Language Variation and Linguistic Invariants: A Unifying Perspective

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1. The One and the Many

A central **question** in syntactic theory is how to reconcile the audible diversity of natural languages (NLs) with the claim that they have a common, biologically determined, form.

1.1 The one Mainstream syntactic theory (MST) attempts this reconciliation by building it into the form of individual expressions which must satisfy general constraints on rules and representations. For any NL L, any *core* expression X of L, MST asks "What is the structure of X?". The initial answer is typically a binary, right branching tree in Spec-[Head-Complement] order, using language independent category symbols and structure building functions (Merge, Move). Variation, rightly, is handled less uniformly: some (?much) is relegated to the *periphery* and ignored; some is acknowledged in parameters (with small ranges e.g. question words remain in situ or front); and some lies in feature variation forcing slightly different patterns of movement (copying). Overt morphology is language specific, not determined by UG and not structurally autonomous (Bobjalik 2002) but a "reflection" of hierarchical category structure (Borer 2005 is an exception here). So MST focuses on the unity pole of the unity **–** diversity continuum.

1.2 The many Here following (K&S 2003), I propose a type of reconciliation that focuses on diversity. Crucially it provides a conceptual notion of structural **invariant** that can be satisfied by non-isomorphic structures. Generally, invariants are not present in the grammar as conditions on rules or representations and are not instantiated by single expressions. Invariants are defined in terms of the *relation* "has the same structure as" not in terms of *the* structure of an expression. Given an expression X, we ask "What expressions have the same structure as X?". In presenting this perspective let me forewarn you of one difference in character between it and MST.

MST focuses on notation and the attendant *elimination of redundancy* (economy). The opening of Kayne 1994 is illustrative (emphasis mine): "It is difficult to attain a restrictive theory of syntax. One way ... is to restrict the space of available syntactic *representations*, for example, by imposing a binary branching requirement, ... The present monograph proposes further severe limitations on the range of syntactic *representations* ...". But restricting notation can be detrimental, as different notations may generalize to new phenomena differently. For example, in the early 1960's context free grammars and categorial grammars were shown to define the same class of languages (Bar-Hillel, Gaifman and Shamir 1960). But it was categorial grammar that generalized to function-argument structure in semantic representation (Lewis 1970, Montague 1973) effectively creating the field of formal semantics as we know it today. In general, the *significant* properties of objects remain invariant under changes of descriptively comparable notation, notational artifacts don't. The truth of *x is hotter than y* does not vary according as we measure temperature in Fahrenheit or Celsius; *x is twice as hot as y* does. So let us focus on regularities of linguistic nature not their notational expression and adopt the slogan

If you can't say something two ways you can't say it.

1.3 Back to the many We begin with two generalizations based on notationally distinct objects. The first is a semantic generalization in propositional logic: (1a) uses standard infix notation for conjunction and disjunction, (1b) uses prefix (Polish) notation. $\vDash$ is "entails".

(1) a. $((P \vee Q) \wedge \neg P) \vDash Q$ b. $\wedge\vee PQ\neg P \vDash Q$

The notational variation is non-trivial: formulas in prefix notation use no parentheses and exhibit no structural ambiguities. Eliminating parentheses from infix notation yields structural, and semantic, ambiguity: it derives $P \wedge Q \vee R$ from the non-equivalent $((P \wedge Q) \vee R)$ and $(P \wedge (Q \vee R))$. But the entailment fact in (1a,b) is *the same*: A disjunction of formulas conjoined with the negation of the first entails the second. And in general the entailments expressible in prefix notation and those in infix notation are the same. The synonymy of $(P \wedge Q)$ and $\wedge PQ$, etc. is determined by independent compositional interpretation. There is no need to derive them from a common "deep structure" or map them to a common "LF".

Second, imagine two intergalactic ethnomathematicians surveying behaviors associated with the notations used at the place-times indicated below. They look at enough cases to avoid performance errors, decide the behaviors are mathematical and wonder if they express the same phenomenon, and if so, just what it is.

