18

Individuals Explained Away

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As a linguist wading into philosophical waters I begin with two semantic observations concerning some intensional common nouns and their modifiers. I provide them with a minimal semantic analysis whose justification is twofold. One, it is linguistically enlightening: it points out some semantic generalizations about English and provides a formally explicit analysis of the relevant entailment patterns. Two, it generalizes standard extensional model theory without adding novel entities such as possible worlds or propositions. Such entities may facilitate the semantic analysis of modal adverbs and propositional attitude verbs, but are not needed for the intensional expressions we study here. Our analysis is explanatory in that it characterizes what we are trying to understand only using notions we already understand, not novel ones we don't.

Our analysis may also have some consequences for Direct Reference Theory (see Almog (2012), Bianchi (2012), Kaplan (1989), Napoli (1995), Wettstein (2004)). Among them is that it eliminates from our naïve ontology a universe of objects we think of singular terms as denoting and unbound pronouns and individual variables in logic as ranging over. It also establishes some boundary points limiting the purview of Direct Reference Theory. And, if nothing else, it may serve as a "bad example" function—"Just look at what happens if you do not adopt a direct reference stance". As well, our use of judgments of entailment is more at home in a Fregean setting than a Direct Reference one (Capuano 2012).

18.1 Some Common Noun Phrases and their Modifiers

We are concerned first with a class of common noun phrases (cnps) we call *agentive*. Some examples are given in (1), accompanied by the indefinite article *a/an* to facilitate readability.

(1) a. a surgeon, a lawyer, a poet, a philosopher, a politician, a sculptor, an author;
 b. a pianist, a flautist, a neurologist, a biologist, a perinatologist, a psychiatrist;

c. a heart surgeon, a poker player, a mountain climber, a race car driver, a portrait painter, a criminal lawyer, a natural language philosopher.

Agentive cnps designate individuals who regularly engage in activities in which they may be more or less accomplished. Such cnps may be syntactically simple—not formed from other words—as in (1a), they may be morphemically complex, as those formed with *-ist* or *-er*, or they may be syntactically compound with an object noun preceding the final noun, as in (1c). Other complexities also exist.

Second, we treat *evaluative* modifiers of such cnps, ones which concern how well participants in the designated activity perform it. Some examples are underlined in (2). Per (2b), they may themselves be syntactically complex.

(2) a. a <u>skillful</u> (heart) surgeon, a <u>talented</u> flautist, a <u>graceful</u> ballet dancer, a <u>competent</u> pool player, a <u>gifted</u> mathematician, a <u>good</u> linguist, a <u>professional</u> boxer, an <u>accomplished</u> mountain climber;
b. a <u>very skillful</u> surgeon, a <u>well-qualified</u> engineer, a heart surgeon <u>wrongly regarded as</u> <u>inept</u>, an <u>exceptionally talented</u> bridge player.

A first semantic property that a semantic analysis of English should account for is that the sentences on the left below entail those to their right:

(3) a. John is a skillful surgeon. a′. John is a surgeon.
b. Sue is a gifted pool player. b′. Sue is a pool player.
c. Kim is a talented flautist. c′. Kim is a flautist.

This entailment pattern is not banal, in that, as Montague (1970) noted, not all adjectival modifiers in such environments support a judgment of entailment: *Ted is an ostensible millionaire* (*an alleged murderer*) does not entail *Ted is a millionaire* (*a murderer*). Our second semantic property is:

(4) *Non-extensionality of evaluative adjectives.* If the heart surgeons and the portrait painters happen to be the same individuals in some context, the skillful heart surgeons and the skillful portrait painters may still be different individuals. *Skillful* is replaceable with other evaluative adjectives and *heart surgeon* and *portrait painter* by other (distinct) agentive cnps, preserving the truth of (4).

We state (3) and (4) more formally. In each context (model) c, we interpret agentive cnps by *properties* p of individuals. Ext(p), *the extension of* p (in c, a qualification we usually omit in what follows) is the set of individuals which have p. And we interpret evaluative adjectives by extensionally *restricting* functions F from properties to properties, meaning that for all properties p, ext(F(p)) $\subseteq$ ext(p). So, noting denotations in a context in bold, ext(**skillful**(**surgeon**)) $\subseteq$ ext(**surgeon**). This accounts for the entailment pattern in (3).

Much trickier, however, is how to account for the non-extensionality facts in (4), though their general statement in (4′) is now straightforward:

(4′) For p,q agentive cnp interpretations and F an evaluative adjective interpretation it may happen that ext(p) = ext(q) but ext(F(p)) ≠ ext(F(q)).

Note that a variety of (non-evaluative) cnp modifiers *are* extensional, failing (4′). In a model in which the heart surgeons and the poker players happen to be the same individuals we *can* infer that the female heart surgeons and the female poker players are the same individuals. Equally the heart surgeons that John shook hands with at the party must be the same individuals as the poker players that John shook hands with at the party, though John may not know that.

18.2 A New Type of Intensionality?

Our judgments in (4) derive from the meaning of evaluative adjectives. The properties a heart surgeon must have to be a skillful heart surgeon are quite different from those a poker player must have to be a skillful poker player, and both are different yet from those making a flautist a skillful flautist. It is easy to see that in a given context a given individual might be both a skillful surgeon and an inveterate but only a mediocre (hence not skillful) poker player. No cross model comparisons are suggested by this judgment, in contrast to our naïve understanding of modal adverbs like *necessarily* and *possibly* or frequency adverbs: *Kim often/rarely walks to school*. So the appeal of a possible worlds approach to non-extensionality is absent in the case of interest to us.

