Universals Across Languages

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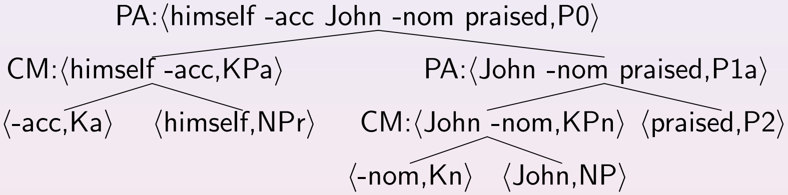
(M) "…abstract fully away from the details of the grammar mechanism – to express syntactic theories purely in terms of the properties of the class of structures they license"

(UG) What significant properties do human languages share?

$$
\begin{aligned}
\Sigma &= \{\text{-nom,-acc,laughed,cried,praised,criticized,John,Bill,himself,and}\},\\
\text{Cat} &= \{\text{NP,NPr,Kn,Ka,P2,P1n,P1a,P0,CONJ}\},\\
\text{Expr} &= \Sigma^* \times \text{Cat},\\
\mathit{Lex} &= \{\ \langle\text{laughed, P1n}\rangle, \langle\text{cried, P1n}\rangle, \langle\text{praised, P2}\rangle, \langle\text{criticized, P2}\rangle, \langle\text{and, CONJ}\rangle,\\
&\quad \langle\text{John, NP}\rangle, \langle\text{Bill, NP}\rangle, \langle\text{himself, NPr}\rangle, \langle\text{-nom, Kn}\rangle, \langle\text{-acc, Ka}\rangle\ \}.\\
\mathcal{F} &= \langle\text{CM,PA,Coord}\rangle,\\
\text{CM} &: \langle\langle\text{s, Kn}\rangle, \langle\text{t, NP}\rangle\rangle \mapsto \langle\text{ts, KPn}\rangle\\
&\quad \langle\langle\text{s, Ka}\rangle, \langle\text{t, NP}\rangle\rangle \mapsto \langle\text{ts, KPa}\rangle\\
&\quad \langle\langle\text{s, Ka}\rangle, \langle\text{t, NPr}\rangle\rangle \mapsto \langle\text{ts, KPa}\rangle\\
\text{PA} &: \langle\langle\text{s, KPn}\rangle, \langle\text{t, P1n}\rangle\rangle \mapsto \langle\text{st, S}\rangle\\
&\quad \langle\langle\text{s, KPa}\rangle, \langle\text{t, P1a}\rangle\rangle \mapsto \langle\text{st, S}\rangle\\
&\quad \langle\langle\text{s, KPn}\rangle, \langle\text{t, P2}\rangle\rangle \mapsto \langle\text{st, P1a}\rangle\\
&\quad \langle\langle\text{s, KPn}\rangle, \langle\text{t, P2}\rangle\rangle \mapsto \langle\text{st, P1n}\rangle\\
\text{Coord} &: \langle\langle\text{s, CONJ}\rangle, \langle\text{t, C}\rangle, \langle\text{u, C}\rangle\rangle \mapsto \langle\text{stu, C}\rangle,\ \ \text{C} \notin \{\text{Kn,Ka,CONJ}\}\\
\text{Kor} &= \langle[\mathit{Lex}], \mathcal{F}\rangle
\end{aligned}
$$



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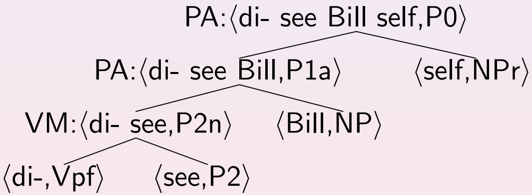
bijection $h : [Lex] \rightarrow [Lex]$ is an **automorphism** iff $\forall f \in \mathcal{F},\ h(f) = f$.

x is **structural** iff $\forall h \in Aut,\ h(x) = x$.

'Structure is what the automorphisms fix'