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(2) Budapest   203 : 7 = 29            Paris, Rio de Janeiro   203 |7
                 63                                              63  29
                         29
    LA, Beijing, Seoul  7| 203          12th c. Rome/India       |  6 |
                            14                              7    |2 0 3|   29
                             63                                  |1 4 3|
                             63                                  |  6 |
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Ah! I see says one, given two numbers they find a third whose product with the second is the first. Huzzah! croaks the second, earthlings divide! So they have found the unifying generalization. Note that even if they had devised algorithms deriving all the notations from the American one they would not have understood what process they were all expressing – the division of 203 by 7 . The explanatory generalization is not just one more notation, but something qualitatively different.

1.3.1 On sameness of structure

We treat a language as a compositionally interpreted set of expressions defined by a grammar $G = (Lex_G, Rule_G)$, where Lex_G is a listed set of expressions (the lexicon) and $Rule_G$ is a set of structure building functions which iteratively map tuples of expressions to expressions. The *language* L(G) *generated* by G is the set of expressions derived from Lex_G by applying the rules finitely many times. A *descriptively adequate* G for a NL L is *sound* (everything it generates is judged by competent speakers to be in L) and *complete* (everything speakers accept is generated).

A grammar G is *isomorphic* to a grammar G', (G ≃ G') iff there is a bijection h:L(G) → L(G') matching the functions F in Rule_G with the F' in $\mathrm{Rule}_{G'}$ so that whenever F derives an expression z from some x and y then F' derives h(z) from h(x) and h(y). An isomorphism h from a G to itself is called an *automorphism* (or a *symmetry*). They are the ways of substituting one expression for another within a language which do not change how expressions are built: h *preserves structure*. F builds z from x and y iff F builds h(z) from h(x) and h(y). The set Aut(G) of automorphisms of G, represents the "structure" of G. It contains the identity map, is closed under composition, $\circ$, and inverses $^{-1}$ and so is a group, called the *automorphism group* (or *symmetry group*) of G.

1.3.2 Invariants An expression s *has the same structure as* an expression t, s ≃ t, iff there is an automorphism h which maps s to t. NB: We have here not referred to "the structure" of an expression, an epistemological plus, as we agree more readily that *John sang* and *Bill danced* have the <u>same</u> structure than we do about what "the" structure of *John sang* is. We define:

Def 1 A relation R on L_G is **invariant** iff h(R) = R, all auts h. I.e. trading all tuples $(s_1,...,s_n) \in R$ for $(h(s_1),..., h(s_n))$ leaves R unchanged. So $w \in L_G$ is **invariant** iff h(w) = w, all auts h.

Thesis $w \in L_G$ is a *grammatical formative* ("function word") iff $w \in \mathrm{Lex}_G$ and w is invariant

So grammatical formatives are the lexical items that are isomorphic only to themselves – if you replace them by something else you change structure (usually yielding ungrammaticality). E.g. in models of grammars we have made up, reflexive pronouns, case, voice, and agreement markers are provably invariant. Here are some generalizations built on this notion of invariant.

2.1 Relation invariants It is a theorem that for all G (not just G for NLs), *is a (proper) (immediate) constituent of* and *c-commands* are invariant relations. Their invariance follows from their (more general than usual) definitions in our format. In contrast, the *is a possible antecedent of* relation is not invariant in all G, but it is for all the G we have constructed to model NLs. This illustrates invariants among non-isomorphic structures. In the G that K&S use for the minimal clause structure in Korean we find expressions like (3b) where anaphors like *self-acc* asymmetrically c-command their antecedents (to see that *self-acc* is an anaphor one must check its semantic interpretation in K&S: it maps a binary relation R to the property {a|aRa}).

(3) a. P0

KPn P1n

KPa

NP Kn P2

NP_{refl} Ka

John -nom self -acc criticized

b. P0

KPa P1a

KPn

NP_{refl} Ka P2

NP Kn

self -acc John -nom criticized

But each of these Ss is interpreted compositionally, and they receive the same interpretation: True iff $(j,j) \in$ CRITICIZE, j the denotation of *John*.