18.3 If Not Possible Worlds, Then What?

We want to say that *surgeon* and *flautist* are interpreted by different properties whether they have the same extension in some situation or not. We shall refer to that part of the meaning of a cnp which is independent of its extension as its intension. But just what sort of object is this? For expressions in general I doubt that there is a uniform notion of intension. Expressions in different grammatical categories differ too greatly in syntactic distribution and in fine-grained semantic interpretation. Witness the diversity of modal interpretations in van Benthem (2010). But here we are just concerned with the intensions of agentive cnps. We shall think of the intension of a property p which interprets such a cnp as what we have to know to know whether an arbitrary individual has p (that is, is in the extension of p). What we have to know to know if someone is a surgeon is different from what we have to know to know if that person is a flautist. So it is easy to see that *surgeon* and *flautist* are interpreted by different properties regardless of whether the same individuals have both. And this is why evaluative adjectives can assign them different values (with different extensions):

a function can take different values at different arguments. Our intuition of cnp intension is, broadly, compatible with Frege's *Sinn* and with much later work in intensional semantics (see Muskens 2007).

18.4 Constructing Non-extensional Models of Agentive Nouns and their Modifiers

We proceed in three steps. STEP 1 reviews standard extensional semantics whose ontological primitives are two: a booleanly structured set of truth values {T,F} in which sentences are interpreted, and an unstructured non-empty universe E in which proper names and individual constants are interpreted. {T,F} is boolean—equipped with functions interpreting *and*, *or*, and *not*. "Truth in a model" and entailment are defined standardly.

The universe E is unstructured—formally its elements are inscrutable entities unrelated to each other in any general way. Common nouns are interpreted by subsets of E—equivalently, elements of the power set of E (the set of subsets of E) here noted P_E.

STEP 2 recasts standard semantics in purely boolean terms. There is no change in entailment relations or truth conditions of sentences because the sets in which expressions of a given category are interpreted on the boolean construal are isomorphic to their classical counterparts (hence they make the same sentences true). But the unstructured universe E is eliminated in favor of a booleanly structured set P of properties which provides interpretations for common nouns (and, up to isomorphism, one-place predicates). P is isomorphic to a power set and thus not structurally different from the set in which common nouns are classically interpreted. But proper name interpretations are now derivative, defined in terms of properties and truth values, the definitions being rooted in primary judgments of entailment and logical equivalence in English (presented shortly). Recasting the ontological primitives in a uniformly boolean way is what enables us to generalize to intensional semantics in STEP 3 by weakening one (perhaps unintended) assumption of standard extensional semantics. It also supports several other semantic generalizations about natural language.

STEP 3 provides a semantics in which distinct properties can have the same extension in a given situation. No appeal to novel primitives, such as possible worlds, is invoked.

STEP 1 (+ basic boolean notation). A standard extensional semantics takes reference and truth as primitive and a standard model consists of a non-empty universe E of objects and a boolean set {T,F} of truth values on which are defined functions like $\wedge$, $\vee$, and $\neg$ which provide interpretations for *and*, *or*, and *not*.

For purposes of later generalization we treat {T,F} as a boolean lattice rather than a boolean algebra. The two are definitionally equivalent, differing only in which notions are taken as primitive and which are defined (Grätzer 1998: 11). The lattice approach, simpler for our purposes, takes as primitive just the material implication relation, noted $\rightarrow$, and given standardly by: $T \rightarrow T$, $F \rightarrow T$ and $F \rightarrow F$. We note that $\rightarrow$ is reflexive, transitive, and antisymmetric ($x \rightarrow y$ and $y \rightarrow x$ implies $x = y$). The $\wedge$, $\vee$, and $\neg$ functions are defined in terms of $\rightarrow$: for x,y in {T,F}, $\wedge\{x,y\}$, usually noted $(x\wedge y)$, is defined as the *greatest lower bound* (glb) of the set {x,y}. This just means that one, the truth value $x\wedge y$ is a lower bound (lb) for {x,y}, that is, $(x\wedge y) \rightarrow x$ and $(x\wedge y) \rightarrow y$, and two, for any lb z for {x,y}, $z \rightarrow (x\wedge y)$. One computes that $(T\wedge T) = T$, that is, T is the glb for {T,T}. Similarly $T\wedge F = F\wedge T = F\wedge F = F$. So $\wedge$ here coincides with the standard truth table for *and*. We note that in any lattice L, the glb of a subset K of L is noted $\wedge K$.

Similarly $x\vee y$, the *least upper bound* (lub) of {x,y}, coincides with the truth table for *or*. One computes that F is the least, or *zero*, element of {T,F}, meaning that for all x in {T,F}, $F \rightarrow x$. Dually T is the greatest, the *unit* element: for all x in {T,F}, $x \rightarrow T$. Also the $\rightarrow$ order is distributive: $(x\wedge(y\vee z)) = ((x\wedge y)\vee(x\wedge z))$ and complemented: for every x in {T,F} there is a y in {T,F} such that $x\wedge y$ is the least element of {T,F} and $x\vee y$ the greatest. For each x this y is provably unique and noted $\neg x$. So $\neg T = F$ and $\neg F = T$, as expected.

We note too that P_E is a boolean lattice where the primitive relation is subset, $\subseteq$. This relation is reflexive, transitive, and antisymmetric. The greatest lower bound $\wedge K$ of any collection of subsets of E is just its intersection—the set of objects that lie in all the sets in K. So $X\wedge Y = X\cap Y$ in the finite case. And the lub of K, noted $\vee K$, is just its union, the set of objects belonging to at least one of the sets in K. So $X\vee Y = X\cup Y$. The least element is the empty set, $\varnothing$, and the greatest is E itself. The complement of a subset X of E is E—X.

So interpretations for sentences and common nouns / one-place predicates (P_1s) are both boolean lattices (of a certain sort, below).[1] Interpretation sets for other categories of expression also have a boolean structure, as the expressions combine quite freely with the boolean connectives *and*, *or*, . . .:

(5) An <u>attractive but not very well-built</u> house is for sale on the corner;
She lives <u>neither in nor near</u> New York City;
Sue <u>both praised and criticized</u> each student;
Kim works <u>neither rapidly nor carefully</u>;
<u>All doctors and most nurses</u> support single-payer health care.