$$
\begin{aligned}
\Sigma &= \{\text{mang-,di-,laughed,cried,praised,criticized,John,Bill,self,and}\},\\
\text{Cat} &= \{\text{NP,NPr,Vaf,Vpf,P2,P2a,P2n,P1n,P1a,P0,CONJ}\},\\
\text{Expr} &= \Sigma^* \times \text{Cat},\\
\mathit{Lex} &= \{\ \langle\text{laughed}, \text{P1n}\rangle, \langle\text{cried}, \text{P1n}\rangle, \langle\text{praised}, \text{P2}\rangle, \langle\text{criticized}, \text{P2}\rangle, \langle\text{and}, \text{CONJ}\rangle,\\
&\quad\ \langle\text{John}, \text{NP}\rangle, \langle\text{Bill}, \text{NP}\rangle, \langle\text{self}, \text{NPr}\rangle, \langle\text{mang-}, \text{Vaf}\rangle, \langle\text{di-}, \text{Vpf}\rangle\ \}.\\
\mathcal{F} &= \langle\text{CM,PA,Coord}\rangle,\\
\text{VM} &: \langle\langle\text{s}, \text{Vaf}\rangle, \langle\text{t}, \text{P2}\rangle\rangle \mapsto \langle\text{st}, \text{P2a}\rangle\\
&\quad \langle\langle\text{s}, \text{Vpf}\rangle, \langle\text{t}, \text{P2}\rangle\rangle \mapsto \langle\text{st}, \text{P2n}\rangle\\
\text{PA} &: \langle\langle\text{s}, \text{P2x}\rangle, \langle\text{t}, \text{NP}\rangle\rangle \mapsto \langle\text{st}, \text{P1y}\rangle, \qquad x \neq y \in \{\text{n,a}\}\\
&\quad \langle\langle\text{s}, \text{P1x}\rangle, \langle\text{t}, \text{NP}\rangle\rangle \mapsto \langle\text{st}, \text{P0}\rangle, \qquad x \in \{\text{n,a}\}\\
&\quad \langle\langle\text{s}, \text{P2a}\rangle, \langle\text{t}, \text{NPr}\rangle\rangle \mapsto \langle\text{st}, \text{P1n}\rangle\\
&\quad \langle\langle\text{s}, \text{P1a}\rangle, \langle\text{t}, \text{NPr}\rangle\rangle \mapsto \langle\text{st}, \text{P0}\rangle\\
\text{Coord} &: \langle\langle\text{s}, \text{CONJ}\rangle, \langle\text{t}, \text{C}\rangle, \langle\text{u}, \text{C}\rangle\rangle \mapsto \langle\text{stu}, \text{C}\rangle,\ \ \text{C} \notin \{\text{Vaf,Vpf,P2,CONJ}\}\\
\text{Toba} &= \langle[\mathit{Lex}], \mathcal{F}\rangle
\end{aligned}
$$


PA:⟨di- see Bill self,P0⟩
PA:⟨di- see Bill,P1a⟩
⟨self,NPr⟩
VM:⟨di- see,P2n⟩
⟨Bill,NP⟩
⟨di-,Vpf⟩
⟨see,P2⟩

- These grammars do not make UG explicit
  - **we need 'deeper' grammatical analyses**
    In general it should be expected that only descriptions concerned with deep structure will have import for proposals concerning linguistic universals. [1, p.209]
  - **we need descriptions that abstract across grammars** ☜
- Automorphisms of these languages differ significantly [4, 6]
- The languages $\langle L, \mathcal{F} \rangle$ are not related by homomorphism ☜

$$\mathcal{F}_{Kor} = \langle CM, PA, Coord \rangle$$
$$\mathcal{F}_{Toba} = \langle VM, PA, Coord \rangle$$

What functions should we have in $\mathcal{F}$?

- Let $explode(\mathcal{F}) = \{\{\langle a, b\rangle\} |\ f_i(a) = b$ for some $f_i \in \mathcal{F}\}$.
  And for any $G = \langle A, \mathcal{F}\rangle$, let $explode(G) = \langle A, explode(F)\rangle$.

  Misses generalizations: defines the same language (and derivation shape unchanged), but *has fewer automorphisms.*

- In Kor, Toba, f_i in $\mathcal{F}$ disjoint, so consider $\langle [Lex], \langle \bigcup \mathcal{F}\rangle\rangle$

  Hides structure: defines the same language (and derivation shape unchanged), but *no new automorphisms.*

- proposal:
  'unify $\mathcal{F}$ to capture gens; then enlarge without changing Aut'

Step 1. balance

- $G = (A, \mathcal{F})$ is *balanced* iff
  - there are no two distinct, compatible, non-empty functions $f_i, f_j \in \mathcal{F}$ such that removing f_i, f_j and adding $f_i \cup f_j$ strictly increases the set of automorphisms, and
  - there are no two distinct, compatible, non-empty functions $g, g' \notin \mathcal{F}$ such that $g \cup g' = f_i$ for some $f_i \in \mathcal{F}$, where the result of adding g and g' yields a grammar with the same automorphisms as G has.

- (like most grammars) Kor, Toba are not balanced

E.g., in Kor, $\text{CM} = \text{CM}_{KnNP} \cup \text{CM}_{KaNP} \cup \text{CM}_{KaNPr}$ where

$$\begin{aligned}
\text{CM}_{KnNP} &: \langle\langle \text{s}, \text{Kn}\rangle, \langle \text{t}, \text{NP}\rangle\rangle \mapsto \langle \text{ts}, \text{KPn}\rangle \\
\text{CM}_{KaNP} &: \langle\langle \text{s}, \text{Ka}\rangle, \langle \text{t}, \text{NP}\rangle\rangle \mapsto \langle \text{ts}, \text{KPa}\rangle \\
\text{CM}_{KaNPr} &: \langle\langle \text{s}, \text{Ka}\rangle, \langle \text{t}, \text{NPr}\rangle\rangle \mapsto \langle \text{ts}, \text{KPa}\rangle
\end{aligned}$$