Lovers of trees Beware! The ordered labeled trees in (3a,b) are isomorphic, but the *expressions* are not isomorphic in our grammar. Reasons: if an aut mapped *john* to *self* and *nom* to *acc* it would map *John nom laughed* to *Self acc laughed*, which is not in our model language.

(4) from Toba Batak is a W. Austronesian anaphora pattern where the distribution of reflexives is conditioned by the verb voice, the reflexive often occurring as what we thought was a “subject”. And as in our model of Korean, lexical items occurring in both Ss have identical interpretations. So again anaphors may asymmetrically c-command their antecedents.

(4) a.

			P0		
		P1n		NP	
	P2a		NP_{refl}		
Vaf	P2				
mang-	see		self	John	

b.

			P0		
		P1a		NP_{refl}	
	P2n		NP		
Vpf	P2				
di-	see		John	self	

Theorem 1 The case markers (nom,Kn) and (acc,Ka) and voice markers (mang,Vaf) and (di,Vpf) are provably invariant in their respective grammars.

Thus is morphology “structural” in exactly the same sense as hierarchical structure – the structure building functions: to wit, all are mapped to themselves by the automorphisms.

Theorem 2 The Anaphor-Antecedent relation is invariant in both grammars (as well as in our model of English, not illustrated here).

Thus we see how a linguistic relation can be universally invariant even though grammars of different languages are not isomorphic in the relevant respects. So it may be that English conditions the distribution of reflexives in terms of constituency, Samoan in terms of linear order, Korean case marking and W. Austronesian voice marking. And we may consider adopting

Axiom 1 The Anaphor-Antecedent relation is invariant in all PHGs (possible human grammars).

Caveat Lector Claiming that a NL is an interpreted pair (Lex, Rule) does not merit the name *theory* – at best it is a common denominator of theories such as Minimalism, HPSG, LFG, RG, ... Axioms would be needed to limit pairs (Lex, Rule) to PHGs. But two features of our approach do have a liberating effect on the design of syntax: (1) the requirement of a compositional semantics, and (2) the theorem that morphology may be structural. We are for example free to condition the distribution of anaphors (defined semantically) directly in terms of case or voice, it is not necessary to derive them from (or reconstruct them to) forms c-commanded by their antecedents.

2.2 Property invariants Is the property of being a lexical item invariant in all G for NL? That is, for all G, does $h(Lex_G) = Lex_G$, all auts h? This identity is a possible empirical truth (not a theorem) even though the lexicons of different NLs usually differ. Similarly, is the property of having a given category C invariant? That is, do all auts map each PH(C), **the set of phrases of category C**, to itself? Taking this as axiomatic would provide a universal structural role for categories. But K&S support empirically that at most the weaker Axiom 2 is justified.

Axiom 2 For all C, PH(C) is invariant under stable automorphisms

A **stable** automorphism is_{def} one that extends to an automorphism of any finite extension of Lex_G Such an extension results from adding a new lexical item isomorphic to one in the old grammar. K&S's model of Spanish for example has two noun classes N_m and N_f and overt agreement of adjectives and dets. There are automorphisms **h** which interchange $PH(N_m)$ and $PH(N_f)$, provided the number of lexical members of each class is the same. If we add one new member to just one of the classes we can no longer interchange them by an aut, so such **h** are unstable – they do not extend to automorphisms of the new grammar.

Axiom 3 Theta Role assignment is invariant

So if x has theta role τ in y then h(x) has τ in h(y), that is, theta role assignment is a function of structure. So if *John* bears different theta roles in *John ran* and *John arrived* then these Ss must be non-isomorphic. **Axiom 3** is strictly weaker than UTAH, which requires that the function be one to one (same theta role $\Rightarrow$ isomorphic sources). But a function can take the same value at different arguments, $(3)^2 = (-3)^2$. So we are not forced to say that active subjects and their corresponding passive agent phrases originate in isomorphic configurations.

