Universals across languages

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0 Introduction

One motivation for model theoretic approaches to syntax is the prospect of enabling us to "abstract fully away from the details of the grammar mechanism – to express syntactic theories purely in terms of the properties of the class of structures they license" (Rogers, 1996). This is a worthy goal: in order to see the significant relations among expressions and their parts more clearly, and to describe similarities among different structures and different languages, we would like to discard those aspects of generative, derivational history which appear just because of our decision to use some particular generative device to specify it. If this is our goal, then although it is known that the derivation trees (or derived trees, or other closely related sets of structures) of various generative formalisms can be defined model-theoretically (Büchi, 1960; Thatcher and Wright, 1968; Doner, 1970; Thomas, 1997), that is not generally what we want. We want something more abstract; we want structures that "abstract fully away from . . . the grammar mechanism." What are those structures? This paper takes some first, standard steps towards an algebraic, group-theoretic perspective on this question.

A generative grammar can be given by a lexicon *Lex* and some generating functions $\mathcal{F}$, defining the language L which is the closure of *Lex* with respect to $\mathcal{F}$. The structure building functions of most grammars are *partial*, that is, they apply to some but not other expressions, and typically the domains of the functions are picked out by "syntactic categories" and "syntactic features." This partiality is a very important part of grammar!

Since the structure building rules in $\mathcal{F}$ define the structure of the language, we **set the stage** for our analysis by requiring the grammars to be "balanced" in a sense defined below, with rules $\mathcal{F}$ that are neither too specific nor too general. (Few of the grammars popular in mainstream syntax are balanced in this sense, but balanced formulations can be defined.) Then, in a **first** step towards a suitably abstract perspective, define the *structural* elements of a language (lexical items, properties, relations) to be those that are fixed by every automorphism of $(L, \mathcal{F})$. Two expressions then have the "same structure" if some automorphism maps one to the other. The automorphisms of course form a group with respect to composition, and so we have an instance of the familiar framework for the study of symmetries (Klein, 1893). This perspective stands well away from particular grammars with which we started, in a number of senses that we briefly explore. Although it conforms at many points with linguists' intuitions about structure, a derivation tree of a particular grammar, if interpreted in the traditional linguistic fashion, can actually be misleading about the "structure" the grammar defines, in our sense.

The automorphisms Aut_G of each grammar G are still very sensitive to small changes in the language though. In order to compare similar but non-identical grammars, we take a **second** step, again using standard concepts, finding homomorphisms that relate structural polynomials of the languages. Then we achieve a perspective in which we can recognize different languages, with different signatures, as related by homomorphisms that preserve certain 'minimal' or 'core' structures of predication and modification, even when they are realized in slightly different ways. This allows a precise formulation of some of the basic common properties that linguists notice in grammars of diverse languages.

1 Grammars and structure

For $Lex \subseteq \mathrm{E}$ and $\mathcal{F} = \langle f_1, f_2, \ldots \rangle$ a sequence of *partial* functions $f_i : \mathrm{E}^n \rightarrow \mathrm{E}$, we regard each $f_i : \mathrm{E}^n \rightarrow \mathrm{E}$ as a set of $n+1$-tuples, as usual. Let $[Lex]_{\mathcal{F}}$ represent the closure of Lex with respect to the functions in $\mathcal{F}$. Then we can regard a grammar $G = (Lex_G, \mathcal{F}_G)$ as defining the language $[Lex_G]_{\mathcal{F}}$ with structure $\mathcal{F}_G$. (When no confusion will result, we sometimes leave off subscripts.)

For example, consider Span=$(Lex, \mathcal{F})$ defined as follows (Keenan and Stabler, 2003, §4.2). Let Σ={every, some, very, gentle, intelligent, -a, -o, man, doctor, woman, obstetrician}, Cat={D, Dm, Df, Nm, Nf, M, A, Am, Af, Agrm, Agrf, NPm, NPf}, and $\mathrm{E} = \Sigma^* \times Cat$ as usual. Then let the lexicon $Lex \subseteq E$ be the following set of 12 elements

$$\begin{array}{lll} Lex = \{ & \langle \text{some}, \text{D} \rangle, & \langle \text{every}, \text{D} \rangle, \\ & \langle \text{very}, \text{M} \rangle, & \langle \text{moderately}, \text{M} \rangle, \\ & \langle \text{intelligent}, \text{A} \rangle, & \langle \text{gentle}, \text{A} \rangle, \\ & \langle \text{-o}, \text{Agrm} \rangle, & \langle \text{-a}, \text{Agrf} \rangle, \\ & \langle \text{man}, \text{Nm} \rangle, & \langle \text{doctor}, \text{Nm} \rangle, \\ & \langle \text{woman}, \text{Nf} \rangle, & \langle \text{obstetrician}, \text{Nf} \rangle \ \}. \end{array}$$

