7** is a kind of naturalness condition: Type <1,1> has a certain algebraic structure, and that structure is respected by each of the six subsets of Type <1,1> we studied above.

We note though that the definite Dets, such as *the* (*ten*) or *John's* (*ten*), do not satisfy the closure condition in **Gen 7**. For example, *neither John's nor Bill's cats* never denotes a principal filter when non-trivial.

A deeper observation is that non-trivial boolean compounds of intersective with co-intersective Dets, as in (29a,b), are themselves neither intersective nor co-intersective.

(29) a. Some but not all students are vegetarians

b. Either just two or three or else all but a couple of students will pass that exam

Theorem (Keenan 1993) The set of conservative functions over E is precisely the boolean closure of the intersective together with the co-intersective functions.

Thus from a boolean perspective we may say that if you understand the intersective and co-intersective functions and you understand how to apply boolean operations you understand all conservative functions.

Leading up to **Gen 8**, conservativity and domain independence tell us that in evaluating [Det N]+P1 we need only consider which objects have the N property and which of those have or lack the predicate property. But for SOME the restriction imposed by the Noun argument is eliminable. We can paraphrase SOME(A)(B) replacing A with the entire universe, compensating by taking a boolean function, intersection, of A and B in the predicate, (30a). Indeed this works for any intersective Det, (30b):

(30) a. SOME(A)(B) = SOME(E)($A \cap B$)

b. For D intersective, D(A)(B) = D(E)($A \cap B$)

Since $A \cap B$ is a subset of E, $E \cap (A \cap B) = A \cap B$, so (30b) holds. And what we have done on the righthand side of (30b) is to replace quantification over A by quantification over the entire universe, compensating by using the predicate property $A \cap B$. We will say then that intersective Dets are *sortally reducible*, meaning that we can eliminate the sort restriction on their domain in favor of quantifying over the entire domain of objects in the model, forming a new Predicate property as a boolean function of the original plus the Noun property. (A *boolean function* on sets is one definable in terms of intersection, union and complement). This replacement is what we do in translating from natural English, (31a), into first order logic, (31b):

(31) a. Some student is a vegetarian b. $\exists$x(student(x) & vegetarian(x)

In (31b) the variable *x* ranges over the entire universe of the model. The formal definition of *sortal reducibility* is:

Def 13 D of type <1,1> is *sortally reducible* iff there is a two place boolean function h such that for all A,B $D(A)(B) = D(E)(h(A,B))$.

Are there Dets reducible by functions other than intersection? Yes. EVERY, and all co-intersective Dets, are sortally reducible using $\rightarrow$, where $A \rightarrow B$ abbreviates $\neg A \cup B$:

(32) a. EVERY(A)(B) iff EVERY(E)($A \rightarrow B$)   "Every A is a B iff every entity is such that if it is an A then it is a B"

b. For D co-intersective, $D(A)(B) = D(E)(A \rightarrow B)$

The truth of (32b) is less immediate than that of (30b), but observe: $E - (A \rightarrow B) = E - (\neg A \cup B) = E \cap \neg(\neg A \cup B) = E \cap (\neg \neg A \cap \neg B) = E \cap (A \cap \neg B) = (A \cap \neg B) = A - B$, so (32b) holds by the co-intersectivity of D. Still **Gen 8** (Keenan 1993, 2000), which assumes **Gen 5**, is surprising:

Gen 8 The sortally reducible Dets are just the intersective and co-intersective ones

From **Gen 8** we infer that (33a) does not have a logical paraphrase of the form in (33b):

(33) a. Most poets daydream

b. For most objects x, [...poet(x)...daydream(x)...], where the bracketed phrase is a boolean compound (in *and*, *or*, *not*,...) of *poet(x)* and *daydream(x)*.

Obviously that boolean compound could not be conjunction, as with intersective Dets, since *Most poets daydream* does not mean *Most objects are both poets and daydream*.