In a standard model these boolean lattices all have two additional properties in common. One, they are *complete*—every subset has a glb (and a lub). And two, they are *atomic*—for every $x \neq 0$ there is an atom $\alpha \leq x$ ($\leq$ is the lattice order relation, relative to which glbs and lubs are taken). An atom α is a smallest non-zero element—that is, $\alpha \neq 0$ and for any $y \leq \alpha$, $y = 0$ or $y = \alpha$.

[1] We prefer to treat P_1s as elements of $[E \rightarrow \{T,F\}]$, functions from E into {T,F}, where the lattice order $\leq$ is given by: $f \leq g$ iff for all $x \in E$, $f(x) \rightarrow g(x)$. But the map sending each subset A of P to that map from E into {T,F} which sends x to T iff $x \in A$ is an isomorphism, so there is no loss in treating P_1s as subsets of E.

The data in (5) prompt a query and a puzzle. The query: Why should we expect that the boolean connectives *and*, *or*, *not*, *neither . . . nor . . .*, and some uses of *but* are syntactically ubiquitous? Combining with expressions of just about any category is unusual, and suggests that their meaning is not specific to the meanings associated with any particular category—be it sentence, n-place predicate, adjective, etc. Rather, we follow Boole's (1854) lead and treat them as reflecting properties of mind, "laws of thought"—ways we conceive of things rather than how things are. Our approach provides an account of this syntactic ubiquity. The set of objects which interpret expressions of any given category are sets with the structure of a boolean lattice (algebra) and *and*, *or*, and *not* are uniformly interpreted by the greatest lower bound, least upper bound, and complement operations in that lattice. (*Neither . . . nor . . .* denotes the Sheffer stroke.) Different categories are associated with different lattices.

The puzzle (solved in STEP 2): the universe E is not endowed with any boolean structure, but singular terms form boolean compounds naturally; they also form such compounds with properly quantified noun phrases, (6b).

(6) a. Neither Kim nor Dana; both Kim and Dana; either Kim or the teacher.
b. <u>John and some student</u> (will take the tickets).
<u>Neither Ed nor any foreign students</u> (were interviewed for the position).
<u>The CEO and all the employees</u> of the company (dislike the IRS).

STEP 2: Eliminating the universe
Consider the interpretation of quantified subjects, as in (7):

(7) a. All poets daydream;
b. Some poets daydream;
c. Most poets daydream.

We interpret these sentences directly using generalized quantifiers (GQs) which map properties to truth values. The quantifiers **all**, **most**, etc. map subsets of E, possible common noun interpretations, to GQs:

(8) For all subsets p,q of E,
a. $\mathbf{all}(p)(q) = T$ iff $p \subseteq q$;
b. $\mathbf{some}(p)(q) = T$ iff $p \cap q \neq \varnothing$;
c. $\mathbf{most}(p)(q) = T$ iff $2 \cdot |p \cap q| > |p|$ (p finite, $|p|$ its cardinality).

So *All poets daydream* is true iff the set of poets is a subset of the set of individuals that daydream, etc. So *all poets*, etc. are of a higher logical type than one-place predicates, consistent with Frege's conception of quantifier. And in a model with an unstructured universe it is not feasible to treat such quantified phrases as denoting elements of E. For example, in a model with just two poets, Kim and Dana, *all poets* is interpreted the same as *both Kim and Dana*, and (9a) has the same truth value as (9b).

(9) a. All poets daydream.
b. Both Kim and Dana daydream.

But if *all poets* denotes an element of E here we can infer that both Kim and Dana are that entity, whence Kim and Dana are the same individual contra the assumption that they constituted *two* poets.

A natural solution to this problem is to interpret proper names as GQs, as in Montague (1970) and Lewis (1970). Here we present an equivalent analysis, but with more immediate linguistic motivation. Consider the mutual entailment patterns below, using ≡ for "is logically equivalent to":

(10) a. Ned is a linguist and a poet ≡ Ned is a linguist and Ned is a poet;
b. Ned is a linguist or a poet ≡ Ned is a linguist or Ned is a poet;
c. Ned is not a linguist ≡ It is not the case that Ned is a linguist.

(10a) says that the GQs which interpret proper names are those that map a glb of properties to what you get when you apply them to the properties separately and take the glb of the results in the truth-value lattice. So proper name interpretations respect glbs. Similarly (10b) says they respect lubs, and (10c) that they respect complements. Thus proper name interpretations, henceforth called *individuals*, are homomorphisms from the property lattice to the truth-value lattice.[2]

This analysis allows us to dispense with E as a primitive in favor of an arbitrary complete, atomic lattice P, whose elements will be called *properties*. P provides denotations for common nouns and does not really differ from the standard model, in which cnps denote in P_E, as a boolean lattice P is complete and atomic iff it is isomorphic to a power set lattice.[3] Subjects, quantified or not, are interpreted as maps from P into {T,F}, with proper names denoting the homomorphisms in this set. Adjectives will denote maps from P into P.

But have we not moved a little rapidly? Does our "elegant" analysis of proper names correspond in *any* sense to what we intend on the classical view? It turns out it does. Consider the Lewis/Montague notion of individual (stripped of its intensional wrappings) as an intermediate step.[4]

[2] The homomorphisms we use here and later are understood to be *complete*. That is, they respect arbitrary greatest lower bounds and least upper bounds: $h(\wedge K) = \wedge\{h(k)|k \in K\}$ and $h(\vee K) = \vee\{h(k)|k \in K\}$. More generally, individuals are homomorphisms from n+1-place predicate denotations to n-place ones: to both praise and criticize John is to both praise John and criticize John; to either praise or criticize John is to either praise John or criticize John, and to praise Bill but not John is to praise Bill and not praise John. But here we are only concerned with individuals as denotations of subjects.