We let $\mathcal{F} = \langle g, m \rangle$, where g gender-marks determiners D and adjectives A as follows, for any $s, t \in \Sigma^*$, writing st for their concatenation:

$$\begin{array}{l} \langle \langle \text{s}, \text{A} \rangle, \langle \text{t}, \text{Agrm} \rangle \rangle \mapsto \langle \text{st}, \text{Am} \rangle \\ \langle \langle \text{s}, \text{A} \rangle, \langle \text{t}, \text{Agrf} \rangle \rangle \mapsto \langle \text{st}, \text{Af} \rangle \\ \langle \langle \text{s}, \text{D} \rangle, \langle \text{t}, \text{Agrm} \rangle \rangle \mapsto \langle \text{st}, \text{Dm} \rangle \\ \langle \langle \text{s}, \text{D} \rangle, \langle \text{t}, \text{Agrf} \rangle \rangle \mapsto \langle \text{st}, \text{Df} \rangle, \end{array}$$

and then phrases are merged together by m as follows,

$$\begin{array}{l} \langle \langle \text{s}, \text{M} \rangle, \langle \text{t}, \text{Am} \rangle \rangle \mapsto \langle \text{st}, \text{Am} \rangle \\ \langle \langle \text{s}, \text{M} \rangle, \langle \text{t}, \text{Af} \rangle \rangle \mapsto \langle \text{st}, \text{Af} \rangle. \\ \langle \langle \text{s}, \text{Am} \rangle, \langle \text{t}, \text{Nm} \rangle \rangle \mapsto \langle \text{st}, \text{Nm} \rangle \\ \langle \langle \text{s}, \text{Af} \rangle, \langle \text{t}, \text{Nf} \rangle \rangle \mapsto \langle \text{st}, \text{Nf} \rangle \\ \langle \langle \text{s}, \text{Dm} \rangle, \langle \text{t}, \text{Nm} \rangle \rangle \mapsto \langle \text{st}, \text{NPm} \rangle \\ \langle \langle \text{s}, \text{Df} \rangle, \langle \text{t}, \text{Nf} \rangle \rangle \mapsto \langle \text{st}, \text{NPf} \rangle \end{array}$$

Lifting any function on E to apply coordinatewise to tuples in E^*, and then pointwise to sets of expressions or tuples of expressions, an *automorphism* $h : [Lex] \rightarrow [Lex]$ of $([Lex], \mathcal{F})$ is a bijection such that for every $f \in \mathcal{F}$, $h(f \restriction [Lex]) = f \restriction [Lex]$.

For x an expression, a tuple of expressions, a set of expressions, or set of tuples, we say x is *structural* iff x is fixed by every automorphism. And x *has the same structure as* y iff there is some automorphism h such that $h(x) = y$.

For any $\mathrm{E} = \Sigma \times Cat$, $Lex \subseteq \mathrm{E}$ and partial functions $\mathcal{F}$, consider the grammar $G = (Lex, \mathcal{F})$. For any $\mathrm{C} \in Cat$ let the phrases of category C

$$\mathrm{PH(C)} = \{ \langle \text{s}, \text{D} \rangle \in [Lex] | \ D = C \}.$$

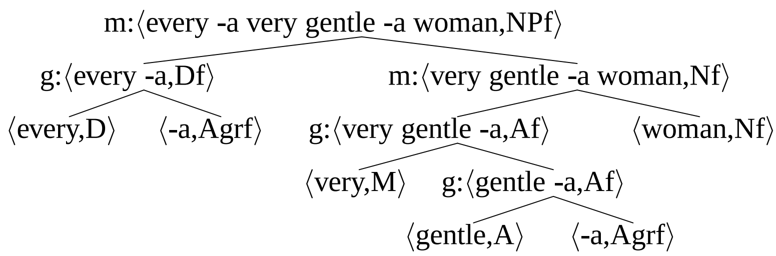


Figure 1: Span derivation of an NPf

Keenan and Stabler (2003) show that with the grammar Span,

- *Lex* is structural, as are $\mathrm{PH}(A), \mathrm{PH}(D), \mathrm{PH}(M)$.

- There is an automorphism that exchanges $\langle \text{every}, \text{D} \rangle$ and $\langle \text{some}, \text{D} \rangle$, exchanging all occurrences of the vocabulary elements *every* and *some* in the strings of each expression but leaving everything else unchanged. The existence of this automorphism establishes that $\langle \text{every -a very gentle -a woman}, \text{NPf} \rangle$ and $\langle \text{some -a very gentle -a woman}, \text{NPf} \rangle$ have the same structure.

- There are other automorphisms that exchange the masculine and feminine phrases. For example, let's define the string homorphism $\cdot^{mf}$ that exchanges these substrings:

  $$\text{-a} \leftrightarrow \text{-o}$$
  $$\text{man} \leftrightarrow \text{woman}$$
  $$\text{doctor} \leftrightarrow \text{obstetrician}$$

  And then let's extend that mapping to exchange the following categories:

  $$\text{Agrm} \leftrightarrow \text{Agrf}$$
  $$\text{Nm} \leftrightarrow \text{Nf}$$
  $$\text{Dm} \leftrightarrow \text{Df}$$
  $$\text{Am} \leftrightarrow \text{Af}$$
  $$\text{NPm} \leftrightarrow \text{NPf}$$

  Then define the total function $h : [Lex] \to [Lex]$ as follows:

  $$h(s, C) = (s^{mf}, C^{mf}).$$

  This function is an automorphism of Span (Keenan and Stabler, 2003, p.143).

  So $\langle \text{every -a very gentle -a woman}, \text{NPf} \rangle$ and $\langle \text{every -o very gentle -o man}, \text{NPm} \rangle$ have the same structure.