The co-intersective option would say that (33a) means *Most objects are such that if they are poets then they daydream*. But this also is incorrect. In a model with 100 objects, just 15 of which are poets and only 2 of whom daydream, (33a) is clearly False: just 2 of the 15 poets daydream. But for 85 of the 100 objects in the domain the formula *if poet(x) then daydream(x)* is True vacuously, since its antecedent is false. So on this translation (33a) is true in any model in which most objects are not poets, regardless of which daydream. Nor is there any point in looking for more complicated boolean compounds than *and* and *if-then* since **Gen 8** says no other boolean functions will do.

Gen 9 "logical" expressions, including Dets, are those whose denotations are invariant under all permutations of the domain.

Compare the P2s *admires,* and *equals* in the sense of *is identical to*. Who admires who in a given state of affairs is an empirical matter, it depends on how the world is. In a model with universe E = {John, Mary, Sue} any subset of the nine pairs of elements of E is a possible extension for *admire*. But there is a single uniform answer to *what is identical to what*, namely everything is identical to itself and that's it. So *is identical to* always denotes $\{\langle x,x\rangle | x \in E\}$ and has a "logical" character that makes it different from *admires*. Similarly among P1s *exists* has a logical character different from *barks* and among Dets, *every* in *every cat* has a logical character that *John's* in *John's cat* lacks.

To characterize this notion of "logical character" we illustrate it first with an example and then generalize. Imagine a model whose domain has just three elements: Fido, Lassie, and Rover. Suppose that just Fido and Lassie bark. So the extension of *bark* (the set of things it is true of) is {Fido, Lassie}. Now consider a permuation h of the domain which maps Fido to Lassie, Lassie to Rover, and Rover to Fido. Applying this h to the extension {Fido, Lassie} of *bark* yields {h(Fido), h(Lassie)} = {Lassie, Rover}. The extension of *bark* here has changed, so its extension is not invariant under h, and hence not invariant under all permutations of the domain.

But with this same domain the extension of *exists* is {Fido, Lassie, Rover}, and replacing each of these objects with its value under h yields {h(Fido), h(Lassie), h(Rover)} = {Lassie, Rover, Fido}, which is the same set we started with. Moreover no matter what permutation h of the domain we pick, the extension of *exists* does not change under replacement of its members by the values under h since the set of those values is everything in the domain, which is the extension of *exists*. Thus *exists* is *permutation invariant* **–** invariant under all permutations of the domain. Similarly *is identical to*, *every*, and many other expressions are permutation invariant. See Keenan (1996), Keenan & Westerstahl (1997) or the more advanced Keenan (2001) for more information concerning how this notion extends to additional categories of expression. And, by **Gen 9**, expressions that we pretheoretically identify as "logical" or "mathematical" always denote permutation invariant objects.

Gen 10 Natural language Dets include many that are not first order definable (FOD)

An expression is FOD if it can be defined by a formula in a first order language. First order Ls are ones whose formulas are built from *atomic formulas* by forming boolean compounds and quantifications ($\forall x\phi$ and $\exists x\phi$ are formulas if ϕ is). *Atomic formulas* consist of an n-place predicate symbol, including the P2 =, and n *terms*, where a term is an individual constant (like '0' in arithmetic) or a variable which ranges over the elements of the domain of the model. Specifically an English Det *d* is FOD iff there is a first order formula ϕ(A',B'), in which the only non-logical predicates are the P1s A'and B', and for all models with universe E and all subsets A,B of E, d_E(A)(B) = True iff ϕ(A',B'), where A' is interpreted as A, and B' as B. For example *at least two* is FOD since

(34) $(\text{AT LEAST TWO})_E(A)(B) = T$ iff $\exists x\exists y(\text{not}(x = y)$ and $A'(x)$ and $B'(y))$.

We care whether natural language expressions are FOD for two related reasons: first, the semantics of first order languages (fols) is very well understood, so if natural languages can be translated, however cumbersomely, into fols this would provide a clear way of representing at least their logical meaning. And second, in fols the entailment relation, $\vDash$, is syntactically characterizable. That is, whenever $\phi \vDash \psi$ for some sentences ϕ,ψ there is a proof from ϕ to ψ, $\phi \vdash \psi$. A proof is a syntactic object, a sequence of formulas with some marked as premisses and the others derived from previous ones by syntactic rule (for example, *Modus Ponens* which says that from P and $P \rightarrow Q$ derive Q). But many natural ways of increasing the expressive power of fols, as by allowing quantification over possible P1 denotations, results in a language in which $\vDash$ is not syntactically characterizable (see Lindstrom 1969).