[3] Given P(A), the power set of A, its boolean relation is just subset, $\subseteq$. Its atoms are the unit sets {b} for each $b \in A$ and for K any collection of subsets of A, $\wedge K$ is provably $\cap K$, the collection of b which lie in each set in K, and $\vee K$ is $\cup K$, the x which are in at least one of the sets in K. Going the other way, if P is complete and atomic, then the map sending each $p \in P$ to $\{\alpha \in P| \alpha \leq p\}$ is an isomorphism from P to P(Atom(P)), the power set of the set of atoms of P.

[4] Lewis/Montague treat *John* as $\lambda p.p(j)$, which maps a property q to the truth value q(j). This is the function I_j, where we write I_b for arbitrary b in E below.

(11) For all $b \in E$ define the function I_b from P_E into {T,F} by: $I_b(p) = T$ iff $b \in p$.

Classically *John is a poet* is interpreted by T if the entity, say j, denoted by *John* is in the set **poet** denoted by *poet*. Now we interpret *John is a poet* by I_j(**poet**), which is T iff j ∈ **poet**. So there is no change in truth conditions, only an organizational change in the function-argument structure of *John is a poet*, *John* now denoting the function and the P_1 *is a poet* the argument. This interpretation of proper names as GQs still defines them in terms of a primitive universe.

But observe now that the I_bs are homomorphisms from P_E into {T,F}:

(12)

$I_b(p \cap q) = T$	iff $b \in p \cap q$	def I_b
	iff $b \in p$ and $b \in q$	def $\cap$
	iff $I_b(p) = T$ and $I_b(q) = T$	def I_b
	iff $(I_b(p) \wedge I_b(q)) = T$	def $\wedge$ in the {T,F} lattice.

Thus we see that I_bs map glbs of properties to glbs of truth values. Similarly they map lubs to lubs and complements to complements. So I_bs are homomorphisms from properties to truth values. And the converse holds as well: the (complete) homomorphisms are exactly the I_bs.[5] So now we define proper name denotations as the complete homomorphisms from P into {T,F} with no reference to any E, so E can be eliminated from our semantic primitives.

Observe the close connection between entities on the classical view and atomic properties on the boolean view. Given P complete and atomic, the map sending each q in P to the set of atoms $\alpha \leq q$ is an isomorphism from P to the power set of the atoms of P. So the atomic properties in P play the role entities do in classical semantics.

Further, for α an atom in P, define I_α as that GQ which maps a property p to T iff $\alpha \leq p$. One checks that the I_αs are just the complete homomorphisms from P into {T,F}.[6] If P is a power set lattice, then an atom is just a unit set, {b}, and {b} ⊆ **poet** iff b ∈ **poet**, so I_b(**poet**) in Lewis/Montague says the same as $I_{\{b\}}$(**poet**) on the boolean view, so individuals on the boolean view correspond one for one to those on the Lewis/Montague construal of the classical view.

Also we should solve our puzzle—that boolean compounds of proper names were embarrassing on the classical view since a bare E does not support boolean structure, yet we form boolean compounds of proper names and interpret them naturally. But it is now easy to see that boolean compounds of proper names do not behave as homomorphisms. For example, *both John and Bill* does not respect negation, as (13a) and (13b) are not logically equivalent:

[5] Let h be a complete homomorphism from P into {T,F}. All homomorphisms map the unit A to the unit T of {T,F}, and $A = \cup\{\{b\}|b \in A\}$ so $h(\cup\{\{b\}|b \in A\}) = \vee\{h\{b\}|b \in A\}$, so for some b, $h(\{b\}) = T$. h can't map two different unit sets to T, otherwise it maps their intersection to T, but their intersection is empty, ∅, which all homomorphisms map to F. So h maps exactly one {b} to T. If $\{b\} \subseteq B$, then $\{b\} \cap B = \{b\}$ so $h(\{b\} \cap B) = h(\{b\}) \wedge h(B) = T$, so $h(B) = T$. If $b \notin B$, then $\{b\} \cap B = \varnothing$ and $h(\{b\} \cap B) = h(\{b\}) \wedge h(B) = T \wedge h(B) = F$, so $h(B) = F$. Thus $h = I_{\{b\}}$ since h holds of exactly the sets that {b} is a subset of.

[6] The proof is basically the same as in note 5, with α replacing {b} and ≤ replacing ⊆.

(13) a. Both John and Bill did not pass the exam;
b. It is not the case that both John and Bill passed the exam.

But boolean compounds of proper names do behave semantically as boolean functions of generalized quantifiers.[7]

An unexpected dividend of our abstract approach to proper names is that we can account for why both linguists and philosophers so often take them to be the basic subject phrases. Our account is the observation that all GQs—functions from an arbitrary complete atomic boolean lattice into {T,F}—are boolean functions of individuals (Keenan and Faltz 1985: 92; Keenan and Westerståhl 1997). Only individuals are a parameter here as boolean functions are properties of mind. So the full class of quantified subject phrases is determined by the choice of individuals. Here is a quick proof using variables in 'I' to range over individuals.

THEOREM 1 All generalized quantifiers are boolean functions of individuals.

a. For all $p \in P$, let f_p be that map from P into {T,F} which maps p to T and all other $q \in P$ to F. f_p is, informally, "every p and no non-p". Formally, $f_p = \wedge\{I|I(p) = T\} \wedge \neg\vee\{I|I(\neg p) = T\}$. So f_p is a boolean function (defined in terms of $\wedge$, $\vee$, and $\neg$) of individuals.
b. For any H from P into {T,F}, $H = \vee\{f_p|H(p) = T\}$. So H is a boolean function of individuals. ☐

This theorem may seem linguistically unnatural, as complex subjects are often not built syntactically as boolean compounds of proper names or singular terms (though some are: *John and either Bill or Sue*, etc.). But the theorem only says that the *functions* denotable by subjects are boolean functions of ones denotable by proper names. And it is worth noting that cross-linguistically proper names are often (always?) historically derived from property-denoting expressions.