For any $G = (Lex, \mathcal{F})$, let Aut_G be the set of automorphisms of G. Clearly, $\langle \mathrm{Aut}_G, \circ \rangle$ is a group, since Aut_G includes the identity on $[Lex]$ which is also the identity with respect to composition of automorphisms, and for any automorphism, its inverse is also an automorphism (Grätzer, 1968; Plotkin, 1972).

It will be convenient to introduce some 'auxiliary' functions. An n-ary *projection function* is a total function $\varepsilon_i^n : \mathrm{E}^n \to \mathrm{E}$, for $0 < i \leq n$, defined by

$$\varepsilon_i^n(x_1, \ldots, x_i, \ldots, x_n) = x_i.$$

The set $poly(G)$ of *polynomials over* $G = (A, \mathcal{F})$ is the smallest set containing the projection functions and such that if $p_1, \ldots, p_n$ are n-ary (partial) polynomials, and m-ary (partial) $f \in \mathcal{F}$, then $f(p_1, \ldots, p_m)$

is an n-ary (partial) polynomial, whose domain is the set of $s \in \mathrm{E}^n$ such that, for $0 < i \leq m$,

$$s \in \mathrm{dom}(p_i) \text{ and } \langle p_1(s), \ldots, p_m(s) \rangle \in \mathrm{dom}(f),$$

and where the values of the polynomial are given by

$$f(p_1, \ldots, p_m)(s) = f(p_1(s), \ldots, p_m(s)).$$

So for example, the expression $\langle$every -a very gentle -a woman$, \mathrm{NPf}\rangle$, derived in Figure 1, is the value of the 6-ary polynomial

$$m(g(\varepsilon_1^6, \varepsilon_2^6), m(m(\varepsilon_3^6, g(\varepsilon_4^6, \varepsilon_5^6)), \varepsilon_6^6))$$

applied to this element of Lex^6: $\langle\langle$every$, \mathrm{D}\rangle$, $\langle$-a$, \mathrm{Agrf}\rangle$, $\langle$very$, \mathrm{M}\rangle$, $\langle$gentle$, \mathrm{A}\rangle$, $\langle$-a$, \mathrm{Agrf}\rangle$, $\langle$woman$, \mathrm{Nf}\rangle\rangle$. Putting the arguments in alphabetical order and eliminating redundancies, we can get the same value with this polynomial

$$m(g(\varepsilon_2^5, \varepsilon_1^5), m(m(\varepsilon_4^5, g(\varepsilon_3^5, \varepsilon_1^5)), \varepsilon_5^5))$$

applied to this element of Lex^5: $\langle\langle$-a$, \mathrm{Agrf}\rangle$, $\langle$every$, \mathrm{D}\rangle$, $\langle$gentle$, \mathrm{A}\rangle$, $\langle$very$, \mathrm{M}\rangle$, $\langle$woman$, \mathrm{Nf}\rangle\rangle$.

To these standard polynomials, we add *incorporations*, which are defined as follows. When there is an n-ary polynomial $f(p_1, \ldots, p_m)$ where every n-tuple in its domain has a unique, common i'th element $e \in [Lex]$, it follows that this element is *structural* in the sense defined above. In that case, the $n-1$-ary polynomial expressed by the term that results from replacing all occurrences of ε_i^n by e in $f(p_1, \ldots, p_m)$ is also a polynomial, taking the same value on $n-1$-tuples as the original polynomial did when it had e in the i'th coordinate.

Each polynomial is represented by a term. Let's say that elements of $\mathcal{F}$ and the projection functions by themselves have *term depth* 0. And for any polynomial term $f(p_1, \ldots, p_n)$, let it's *term depth* be 1 more than the maximum depth of terms $p_1, \ldots, p_n$. Let the *depth* of any polynomial p be the minimum term depth of polynomials defining the function p.

Given any grammar $(Lex, \mathcal{F})$, it is clear that $(Lex, poly(G))$ has the same automorphisms. The addition of the polynomials does not change structure, even though it gives every expression a 1-step derivation (Keenan and Stabler, 2003, p.58).

We see in this setting that the mere fact that two expressions have structurally different derivations does not show that they have different structures. One and the same expression can have infinitely many derivations. Even two expressions with isomorphic derivations with the same categories, differing only in their strings, can differ in structure if the generating functions can be sensitive to the strings.

2 Balanced grammars

In Span, the categories serve to pick out the domains of the structure building functions. Let's say that $G = (Lex, \mathcal{F})$ is *category closed* iff for any $s_1, \ldots, s_n, t_1, \ldots, t_n \in [Lex]$ and for $0 < i \leq n$, if s_i and t_i have the same categories, then for all $f \in \mathcal{F}$

$$\langle s_1, \ldots, s_n \rangle \in \mathrm{dom}(f) \text{ iff } \langle t_1, \ldots, t_n \rangle \in \mathrm{dom}(f).$$

Let's say that G is *category functional* iff for all $f \in \mathcal{F}$ and for any $s_1, \ldots, s_n, t_1, \ldots, t_n \in \mathrm{dom}(f)$, if, for $0 < i \leq n$, s_i and t_i have the same categories, then $f(s_1, \ldots, s_n)$ and $f(t_1, \ldots, t_n)$ have the same category.