Now Barwise and Cooper (1981) argue that MOST is not FOD, even if the domain E is required to be finite. This entails that the Det_2 *more...than...* is not FOD since if it were we could define MOST by MOST(A)(B) = MORE $(A\cap B)$ THAN $(A - B)(E)$. Typically proportionality Dets and cardinal Det_2s are not FOD. See Westerståhl (1989).

It is worth noting here that the notions of sortal reducibility and first order definability overlap but are independent. Dets like *just finitely many* and *all but finitely many* are, respectively, intersective and co-intersective, but not FOD. The "Aristotelian" universal quantifier *all the*, which behaves like *all* except that it presupposes that the Noun argument is non-empty, is FOD per (35b) but is not sortally reducible (since it is neither intersective nor co-intersective). Similarly the definite description operator *the one* is FOD, (35d), but not sortally reducible.

(35) a. (ALL THE)(A)(B) = T iff $A \neq \emptyset$ and $A \subseteq B$

b. All the A' are B' iff $\exists x(A'x)$ & $\forall x(A'x \rightarrow B'x)$

c. (THE ONE)(A)(B) = T iff $|A| = 1$ and $A \subseteq B$

d. The one A' is a B' iff $\exists x(A'x)$ & $\forall y(A'y \rightarrow y = x)$ & $\forall x(A'x \rightarrow B'x)$

We close by noting a few quantificational expressions which are less well studied than those given above. The first example is surprising.

3. Crossing the Frege Boundary: DPs as arity reducers $P_{n+1} \to P_n$

We have presented DPs as subjects, combining with P_1s to form P_0s (Ss) and interpreted as functions from P_E, P1 denotations, to {T,F}, P0 denotations. But DPs combine generally with P_{n+1}s to form P_ns.

(36) a. John interviewed every student / more men than women / no candidate but Mary
b. We told every detective several lies
c. We sent no offender more than two warnings

In (36a) the DP *every student* combines with the P2 *interviewed* to form the P1 *interviewed every student* so it is natural to interpret it here as a function mapping binary relations (P2 denotations) to sets (P1 denotations). Since we already interpret *every student* as a map from sets to truth values we now just extend this function by adding some new objects to its domain – the binary relations. What is linguistically crucial here is that the value this function assigns to a binary relation is determined by the values it assigns to the sets. To say that John interviewed every student is to say that EVERY(STUDENT) holds of the *set* of objects that John interviewed. So once we have determined for each set A the value that EVERY(STUDENT) assigns to A then we have determined what set it assigns to each binary relation R. Here is the explicit definition, writing xR for $\{y|\langle x,y\rangle \in R\}$, the set of y such that x bears R to y:

Def 14 For F a map from P_E into {T,F} and R a binary relation over E,

$$F(R) = \{x \in E|\ F(xR) = T\}$$

Corollary For every F from P_E into {T,F} there is exactly one map from n+1-ary relations to n-ary ones, all n, satisfying the equation in Def 14.

This guarantees that the P1 *interview every student* denotes a set, the set of objects x which are such that EVERY(STUDENT) is true of the set of things that x interviewed. One sees clearly in Def 14 that the value F assigns to R is decided by the value it assigns to the *sets* xR. If F maps xR to T then $x \in F(R)$, otherwise it isn't.

Now Def 14 is misleading in one respect – it suggests that we first define a GQ on the subsets of E and then in some second step *extend* that definition. But in fact we just need to give one properly general definition to begin with and generalizing to all n is nearly trivial. For x an n-tuple of objects, and R an n+1-ary relation, write xR for $\{y|\langle x,y\rangle \in R\}$. Note that $\langle x,y\rangle$ here is an n+1-tuple. Then $F(R) =_{df} \{x \in E^n | F(xR) = T\}$. In this way we treat DPs as *arity-1 reducers*, maps from n+1-ary relations to n-ary ones, all n, whose values at any n+1-ary relation is uniquely determined by its values on the unary relations (the subsets of E). See Keenan & Westerståhl (1997).