2.4 Greenberg Duality. Two languages are word order duals if the expressions of one are the mirror images of those of the other. A single lexical item is its own dual. A rigid SXOV language is (isomorphic to) the dual of a rigid VOXS language (same Lex). But every L has a dual, regardless of the "consistency" of its word order paradigms. More carefully now, the dual v^d of a sequence $v = \langle v_1,v_2,...,v_n \rangle$ of lexical items is just its mirror image, $\langle v_n, ..., v_2, v_1 \rangle$. The dual K^d of a set K of expressions is the set whose elements are the duals of the members of K. If F is in $Rule_G$ then F^d is that function whose domain is $\{\langle w_1,...,w_n \rangle | \langle w_1{}^d,...,w_n{}^d \rangle \in Dom(F)\}$. And F^d maps $(w_1,...,w_n)$ to the dual of $F(w_1{}^d,...,w_n{}^d)$. We define G^d to be that grammar with the same lexical items as G and whose rule set is $\{F^d | F \in Rule_G\}$. And we prove:

Theorem 3.1 $L(G)^d = L(G^d)$
3.2 $G \simeq G^d$, the map sending each $w \in L(G)$ to w^d is an isomorphism (surprisingly).

Axiom 4 The set PHG of possible human grammars is closed under isomorphism

The axiom just says that if $G \in PHG$ and $G \simeq G'$ then $G' \in PHG$. Justification: UG only selects for structure not content and thus cannot distinguish between isomorphic variants.

Corollary: PHG is closed under duals (From Axiom 3 and Theorem 2.2)

A possible consequence of the Duality Corollary is that it should make us hesitant to accept Kayne's Antisymmetry axiom, which as I understand it, forces right branching structures. If only those grammars were acceptable then PHG would not be closed under duals. But since there are left branching Ls (Malagasy) the force of the axiom must be weakened in some way.

The Corollary suggests another comparison with MST. Namely, eliminating redundancy is overrated. Mathematicians like axioms of a theory to be independent (= non-redundant – no one follows from the others). But – here comes another slogan – **Symmetry trumps redundancy**. The axiomatic definitions of groups and of boolean algebras are closed under duals (the identity element axioms, distributivity) though any one of the pair of axioms follows from the others.

2.5 Degrees of Invariance We naturally generalize *invariance* to a (scalar) relation by:

Def 2 w is *more invariant than* w' iff the number of automorphisms that fix w is greater than the number that fix w' (An aut h "fixes" an expression s iff it maps s to itself; write Card(w) for the number of automorphisms that fix w. Card(w) is finite as long as Lex is finite).

Using this notion for example we can abstractly represent the grammaticization of an expression w over time by saying that Card(w) is an increasing percentage of the automorphisms of G which fix w. When that percentage reaches 100 w is fully grammaticized. We may now also say that conjunctions and prepositions are more grammaticized than common nouns.

3. A Goal of Descriptive Linguistics Classify human grammars by their Symmetry Groups. How many ways are there to build a Predicate-Argument system? A Modifier System?

Empirical considerations show that G for NL are not cyclic (and often not Abelian), so they have non-trivial subgroups. A first guess: each set PH(C) of phrases of category C determines a subgroup of G and their product is isomorphic to the subgroup of stable automorphisms. But before pursuing such claims we need to codify in terms of invariants standard structural notions like *paradigm*, *inflectional morphology*, *allomorph*, *subcategory*, *extraction* and *copy rules* (Kobele 2006). Codifying simple agreement led to the important notion of stable automorphism. These other structural notions might enrich the group structure of NL G in similar ways.

4. Historical Background

Felix Klein's Erlangen dissertation in 1872 stood Euclidean geometry on its head, treating it as the invariants under translations, rotations, and reflections. Other geometries are invariants of other transformations. Emmy Noether (Cole 1997:183), at Erlangen in the early 1900s, relates symmetry and conservation laws in relativity theory. Tarski (1986, 1966) later treats *logical* objects as ones invariant under all operations. See Keenan 2001. Roman Jakobson (1963) speaks of mathematical invariants in Linguistics. Symmetries are widely used in physics (e.g. predicting new elementary particles; Sternberg 1995) and classification of molecules in physical chemistry (Cotton 1990). Helpful references are Gardner 2005, Stewart and Golubitsky 1999, and Weyl 1952.

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