Span is category closed and category functional. We will restrict attention to grammars with these properties in the sequel except when explicitly indicated. Imposing these conditions requires that syntactic categories be explicit in a sense, reflecting all properties relevant to the application of structure building functions.

It will also be useful to require that our grammars make their operations appropriately explicit in the signature, in a sense we now define. For any partial functions $\mathcal{F}$, let $explode(\mathcal{F}) = \{\{\langle a,b\rangle\}| \ f_i(a) = b$ for some $f_i \in \mathcal{F}\}$. And for any $G = (A,\mathcal{F})$, let $explode(G) = (A, explode(F))$. (The order of the functions in $explode(\mathcal{F})$ will not matter for present purposes.) Then for any grammar G, the grammar $explode(G)$ defines the same language, but will often have fewer automorphisms. In $explode(G)$, every expression that is in the domain or range of any function is structural. So the only non-trivial automorphisms, if any, are those that exchange lexical items not in the domain or range of any function.

The grammar explode(Span) has infinitely many generating functions, and is "unbalanced" in the sense that there are regularities in m and g that we see in the automorphisms of Span, but not in automorphisms of explode(Span).

Let's say functions f,g are *compatible* iff they agree on any elements common to both of their domains; so functions with disjoint domains are always compatible. Since the functions g and m of Span are compatible, consider the grammar collapse(Span)=(Lex,$\langle g \cup m\rangle$) with a single generating function. This grammar is "unbalanced" too, in the sense that while collapse(Span) and Span have the same automorphisms, taking the union of g and m does not reveal anything new.

Let's say that a grammar $G = (A,\mathcal{F})$ is *balanced* iff both

- there are no two distinct, compatible, nonempty functions $f_i, f_j \in \mathcal{F}$ such that removing f_i, f_j and adding $f_i \cup f_j$ strictly increases the set of automorphisms, and
- there are no two distinct, compatible, nonempty functions $g, g' \notin \mathcal{F}$ such that $g \cup g' = f_i$ for some $f_i \in \mathcal{F}$, where the result of adding g and g' to $\mathcal{F}$ yields a grammar with the same automorphisms as G has.

Balance matters. As noted above, it affects the automorphisms. And it affects grammar type, the signature. In the present context, balance matters because the elements of $\mathcal{F}$ determine the available structural polynomials that are useful in comparing grammars, as explained below.

In addition to the noun phrase grammar Span above, Keenan and Stabler (2003) define a "little English" Eng (p15), a "little Korean" case marking language Kor (p47), a "free word order" case marking language FWK (p54), a little verb-marking language Toba (p67), and a classical categorial grammar CG1 (p105).

Theorem 1 *None of the grammars Span, Eng, Kor, FWK, Toba, or CG1 are balanced.*

Proof: It suffices to show that in each grammar, there is a function $f \in \mathcal{F}$ that can be replaced by distinct nonempty $g1, g2$ such that $f = g1 \cup g2$, without changing the automorphisms. For Span, let $g1 = g \cap (\mathrm{PH}(D) \times \mathrm{E} \times \mathrm{E})$ and $g2 = g \cap (\mathrm{PH}(A) \times \mathrm{E} \times \mathrm{E})$. Then $g1, g2$ are compatible, $g = g1 \cup g2$, and since $\mathrm{PH}(D)$ and $\mathrm{PH}(A)$ are already structural in Span, the automorphisms of Span are unchanged by the addition of g by $g1$ and $g2$. The other grammars mentioned above have similarly over-unified structure-building functions. $\square$

Define the grammar bal(Span) with $\Sigma, \mathrm{Cat}, Lex$ unchanged from Span, but where $\mathcal{F} = \{f_i \cap S| \ f_i \in \mathcal{F}_{Span}$, and $S \in \mathrm{PH}_{Span}$ is an invariant of $Span\}$. Then $\mathcal{F}$ includes, for example, a function $g1$ that gender-marks only determiners D, as follows,

$$\langle\langle \mathrm{s,D}\rangle, \langle \mathrm{t,Agrm}\rangle\rangle \mapsto \langle \mathrm{st,Dm}\rangle$$
$$\langle\langle \mathrm{s,D}\rangle, \langle \mathrm{t,Agrf}\rangle\rangle \mapsto \langle \mathrm{st,Df}\rangle.$$

$\mathcal{F}$ includes a function $g2$ that gender-marks adjectives A:

$$\langle\langle \mathrm{s,A}\rangle, \langle \mathrm{t,Agrm}\rangle\rangle \mapsto \langle \mathrm{st,Am}\rangle$$
$$\langle\langle \mathrm{s,A}\rangle, \langle \mathrm{t,Agrf}\rangle\rangle \mapsto \langle \mathrm{st,Af}\rangle.$$

$\mathcal{F}$ includes a function $m1$ that produces complex Am,Af:

$$\langle\langle \text{s}, \text{M}\rangle, \langle \text{t}, \text{Am}\rangle\rangle \mapsto \langle \text{st}, \text{Am}\rangle$$
$$\langle\langle \text{s}, \text{M}\rangle, \langle \text{t}, \text{Af}\rangle\rangle \mapsto \langle \text{st}, \text{Af}\rangle.$$