This interpretation of DPs crosses the Frege boundary opening up new avenues of generalization. The Fregean solution to the problem of multiple quantification consists of iterated application of unary quantification (GQs). Thus in the recursive truth conditions of (37a), more enlighteningly revealed in (37b) which separates variable binding from quantification, we only evaluate the quantifiers against sets, never binary relations.

(37) a. Every teacher praised some student

b. (every teacher)λx(some student)λy(x praised y)

In (37b) *x praised y* is a P_0 (Formula), *λy.x praised y* is a P_1, *(some student)λy.x praised y* is a P_0, *λx(some student)λy(x praised y)* a P_1, and the entire expression (*every teacher*)λx(*some student*)λy(x *praised* y) a P_0. So every time we add a quantifier we convert a P_1 to a P_0, and semantically we map a set to a truth value. So on standard

Fregean analyses, as in Heim & Kratzer (1998:Ch 7) details of formalism aside, quantifiers are unary – they always take a set (unary relation) as argument to yield a truth value (0-ary relation) as value. But this approach has limitations as regards the semantic analysis of natural language.

First it eliminates the restrictions on the domain of quantification in favor of quantifiying over the whole domain. But, **Gen 8** and **Gen 10**, there are limits on our ability to do this with proportionality quantifiers. Second, and more interestingly, it only works for non-subject DPs which can also serve as subject DPs. But there are DPs which only occur as objects or whose interpretations as objects are different from their interpretations as subjects. Reflexives and reciprocals are examples of the former:

(38) a. Bill praised himself / everyone but himself / both himself and the teacher
b. The candidates criticized each other / each other and each other's wives

(39) a. *Himself praised Bill
b. *Each other criticized the candidates

To illustrate with reflexives, we may interpret the P1 *praised himself* by SELF(PRAISE), where for all binary relations R, SELF(R) $=_{def}$ $\{x \in E| (x,x) \in R\}$. Similarly (EVERYONE BUT SELF)(R) $=_{def}$ $\{x \in E|$ for all $y \neq x \in E, (x,y) \in R\}$. And from Keenan (1989):

Theorem For E with at least two elements, there is no function F from P_E into {T,F} such that for all binary relations R, F(R) = SELF(R)

Comparable claims hold for the denotation of *everyone but himself* and *both himself and the teacher*. Thus these DPs, which do not occur as main clause subjects, denote ways of mapping binary to unary relations which are not obtainable per Def 14 from subject denoting DPs. The existence of such maps is unsurprising, but a few cardinality facts will show us just how unsurprising.

In a model with just two individuals there are $2^2 = 4$ elements of P_E, and $2^4 = 16$

GQ_Es. In contrast the number of maps from binary relations to sets is over 4,000 million! (There are 4 pairs in $E \times E$ and thus $2^4 = 16$ elements of $P_{E \times E}$; there are 2^2 elements of P_E, hence $2^{(2 \cdot 16)} = 2^{32} \approx 4 \cdot 10^9$ maps from binary relations to subsets of E). So there are *massively* more things in principle denotable by direct objects than subjects. And one (proper) subset of these are denotable by reflexive DPs of the sort noted above.

Of course there are known solutions to representing reflexives that do not involve maps from binary to unary relations. We might represent *Every student admires himself* by (*every student*)λx(x *admires* x), which determines the correct truth conditions. But this solution is linguistically unnatural: it means that *himself* is not assigned a meaning; nor does it have a category in the logical syntax, so there is no reason to expect it to coordinate with non-reflexive DPs as the last example in (38a). Worse, there is no reason to expect the ungrammaticalities in (39a,b). But on the view presented here there is such an expectation: *himself* denotes a function SELF which takes binary relations as arguments, and in Ss with the structure [*himself*][$_{P1}$ *laughed / praised Bill*], *himself* does not form a constituent with anything interpreted as a binary relation[3], so either *himself* there must be assigned a different meaning or it is uninterpretable.