Think of Amerindian names, often given in English glosses: *Sitting Bull*, *Thunder Cloud*, *Dances with Wolves*. A few are borrowed: *Cochise* from Apache *k'uu-ch'ish* 'oak', *Mississippi* from Ojibwa *mshi-* 'big', and *ziibi* 'river'. *Schuylkill* in the *Schuylkill* River in Philadelphia comes from Dutch *schuil* 'hidden' and *kill* 'river'. Malagasy (Austronesian; Madagascar) forms proper names by prefixing property-denoting expressions with a particle (*Ra-* for adults, *I-* for children and foreigners). *Ramanandraibe* = '*Ra+manana+ray+be*, Ra+has+father+big'; *Ramasinaivo* = '*Ra+masina+naivo*, Ra+power+second-born'; *Randriamasimanana* = '*Ra+andriana +masina+manana*, Ra+noble caste+power+has'.

[7] In general (as with P_1s), when B is any boolean lattice and A any non-empty set, $[A \rightarrow B]$, the set of functions from A into B is a boolean lattice which inherits the structure of B pointwise: $f \leq g$ iff for all $x \in A$, $f(x) \leq g(x)$. Provably $(f \wedge g)$ maps each x to $f(x) \wedge g(x)$, $(f \vee g)$ maps each x to $f(x) \vee g(x)$ and $(\neg f)(x) = \neg(f(x))$. The zero function maps each x to the zero in B, and the unit to the unit in B. So generalized quantifiers, maps from P into {T,F}, have a complete atomic boolean structure pointwise.

In English we find some bird and mammal names: *Crow*, *Crane*, *Partridge*, *Fox*, *Wolf*; in Hebrew *Aryeh* 'lion', *Dov* 'bear', *Snait* 'squirrel', *Yael* 'ibex'. Family names (the most recently innovated) in European languages are often transparent: occupations—*Shoemaker*, *Carpenter*, *Taylor*, *Smith*, *Wright*, *Tanner*, *Baker*, *Butcher*; color names—*Brown*, *Black*, *Green*, *White*, *Gray*; descendants' designation—*Fitzwilliam* (*fils* 'son' of William), *MacArthur* 'son of Arthur', *Robertson*, *Jackson*, *Johnson*—*Jonsson* in Swedish, which retains the genitive 's', as does *Siggursdottir* 'Siggur's daughter' in Icelandic. *Benjamin* is Hebrew *ben* 'son of' + *yamin* 'right hand'. *Edward* derives from Anglo Saxon *ead* 'wealth, happiness' and *weard* 'guardian', hence 'wealth protector'. Even *John* (*Johan*, *Ivan*, *Yiannis*, *Sean*, *Juan*) derives from Hebrew *Yehohanan* 'God has favored'. *Jones* is derived from *John's* (*child*). In Irish we find *Barry* from Old Irish *Bairre*, short for *Fionbarr* from *fionn* 'white, fair' and *barr* 'head', *Fergus* Anglicized from Irish *Fearghas* from *fear* 'man' and *gas* 'vigor'. From Hindi—*Kumar* 'boy / son / prince', *Shankar* 'conferer of welfare', *Siddartha* 'one who has accomplished his goal'. From Arabic—*Fahd* 'panther, leopard', *Mubarak* 'blessed, fortunate', *Mahmud* 'praiseworthy'. From Chinese—*Cheng* 'accomplished', *Yu* 'jade', *Zedong* (as in Chairman *Mao*) 'East of the marsh'. Japanese—*Jiro* 'second son', *Masao* 'correct man', *Midori* 'verdant'. (See Hanks et al. 2007.)

18.5 An Objection: Just Jazzy Mathematics?

Is our boolean construal of classical model theory anything more than mathematical gymnastics, sound and fury signifying little? Indeed our boolean construal is simply one, mathematically explicit, way of formulating standard extensional semantics. But it is this format, rather than the usual set-theoretical one, that generalizes to a non-extensional semantics. We just relax the atomicity requirement on P, so there will be non-zero properties which dominate no atom. Explicitly:

STEP 3 A semantic analysis of cnps is extensional if properties p,q are the same whenever they have the same extension. Formally:

(14) *Extensionality*: For all $p,q \in P$, if $I(p) = I(q)$, all individuals I, then $p = q$.

THEOREM 2 Our STEP 2 formulation of P is extensional.
proof: Assume $I(p) = I(q)$, all I. Then for any atom α, $I_\alpha(p) = I_\alpha(q)$, so $\alpha \leq p$ iff $\alpha \leq q$. So p and q dominate the same atoms and for each x in an atomic boolean lattice L, $x = \vee\{\alpha \in Atom(L) | \alpha \leq x\}$, whence $p = \vee\{\alpha \in Atom(P) | \alpha \leq p\} = \vee\{\alpha \in Atom(P) | \alpha \leq q\} = q$. □
(In a power set lattice L, the condition $x = \vee\{\alpha \in Atom(L) | \alpha \leq x\}$ just says that a set x is the union of its unit sets).

Now, to build a non-extensional analysis we relax the atomicity requirement on P as follows:

(15) A model for our fragment of English is a pair (P,{T,F}), with {T,F} as before, and P a complete non-atomic boolean lattice with at least one atom.

But, do such Ps exist? And if so, what do they look like? Can they have as many atoms (individuals) as we like? All finite boolean lattices are isomorphic to power sets and thus atomic. So P must be infinite (even if it has only finitely many atoms). This may seem "technical", but it is linguistically motivated below.

It turns out that there is a minimal atomless boolean lattice—one with a linguistic character whose elements can be thought of as information packets—logical propositions—and whose order relation is "is more informative than". And given any non-zero packet p we can always find another non-zero one q which is strictly more informative than p: $0 < q < p$. So this lattice is atomless.