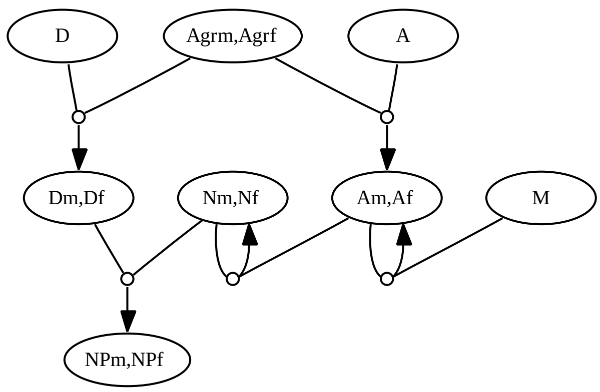
$\mathcal{F}$ includes a function $m2$ that produces complex Nm,Nf:

$$\langle\langle \text{s}, \text{Am}\rangle, \langle \text{t}, \text{Nm}\rangle\rangle \mapsto \langle \text{st}, \text{Nm}\rangle$$
$$\langle\langle \text{s}, \text{Af}\rangle, \langle \text{t}, \text{Nf}\rangle\rangle \mapsto \langle \text{st}, \text{Nf}\rangle.$$

And $\mathcal{F}$ includes a function $m3$ producing noun phrases:

$$\langle\langle \text{s}, \text{Dm}\rangle, \langle \text{t}, \text{Nm}\rangle\rangle \mapsto \langle \text{st}, \text{NPm}\rangle$$
$$\langle\langle \text{s}, \text{Df}\rangle, \langle \text{t}, \text{Nf}\rangle\rangle \mapsto \langle \text{st}, \text{NPf}\rangle.$$

The grammar bal(Span) is obviously still category closed and category functional, and we conjecture that it is balanced in the sense defined above. We can diagram the relations between the functions of bal(Span), showing the functions as small circles between ovals that include their domains and ranges:



Note that this kind of graph does not provide full information about the indicated functions. It does not show for example how the determiner and noun genders must match, so that gender in effect splits the nominal system into two similar systems.

It is easy to see that combinatory categorial grammars (Steedman, 1989) are unbalanced in the way Span is. Standard tree adjoining grammars (TAGs) (Joshi and Schabes, 1997) with $\mathcal{F} = \langle \text{substitution, adjunction}\rangle$ are unbalanced too, as are minimalist grammars (Chomsky, 1995) with $\mathcal{F} = \langle \text{merge, move}\rangle$ and variants. These grammars can usually be converted into "balanced" forms by adding, for each generating function f, the set of functions obtained by restricting f to each of the structural subsets of its domain. This makes structural distinctions more explicit in $\mathcal{F}$, and thereby increases the options for building the polynomials which we will exploit in the next section.

3 Comparing grammars

We have seen that grammars of the sort defined in §1 are partial algebras that define groups of automorphisms. We introduce some standard notions for comparing different languages. Following (Grätzer, 1968, ch.2), we define three different notions of homomorphism for our partial algebras. Function $h : A \to B$ is a *homomorphism* from $(A, \langle f_1, \ldots\rangle)$ to $(B, \langle g_1, \ldots\rangle)$ iff for $0 < i$, both

1. whenever $\langle s_1, \ldots, s_n\rangle \in \text{dom}(f_i)$, $\langle h(s_1), \ldots, h(s_n)\rangle \in \text{dom}(g_i)$, and
2. $h(f_i(s_1, \ldots, s_n)) = g_i(h(s_1), \ldots, h(s_n))$.

A homomorphism is *full* iff for $0 < i$ and for all $s_1, \ldots, s_n, s \in A$,

a. $\langle h(s_1), \ldots, h(s_n)\rangle \in \text{dom}(g_i)$ and

b. $g_i(h(s_1), \ldots, h(s_n)) = h(s)$

imply that there are $t_1, \ldots, t_n, t \in A$ such that

c. $h(s_1) = h(t_1), \ldots, h(s_n) = h(t_n), h(s) = h(t)$, and

d. $\langle t_1, \ldots, t_n\rangle \in \text{dom}(f_i)$, $f_i(t_1, \ldots, t_n) = t$.

And a homomorphism is *strong* iff for $0 < i$,

$$\langle s_1, \dots, s_n \rangle \in \text{dom}(f_i) \text{ iff } \langle h(s_1), \dots, h(s_n) \rangle \in \text{dom}(g_i).$$

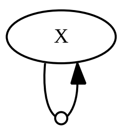
To compare grammars with different types, different signatures, we find polynomials that can be homomorphically related. Let's say that there is a (full, strong) *polynomial homomorphism* of $(A, \mathcal{F})$ into $(B, \mathcal{G})$ iff there are polynomials $P_1, P_2, \dots,$ over $(A, \mathcal{F})$ such that there is a (full, strong) homomorphism from $(A, \langle P_1, P_2, \dots \rangle)$ to $(B, \mathcal{G})$.