Now DPs as arity-1 reducers suggests a more dramatically non-Fregean query: Does English present arity-k reducers for k > 1? An arity-2 reducer for example would directly map binary relations to truth values, "saturating" two arguments of a predicate. The "composition" of two arity-1 reducers as in (40b) is an arity-2 reducer, but one that, by construction, is reducible to iterated application of two arity-1 ones:

(40) a. No teacher read every play

b. (NO(TE) ∘ EVERY(PL))(READ) = NO(TE)(EVERY(PL)(READ))

Does English present arity-2 expressions not paraphrasable as the composition of two arity-1 ones? Keenan (1992) shows that the combination of the subject and object DPs in (41) determine arity-2 functions not equal to F∘G for any arity-1 functions F,G. One is given in (42), which interprets the subject-object pair in (41a).

(41) a. Different people like different things

b. Each student answered a different question (on the exam)

c. John criticized Bill but no one else criticized anyone else

d. Which students read which books?

(42) $(\mathrm{DIFF}(A),\mathrm{DIFF}(B))(R) = \text{True iff } |A| \geq 2 \ \& \ \forall x \neq y \in A, \ xR{\cap}B \neq yR{\cap}B$

The full range of expressible arity-2 reducers in English is unknown. So, much remains to be discovered even in English, the language in which quantifiers have been the most extensively studied. And we are just beginning to study quantification in less well known languages (Bach et al 1995, Matthewson 2001) as well as in contexts other than that of NPs and Dets, the most prominent here being temporal and event quantification using adverbial quantifiers: *Sue / often / always / occasionally / seldom / rarely / never visits cathedrals on holidays* (de Swart 1996), or *Mary winked at John during every meeting one Monday* (Pratt and Francez 2001).

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Footnotes

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1 Or somewhat more naturally as functions from individuals of the model to truth values. So the property expressed by *read the Times* would map to T those individuals who read the Times, and to F all those who don't. Here we use the more familiar set rather than property approach.

2. Writing INT_E ($CARD_E$) for the set of intersective (cardinal) functions from P_E to GQ_E , we note

Proposition For all E, $CARD_E \subseteq INT_E$, $|CARD_E| = 2^{|E|+1}$ and $|INT_E| = 2^k$, for $k = 2^{|E|}$.

So in a model with $|E| = 4$, there are $2^5 = 32$ possible denotations for cardinal Dets but $2^{16} = 65,536$ possible denotations for intersective Dets! (Keenan & Moss 1985)

3. These are awkward to represent on views which treat comparison as a sentence level operator.

4. DPs such as (*both Jack and neither Manny nor Jack*) are true of no properties

5. Heim (1987) and Safir (1982) doubt the grammaticality of such Ss. I have added emphatic elements like *anyway* which seem to improve it. About two thirds of the speakers I've consulted accept it easily.

6. Taking the $\leq$ as a primitive boolean order we would define $(x \wedge y)$ as the greatest lower bound of $\{x,y\}$ and $(x \vee y)$ as its least upper bound. Taking $\wedge$, $\vee$ and $\neg$ as primitive we must require that they satisfy the boolean axioms: both $\wedge$ and $\vee$ are commutative and associative, each distributes over the other, etc.

7. In any boolean algebra the meet, join and complement operations satisfy a battery of axioms, so the generalizations this interpretation supports are extensive. For example meets distribute over joins: $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ in all boolean algebras. So for Sentences P,Q,R we predict that P & (Q or R) is logically equivalent to (P & Q) or (P & R). Similarly *every student and either some teacher or some dean* $\equiv$ *(every student and some teacher) or (every student and some dean)*, etc. All the sets in which our expressions of interest denote have a boolean structure.

8. D of type <<1,1>,1> is Cons iff D(A,A')(B) = D(A,A')(B') whenever B and B' have the same intersection with each of the N properties. See Keenan & Westerståhl 1997.

9. Or a function with SELF among its arguments.