Consider the Lindenbaum–Tarski lattice LT derived from Sentential Logic (SL). Formulas in SL are built from denumerably many "atomic" formulas P_1, $P_2, \ldots$ by iteratively forming conjunctions, disjunctions, and negations. A *model* for SL is function v mapping the atomic formulas into {T,F} and extended to a function V on all formulas by setting $V(\phi \,\&\, \psi) = V(\phi) \wedge V(\psi)$ and $V(\text{not } \phi) = \neg V(\phi)$. Now let ϕ be an SL formula which has a model, that is, an assignment of values to the atomic formulas which makes ϕ true. Since only finitely many atomic formulas occur in ϕ let $P_k, P_{k+1}, \ldots$ be a denumerable sequence of distinct atomic formulas not occurring in ϕ. Then $(\phi \,\&\, P_k)$ has a model, as does $((\phi \,\&\, P_k) \,\&\, P_{k+1})$, etc. Every later formula has a model and properly entails (entails and is not entailed by) the previous ones. So we have an *infinite descending chain* (sequence) of formulas each of which gives strictly more information about the model than its predecessor, but no formula in the sequence gives complete information, i.e., decides the truth of all the atomic formulas. Each formula leaves the truth of all but finitely many atomic formulas open. So infinite descending chains are comprehensible. They are also prominent in English cnps. Consider the following infinite sequence of cnps:

(16) a. h(1) = doctor;
b. for all n > 0, h(n+1) = doctor who knows a h(n).

So h(2) is *doctor who knows a doctor*, h(3) *doctor who knows a doctor who knows a doctor*, etc. If an individual I has the property expressed by any of these, then I has the property expressed by each of its predecessors in the enumeration.

The entailment relation, however, is not a boolean order relation since, while reflexive and transitive, it is not antisymmetric: there are distinct formulas each of which entails the other. Indeed each formula ϕ is logically equivalent to denumerably many distinct formulas (consider ϕ, $(\phi \,\&\, \tau)$, $((\phi \,\&\, \tau) \,\&\, \tau), \ldots$ τ any tautology). So we trade in each formula ϕ for its logical equivalence class $[\phi]$, the set of formulas whose truth value in any model M is the same as that of ϕ in M. Think of $[\phi]$ as the logical proposition expressed by ϕ. Define a $\leq$ relation on the set of these classes by setting $[\phi] \leq [\psi]$ iff ϕ entails ψ (that is, ψ is true in all models in which ϕ is true). This is well defined and a properly boolean partial order (Bell and Slomson 1971: 40–2). The zero is the class of a contradiction, the unit the class of a tautology, and $[\phi]\wedge[\psi] = [\phi \,\&\, \psi]$, $[\phi]\vee[\psi] = [\phi \text{ or } \psi]$, and $\neg[\phi] = [\text{not } \phi]$.

The sequence $[\phi]$, $[(\phi \,\&\, P_k)]$, $[((\phi \,\&\, P_k) \,\&\, P_{k+1})]$, . . . of equivalence classes of the proper entailment chain above is an infinite descending chain: each later class lies strictly between zero and its predecessor. So in LT each non-zero element heads an infinite descending sequence so LT is atomless, and increasing information still corresponds to proper entailment.

Worth noting here is that LT is, mathematically, unique. First, no atomic formula entails any other one, so the set of equivalence classes of atomic formulas is denumerable. And since SL itself is denumerable, the cardinality of the set of equivalence classes of all formulas in SL is at most denumerable. And it is a general theorem (Givant and Halmos 2009: 135) that all denumerable atomless boolean lattices are isomorphic.

LT does, however, have one technical shortcoming (for our purposes) which is standardly overcome in a technical way. Namely, LT is not complete. But there is a minimal uniform way of adding elements to any boolean lattice L to form a complete one—the Tarski–MacNeille extension—here noted simply L*. To form L*, we only add in lubs for subsets of the original which do not have one. So we do not replace any old lubs and do not add any elements below those in the original. So if A is atomless, so is A*. For details of the construction see Givant and Halmos (2009), theorem 22. So LT* is complete and atomless.

Now the last step to yield a complete non-atomic lattice with k atoms, any cardinal $k > 0$, is easy. Let A be a set of cardinality $k > 0$ (k may be infinite). Then P(A) is a complete atomic boolean lattice with k atoms and the cross-product lattice $P(A)\times LT^*$ is a complete non-atomic boolean lattice with k atoms.

Cross-product lattices $B\times D$ behave as expected: the relation and the boolean operations are run in parallel. E.g., $(b,d) \leq (b',d')$ iff $b \leq_B b'$ and $d \leq_D d'$; $(b,d) \wedge (b',d') = (b\wedge_B b', d\wedge_D d')$, $\neg(b,d) = (\neg_B b,\neg_D d)$, and $0 = (0_B,0_D)$ and $1= (1_B,1_D)$. $\text{Atom}(B\times D) = \{<b,0>|b \in \text{Atom}(B)\} \cup \{<0,d>|d \in \text{Atom}(D)\}$ and $B\times D$ is complete iff both B and D are.

Thus $P(A)\times LT^*$ is complete with $|A|$ many atoms, the pairs $<\alpha,0>$, α an atom of P(A), and 0 the zero element of LT* (determined by the equivalence class of a formula with no models, e.g., a contradiction). And again to finesse charges of merely engaging in mathematical gymnastics we note the theorem and corollary below, where for b a non-zero element of a boolean lattice $\downarrow b$ is $\{x \in B|x \leq b\}$. $\downarrow b$ is itself a boolean lattice in which $\wedge$, $\vee$, and 0 coincide with those in B, the unit element is b and complements are taken relative to b: $\neg_{\downarrow b}x = b\wedge\neg_B x$.

Theorem 3 For B a boolean lattice and $0 < b < 1$, B is isomorphic to $\downarrow b\times\downarrow\neg b$, the map sending each x in B to $<x\wedge b, x\wedge\neg b>$ is an isomorphism.

Corollary 4 For P complete, non-atomic but with at least one atom, $P \simeq \downarrow\vee\text{Atom}(P)\times\downarrow\neg\vee\text{Atom}(P)$, where Atom(P) is the set of atoms of P.