Let's define a minimal recursive language $\mathcal{R} = (Lex, \mathcal{F})$ as follows.

$$\Sigma=\{\text{a,b,w}\}, \text{ and}$$
$$\text{Cat}=\{\text{X,W}\},$$

$$Lex = \{\ \langle \text{a}, \text{X} \rangle,\ \langle \text{b}, \text{X} \rangle,\ \langle \text{w}, \text{W} \rangle\ \}$$

and $\mathcal{F} = \langle m \rangle$, where m is the identity function on PH(X). Keenan and Stabler (2003, p165) propose that grammatical constants often play a special role in the grammar – these include many 'grammatical morphemes' etc. The grammar $\mathcal{R}$ includes two elements to indicate that the recursion involves a category that includes non-constant elements. And we include $\langle \text{w}, \text{W} \rangle$ to indicate there can be elements that do not participate in the recursion. $\mathcal{R}$ has the following diagram:



And let's define a minimal "one step" language $\mathcal{O} = (Lex, \mathcal{F})$ as follows.

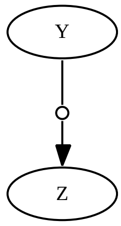
$$\Sigma=\{\text{c,d,e,f}\}, \text{ and}$$
$$\text{Cat}=\{\text{Y,Z}\},$$

$$Lex = \{\ \langle \text{c}, \text{Y} \rangle,\ \langle \text{d}, \text{Y} \rangle\ \},$$

and $\mathcal{F} = \langle n \rangle$, where, for any $s, t \in \Sigma^*$, n maps expressions as follows:

$$\langle \text{c}, \text{Y} \rangle \mapsto \langle \text{e}, \text{Z} \rangle$$
$$\langle \text{d}, \text{Y} \rangle \mapsto \langle \text{f}, \text{Z} \rangle.$$

We can diagram $\mathcal{O}$:



Theorem 2 *There is a full polynomial homomorphism from $\mathcal{O}$ to $\mathcal{R}$, but no strong one.*

Proof: Consider the function $h : [Lex_{\mathcal{O}}] \to [Lex_{\mathcal{R}}]$ given by the following mappings:

$$\langle c, Y \rangle \mapsto \langle a, X \rangle \qquad \langle d, Y \rangle \mapsto \langle b, X \rangle$$
$$\langle e, Z \rangle \mapsto \langle a, X \rangle \qquad \langle f, Z \rangle \mapsto \langle b, X \rangle.$$

This is a homomorphism from $([Lex_{\mathcal{O}}], \langle n \rangle)$ to $([Lex_{\mathcal{R}}], \langle m \rangle)$ since whenever $s \in \text{dom}(n)$, $h(s) \in \text{dom}(m)$, and $h(n(s)) = m(h(s))$. This homomorphism is *full*, since whenever $h(s) \in \text{dom}(m)$ and $m(h(s)) = s'$, there are $t, t' \in [Lex_{\mathcal{O}}]$ such that $h(s) = h(t), h(s') = h(t')$, and $t \in \text{dom}(n)$, $n(t) = t'$. For example, $h(e, Z) = \langle a, X \rangle \in \text{dom}(m)$ and although $\langle e, Z \rangle \notin \text{dom}(n)$ there are elements $\langle c, Y \rangle, \langle e, Z \rangle \in [Lex]_{\mathcal{O}}$ such that $h(e, Z) = h(c, Y), h(e, Z) = h(e, Z)$, such that $\langle c, Y \rangle \in \text{dom}(n)$ with $n(c, Y) = \langle e, Z \rangle$. However, homomorphism h is not *strong* since it is not the case that

$$s \in \text{dom}(n) \text{ iff } h(s) \in \text{dom}(m).$$

In particular, $h(e, Z) \in \text{dom}(m)$ but $\langle e, Z \rangle \notin \text{dom}(n)$. Not only is h not a strong polynomial homomorphism from $\mathcal{O}$ to $\mathcal{R}$, but it is easy to see that no such thing exists, since in $\mathcal{R}$, everything is in the range of m is also in its domain, while in $\mathcal{O}$, n maps elements from its domain to things outside that domain. □

4 Predication and modification

Human languages differ in their most basic constituent order, and in their argument and agreement marking properties, as for example Keenan and Stabler (2003) illustrate with tiny fragments of Spanish, English, Korean, free word order languages, and Toba Batak. In these languages and, we claim, human languages, certain semantically identified relations are structural. But one puzzle left unanswered in that work was: How can we describe the significant syntactic similarities among languages as different as these, in a clear and illuminating way? We might like to say, for example, that human languages all have transitive and intransitive predication; all languages have modification of both arguments and predicates, and so on.

One surprisingly popular idea – see for example (Chomsky, 1965, p.209n) and (Chomsky, 1976, p.56) – is that the reason that these similarities are not clear is that we are considering descriptions of languages that are not 'deep' enough. With deeper descriptions of each language, perhaps a 'universal core' would reveal itself in every grammar. But we do not need to assume that in order to find common properties of genuinely different languages. This perspective explains the slightly unusual title of this paper; rather than assuming that there is a particular universal grammar explicit in the the core of every language (together with some extraneous switch-settings and peripheral material), we will take the much weaker and more natural position that although languages genuinely differ, they can have significant algebraic similarities. We have set up some tools to make such ideas precise.

Let's define a minimal predicative language $\mathcal{P} = (Lex, \mathcal{F})$ as follows:

$$\Sigma=\{a,b,p,q,r,s,w\},$$
$$\text{Cat}=\{P0,P1,P2,W\},$$

$$Lex = \{ \langle a, D\rangle, \langle b, D\rangle, \langle p, P1\rangle, \langle q, P1\rangle, \langle r, P2\rangle, \langle s, P2\rangle, \langle w, W\rangle \},$$

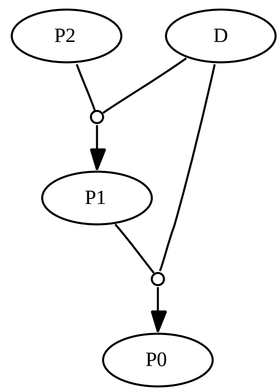
and $\mathcal{F} = \langle m1, m2\rangle$, where $m1$ saturates unary 'predicates' as follows, for any $s,t \in \Sigma^*$,

$$\langle\langle s, D\rangle, \langle t, P1\rangle\rangle \mapsto \langle st, P0\rangle,$$

and $m2$ maps binary 'predicates' to unary ones,

$$\langle\langle s, D\rangle, \langle t, P2\rangle\rangle \mapsto \langle st, P1\rangle.$$

We can diagram $\mathcal{P}$:



And let's define a minimal modifier language $\mathcal{M} = (Lex, \mathcal{F})$ as follows.

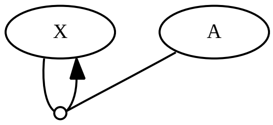
$$\Sigma=\{a,b,p,q,w\}, \text{ and}$$
$$\text{Cat}=\{A,X,W\},$$

$$Lex = \{ \langle a, A\rangle, \langle b, A\rangle, \langle p, X\rangle, \langle q, X\rangle, \langle w, W\rangle \},$$

and $\mathcal{F} = \langle m\rangle$, where m 'modifies' some elements X as follows, for any $s,t \in \Sigma^*$,

$$\langle\langle s, A\rangle, \langle t, X\rangle\rangle \mapsto \langle st, X\rangle.$$

We can diagram $\mathcal{M}$:



As noted above, Keenan and Stabler (2003) define the languages Span, Eng, Kor, FWK, Toba, and CG1. Theorem 1 points out that none of these is balanced, but a balanced grammar bal(Span) is provided above, and balanced grammars of the other languages are also easy to formulate.

Theorem 3 *There is a strong polynomial homomorphism from bal(Span) to $\mathcal{M}$.*

Proof: Define $h : [Lex_{bal(Span)}] \rightarrow [Lex_{\mathcal{M}}]$ as follows:

$$h(s) = \begin{cases} \langle \mathrm{a}, \mathrm{A} \rangle & \text{if } s = \langle \mathrm{x}, \mathrm{Am} \rangle \text{ for any } x \in \Sigma^* \\ \langle \mathrm{b}, \mathrm{A} \rangle & \text{if } s = \langle \mathrm{x}, \mathrm{Af} \rangle \text{ for any } x \in \Sigma^* \\ \langle \mathrm{p}, \mathrm{X} \rangle & \text{if } s = \langle \mathrm{x}, \mathrm{Nm} \rangle \text{ for any } x \in \Sigma^* \\ \langle \mathrm{q}, \mathrm{X} \rangle & \text{if } s = \langle \mathrm{x}, \mathrm{Nf} \rangle \text{ for any } x \in \Sigma^* \\ \langle \mathrm{w}, \mathrm{W} \rangle & \text{otherwise.} \end{cases}$$

Letting $m2$ be the polynomial over Span, this is a homomorphism from $(Lex_{bal(Span)}, \langle m2 \rangle)$ to $\mathcal{M}$ since whenever $\langle s_1, s_2 \rangle \in \mathrm{dom}(m1)$, $\langle h(s_1), h(s_2) \rangle \in \mathrm{dom}(m)$, and $h(m1(s_1, s_2)) = m(h(s_1), h(s_2))$. It is strong since $\langle s_1, s_2 \rangle \in \mathrm{dom}(m1)$ iff $\langle h(s_1), h(s_2) \rangle \in \mathrm{dom}(m)$. □

It's clear that there are other strong polynomial homomorphisms from bal(Span) to $\mathcal{M}$, finding recursion in either the N modifiers or in the A modifiers. It is similarly easy to show that there are strong polynomial homomorphisms from Eng, Kor, FWK, Toba, and CG1 of Keenan and Stabler (2003) to $\mathcal{P}$. We propose,

Hypothesis 1 *For every category closed, category functional, balanced grammar for a human language G, there are strong polynomial homomorphisms from G to $\mathcal{P}$, to $\mathcal{M}$, and to $\mathcal{R}$.*

In other grammars of human languages, we find the encoding of predicative, modifier, and other recursive relations sometimes elaborated by marking arguments or predicates, or making other small adjustments, but we expect it will always be easy to find structures like these in any human language.