And for $p \in P$, set $\text{ext}(p) = p \wedge \vee \text{Atom}(P)$ and $\text{int}(p) = p \wedge \neg \vee \text{Atom}(P)$. The extensional part of p is ext(p) and the intensional part int(p). We note that $\downarrow \vee \text{Atom}(P)$ is complete and atomic and $\downarrow \neg \vee \text{Atom}(P)$ is complete but atomless. A property q will be called *extensional* if $q \leq \vee \text{Atom}(P)$, that is, $\text{ext}(q) = q$, and *intensional* if $q \leq \neg \vee \text{Atom}(P)$, that is, $\text{int}(q) = q$. The property expressed by *individual who is John* is extensional, that expressed by *barber who shaves just those barbers who do not shave themselves* is intensional.

We take such P as the primitive denotation set for cnps. Individuals, proper name denotations, are defined as before: the complete homomorphisms from P into {T,F}. Extensions are defined as before: the set of individuals that hold of p. But the resulting semantics fails extensionality, (17b):

(17) a. For all $p,q \in P$, p and q have the same extension iff $\text{ext}(p) = \text{ext}(q)$;
b. There are distinct $p,q \in P$ with the same extension: $\text{ext}(p) = \text{ext}(q)$.

To see (17b), suppose α is an atom of P and $q \neq q'$ in $\downarrow \neg \vee \text{Atom}(P)$. Then $\alpha \vee q$ and $\alpha \vee q'$ are distinct properties with the same extension, $\{I_\alpha\}$.

Note that the set of intensions of common nouns has a boolean structure—it is the lattice (algebra) $\downarrow \neg \vee \text{Atom}(P)$. But we have said nothing about what intensions *are*. We share with Muskens (2007) the view that what is important is the structure of the set—the functions and relations it supports—not the identity of its members. And our set of intensions does have a distinctive boolean structure—a complete atomless one. Our semantic interpretation is analogous to interpreting the language of elementary arithmetic in any set that satisfies the Peano axioms, it does not have to be the set of natural numbers set-theoretically construed. For purposes of arithmetical study we need not say what numbers "are". We do suggest further constraints on intensions shortly and our approach does support that:

(18) Boolean compounds of non-extensional adjectives are non-extensional.

The reader can check this with primary judgments for expressions like *both skillful and accomplished*, *competent but neither gifted nor ingenious*, etc.

18.6 Evaluative Adjectives

Now let us see how the non-extensionality of evaluative adjectives is naturally represented in models of type P. As indicated, evaluative adjectives like *skillful* denote restricting functions from P to P. We impose no further constraints. It follows that properties p,q may have the same extension but those of **skillful**(p) and **skillful**(q) may be different. Here is an example type.

Let abc be an extensional property dominating three atoms a, b, and c. Let σ and τ be intensional properties with $\sigma \nleq \tau$ and $\tau \nleq \sigma$, though they may overlap: $\sigma \wedge \tau \neq 0$.

surgeon and **flautist** empirically meet these latter conditions: what one needs to know to know if x is a surgeon is different from what one needs to know to know if x is a flautist. But there is one common requirement: x must be human (in distinction to **freezer** or **screwdriver**). Let **surgeon** = abc$\vee\sigma$ and **flautist** = abc$\vee\tau$. So **surgeon** and **flautist** are distinct properties with the same extension, abc, but incomparable intensions. Now suppose that **skillful(surgeon)** = ac$\vee\sigma'$ and **skillful(flautist)** = bc$\vee\tau'$, with $0 < \sigma' < \sigma$ and $0 < \tau' < \tau$. So ext(**skillful**(**surgeon**)) = ac $\neq$ bc = ext(**skillful**(**flautist**)) and I_c is both a skillful surgeon and skillful flautist. In this model **surgeon** and **flautist** have the same extension but **skillful(surgeon)** and **skillful(flautist)** do not. And the intension σ' of **skillful(surgeon)** $\leq$ the intension σ of **surgeon**, and similarly for **skillful(flautist)**, so the intensional inclusion relation corresponds to "requires more information than".

Our model also accounts for a slightly subtle property of restricting adjectives. For every restricting map F from P to P, F $\leq$ **id**, the identity map sending each p to itself. This means that the set of restricting maps is $\downarrow$**id**, so negations of adjectives are just interpreted relative to **id**. Thus *Several not very skillful surgeons work here* does not mean merely that several people who are not very skillful surgeons work here, it means more specifically that several surgeons who are not very skillful ones work here.

One incompleteness in our treatment of restricting adjectives is the absence of a precise representation of what skillful surgeons and skillful flautists have in common. Informally such individuals must be "good at what they do". But my judgment is that **being good at** is not cognitively or logically reducible to more primitive notions.

Unreducibility is supported experimentally by Osgood (1952) and Osgood et al. (1957), who did three studies designed to "measure" the meanings of adjectives. Using a set of 50 scales determined by antonymous adjective pairs, such as *good–bad*, *large–small*, . . ., *strong–weak*, *happy–sad*, *sharp–dull*, . . ., *active–passive*, *wide–narrow*, each scale was presented with 7 equal divisions. In one test, subjects were asked to locate other adjectives along these scales. For example, *eager* and *burning* received similar placement on the strong–weak and active–passive scales, but different placement along the hot–cold and dry–wet scales. In another test, subjects ranked each of 20 concepts expressed by nouns—*boulder*, *sin*, *father*, *statue*, *cop*, *America*, . . . —on each scale. The results of each test were subjected to factor analysis to determine which scales gave similar results and were relatively independent of other scales.

All three tests yielded the same three main factors (correlation groups). The largest was one they call evaluative, grouping *good–bad*, *beautiful–ugly*, *sweet–sour*, *tasty–distasteful*, *clean–dirty*, *happy–sad*, *fair–unfair*, among others. The other two were potency and activity, each with about half as many scales as the evaluative one. Potency included *large–small*, *strong–weak*, *heavy–light*, and *thick–thin*. Activity included *fast–slow*, *active–passive*, and *hot–cold*. So on these experiments *evaluativity* was the primary unreducible factor.

There is one case of restrictive adjectives where we can be more explicit about their intensional contribution, those called *intersective* or *absolute* (see Montague (1970), Parsons (1970), Kamp (1975), and Keenan and Faltz (1985)). Examples are *male*, *female*, *blue-eyed*, *six foot tall*. A blue-eyed doctor is a doctor who is a blue-eyed individual, where *individual* denotes 1_P, the maximal property that all individuals have. We define a map F from P into P to be *absolute* iff for all $p \in P$, $F(p) = p \wedge F(1_P)$. Then

Fact For F absolute, $p \in P$, $int(F(p)) = int(p \wedge F(1)) = p \wedge F(1) \wedge \neg\vee Atom(P) = (p \wedge \neg\vee Atom(P)) \wedge (F(1) \wedge \neg\vee Atom(P)) = int(p) \wedge int(F(1))$.

Thus the intension of *blue-eyed doctor* is that of *doctor* plus that of *blue-eyed individual*. We note also, without argument, that on our semantics:

(19) For F,G absolute, $F(G(p)) = G(F(p)) = (F \wedge G)(p)$, all $p \in P$. But if F and G are merely restricting, all three of these properties may be different.

Thus in some models *small* (*expensive house*), *expensive* (*small house*), and (*small and expensive*) *house* all have different extensions (Keenan and Faltz 1985). But in all models *blue-eyed six-foot-tall doctor* and *six-foot-tall blue-eyed doctor* have the same extension—they denote the same individuals.

18.7 Value Judgment Quantifiers

Some generalized quantifiers, functions which, like adjectives, take properties as arguments, have an evaluative component and are also non-extensional. Individuals and their boolean compounds remain extensional on our semantics: If John, or every student, is a vegetarian and the vegetarians and the flautists are the same individuals, then John, or every student, is a flautist. But there are value judgment quantifiers and they are, unsurprisingly, not extensional. In a situation in which the doctors and the lawyers are the same, (20a) and (20b) may both be true (Keenan and Stavi 1986):

(20) a. Not enough doctors came to the meeting;
b. More than enough (even too many) lawyers came to the meeting.

Imagine a meeting of the local medical association whose bylaws require 100 doctors for a quorum, but just one lawyer, to take the minutes. Only 50 doctor–lawyers show up—not enough doctors, more than enough lawyers. Other value judgment quantifiers are *too many*, *too few*, *surprisingly many*, *disappointingly few*, and in my opinion, simply *many* and *few*.

So intensional quantifiers are a natural follow-up to this study. But even with adjectives, we have only scratched the surface.

Clark (1970) notes the difference between type-building modifiers and mere attributive adjectives: *beef stew* vs. *red balloon*. Rotstein and Winter (2004) note a distinction between total *vs.* partial adjectives, as in *somewhat dirty*, which is natural, *vs. somewhat clean*, which is not. Rett (2007) notes that some antonymous scalar adjectives may differ in evaluativity in the same construction: *x is as tall as y* is noncommittal regarding whether x and y are tall, but *x is as short as y* implies that both x and y are short. Burnett (2012) distinguishes between restricting adjectives like *tall* vs. *empty*, where *tall* can adjust its "threshold" and *empty* cannot. Given two glasses, one noticeably taller than the other but neither particularly tall, I might naturally ask you to hand me the tall one (even though if more glasses are added to the sample I might no longer call my original choice a tall one). In contrast, given two glasses of the same size, one half full with water, the other only one-quarter full, I cannot ask you to hand me the empty one.

18.8 Further Conditions Constraining Intensions

I suggest three further conditions we might plausibly impose on intensional models. The first is a local condition, one that concerns properties of intensions in each model. The second two are global, ones involving cross-model comparison of intensions.

1. *Intensions determine extensions*. In each model, if cnps d and d′ have the same intension, they have the same extension. This property follows on the formally explicit analysis of Moschovakis (2006), where intensions are extension-computing algorithms. But it does not follow on our simple boolean approach.

2. *Intensions are constant*. We think naturally of the extension of cnps like *doctor* as varying from one situation to another. But, naïvely, what we need to know to know if someone is a doctor does not seem to so vary. So let us, tentatively, require that for an agentive cnp d, the intension of the denotation of d is the same in all models (contexts). Its extension of course may vary. Taking P to be $P(A) \times LT^*$ where LT^* is fixed and only A can vary suffices. (We might experiment using LT^* built from languages other than Sentential Logic.) Note too that our other primitive, {T,F}, does not vary across models.

3. *Distinct underived cnps have different intensions* unless stipulated otherwise. In general it is difficult to find true synonyms among underived common nouns (Carstairs-McCarthy 1998).[8] Examples of synonyms in the literature are pairs like *oculist/eye-doctor, bachelor/unmarried man*. But in each of these pairs at least one of the terms is morpho-syntactically complex. Candidates for complex cnps with the same intension are ones formed from absolute adjectives: *female surgeon* and *surgeon who is female*. Also *blue-eyed six-foot-tall surgeon* and *six-foot-tall blue-eyed surgeon*. A richer type of candidate would be *famous author and critic* and *famous critic and author*. Also reasonable are pairs like *heart surgeon who is a portrait painter* and *portrait painter who is a heart surgeon*.

[8] One curious exception to this claim are scatological terms. Doubtless we can all think of several four-letter synonyms for male and female sex organs.

18.8 Conclusion

We have constructed a linguistically revealing semantics of (a fragment of) natural language which dispenses with a universe in favor of a booleanly structured set of properties. So we know it is possible. And our model generalizes to one in which we represent non-trivial entailments (*Ann is a skillful surgeon* ⊨ *Ann is a surgeon*) with properly intensional modifiers such as *skillful*.[9]

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