Hypothesis 2 *There are low complexity polyomials satisfying Hypothesis 1, polynomials with depths in the range of 2 or 3.*

Keenan and Stabler (2003) observe that the automorphism *mf* of Span has a different status than the automorphisms that permute elements inside of each category. One difference noted there is that *mf* is disrupted by the addition of a single new element of category Nf; with this change, the categories Nm and Nf become structural. But now we can notice in addition, that for *mf* (as for any other element of *Aut*), given any polynomial homomorphism *h* from *Span* to *A*, $h(mf)$ is category preserving. This is an immediate consequence of the fact that $\mathcal{M}$ itself does not have enough structure to distinguish masculine and feminine systems of modification, and provides a precise sense in which we can see that the agreement marking that introduces the category changing automorphisms into the modification systems, does not participate in the modification system; it is collapsed by every strong polynomial homomorphism to $\mathcal{M}$ into a category preserving automorphism. Extending Span to include predication, we find the agreement distinctions similarly collapsed in strong polynomial homomorphisms from that system to $\mathcal{P}$.

5 The syntactic status of sentences

Do sentences, semantically identified as the bearers of propositional content, have any special syntactic status across languages? Given a grammar of an arbitrary natural language with the categories and lexical items renamed, and without any semantics, could you tell which categories were clauses? Here we tentatively propose one positive idea, and another universal claim about human languages.

We defined a trivial one-step grammar $\mathcal{O}$ above. Every reasonable non-empty possible human grammar will have a strong polynomial homomorphism to $\mathcal{O}$. Similarly, human grammars will typically have strong polynomial homomorphisms to the similar "two-step" grammar, and the "three step" grammar. But one idea about human languages is that there is a limit to the number of steps that can be taken without any recursion, and that clausal categories have a distinctive status from this perspective, as follows.

For any grammar $G = (Lex, \mathcal{F})$, define $Lex_0 = Lex$, and $Lex_{n+1} = Lex_n \cup \{f(\vec{e}) | \ \vec{e} \in Lex_n^* \cap \mathrm{dom}(f), f \in \mathcal{F}\}$. Clearly the language $[Lex] = \bigcup_{i \geq 0} Lex_i$. Standard notions of immediate constituency ICON and related notions can be defined as follows. Let $e\,\mathrm{ICON}\,e'$ iff there is $\langle d_1, \ldots, d_n \rangle \in [Lex]^*$ such that $e = d_i$ for some $0 < i \leq n$ and there is some $f \in \mathcal{F}$ such that $e' = f(d_1, \ldots, d_n)$. Then let PCON be the transitive closure of ICON, and let CON be the reflexive, transitive closure of ICON.

Let's distinguish those expressions whose derivations do not include any recursion, in any category, $e \in [Lex]$ is *non-recursive*, $\mathrm{NR}(e)$ iff there are no $d, d' \in [Lex]$ such that $\mathrm{Cat}(d) = \mathrm{Cat}(d')$, $d\,\mathrm{PCON}\,d'$, and $d'\,\mathrm{CON}\,e$. Now we can define the height of the most complex but non-recursive elements of a category. The *non-recursive height* of category C,

$$nrh(C) = \max_i \exists e \in Lex_i,\ \mathrm{NR}(e),\ \mathrm{Cat}(e) = C.$$

Then we can know say what it is to be the most complex category without recursion, as follows: C is non-recursively maximal iff there is no $C' \in Cat$ such that $nrh(C') > nrh(C)$.

It is easy to show that the set of expressions that have a non-recursively maximal category is a structural set, in the sense defined above. In the example grammars Eng, Kor, FWK, Toba, CG1 of Keenan and Stabler (2003), mentioned above, there is a unique non-recursively maximal category, the 'sentence' category (named P0 or S in those grammars).

Hypothesis 3 *In every category closed, category functional, balanced grammar for a human language, there are non-recursively maximal categories that hold of the expressions semantically identified as bearers of propositional content ('clauses').*

Note that in a grammar that marks extractions of X with /X features in the category system, if it allows extractions of arbitrarily many Xs, there can easily fail to be any non-recursively maximal category. If any human language allows unboundedly many elements to be extracted from a single constituent – contra the kinds of limits in TAGs (Joshi and Schabes, 1997) and minimalist grammars (Stabler, 1997), etc. – then this last hypothesis will need to be reformulated. A more careful consideration of these questions must be left to another place.

6 Conclusions

We have defined an approach to language that is suitably abstract for stating the purely syntactic component of semantically loaded universals of language like these:

- All human languages exhibit transitive and intransitive predication.
- All human languages exhibit modification of at least one category.
- All human languages have recursion.

To capture the purely syntactic part of these, we propose,

Hypothesis 1 For any category closed, category functional, balanced grammar G for a human language, there are strong polynomial homomorphisms from G to $\mathcal{P}$, to $\mathcal{M}$, and to $\mathcal{R}$.

Hypothesis 2 There are low complexity polynomials satisfying Hypothesis 1, polynomials with depths in the range of 2 or 3.

Finally, we propose more tentatively that clausal categories are maximal in a certain sense:

Hypothesis 3 In every category closed, category functional, balanced grammar for a human language, there are non-recursively maximal categories that hold of the expressions semantically identified as bearers of propositional content ('clauses').

It should be possible to use this kind of approach to articulate precise versions of a range of familiar universal claims about syntax. As these claims become more precise, it may be possible to establish whether they are really correct. Notice that these claims are not tied to an particular grammar formalism. For example, we already observed that a particular grammar $G = (Lex, \mathcal{F})$ satisfies these hypotheses iff $G = (Lex, poly(G))$ does. It does not matter which grammar we select from any of the infinitely many that define the same automorphisms.

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