Step 2. close wrt projection and composition

- An n-ary *projection function* is a total function $\epsilon_i^n : \text{Expr}^n \rightarrow \text{Expr}$, for $0 < i \leq n$, defined by

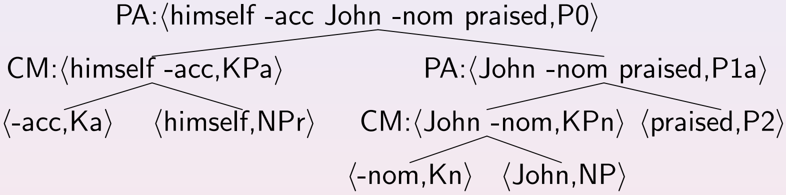
$$\epsilon_i^n(x_1, \ldots, x_i, \ldots, x_n) = x_i.$$

- The *polynomials over* $(A, \mathcal{F})$ = smallest set containing the projection functions and such that if $p_1, \ldots, p_m$ are n-ary polynomials, and n-ary $f \in \mathcal{F}$, then $f(p_1, \ldots, p_m)$ is also an n-ary polynomial, whose domain

$$\text{dom}(f(p_1, \ldots, p_m)) = \{s \in \text{Expr}^n \mid \begin{array}{l} s \in \text{dom}(p_i)\ (0 < i \leq m) \text{ and} \\ \langle p_1(s), \ldots, p_m(s) \rangle \in \text{dom}(f)\}, \end{array}$$

and where the values of the polynomial are given by

$$f(p_1, \ldots, p_n)(s) = f(p_1(s), \ldots, p_m(s)).$$



The expression $\langle\text{himself -acc John -nom praised}, \text{P0}\rangle$ is the value of

$$\text{PA}(\text{CM}(\epsilon_1^5, \epsilon_2^5), \text{PA}(\text{CM}(\epsilon_3^5, \epsilon_4^5), \epsilon_5^5))$$

applied to this element from Lex^5:

$$\langle\langle\text{-acc}, \text{Ka}\rangle, \langle\text{himself}, \text{NPr}\rangle, \langle\text{-nom}, \text{Kn}\rangle, \langle\text{John}, \text{NP}\rangle, \langle\text{praised}, \text{P2}\rangle\rangle.$$

Step 3. close wrt incorporation of constants

- When $\forall\langle s_1,\ldots,s_n\rangle \in \text{dom}(f_i)$, $s_j = s$, then s is structural. In that case, define the $(n-1)$-ary **incorporation**

$$f_i(\epsilon_1^{n-1},\ldots,\epsilon_{j-1}^{n-1},s,\epsilon_j^{n-1},\ldots,\epsilon_{n-1}^{n-1})(s_1,\ldots,s_{n-1}) = f_i(s_1,\ldots,s_{i-1},s,s_j,\ldots,s_{n-1}).$$

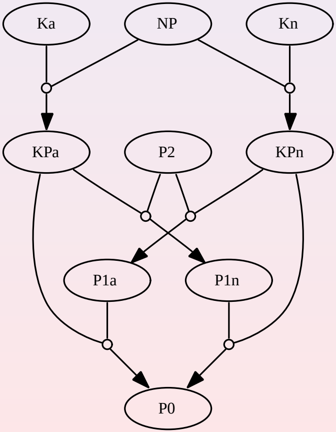
- E.g. Given $\text{CM}_{KnNP} : \langle\langle \text{s},\text{Kn}\rangle,\langle \text{t},\text{NP}\rangle\rangle \mapsto \langle \text{ts},\text{KPn}\rangle$, we have

$$\text{CM}_{KnNP}(\langle\text{-nom},\text{Kn}\rangle,\epsilon_1^1) : \langle \text{t},\text{NP}\rangle \mapsto \langle \text{t -nom},\text{KPn}\rangle$$

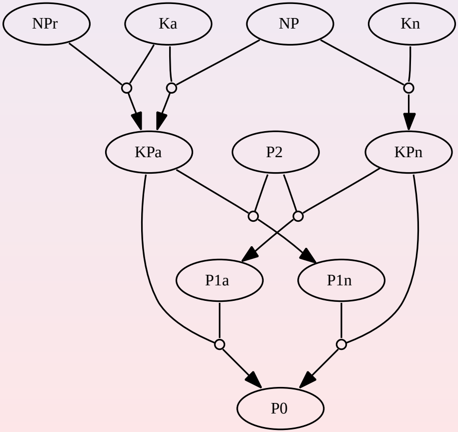
- **Thm:** Closing wrt projection, polynomials, incorporation does not change Aut

- Let $bal(A, \mathcal{F}) = \langle A, \mathcal{G} \rangle$ where $\mathcal{F}$ is balanced and $\mathcal{G}$ is closed with respect to polynomials, unions of compatible functions, and incorporations – a "clone" [5, 9]

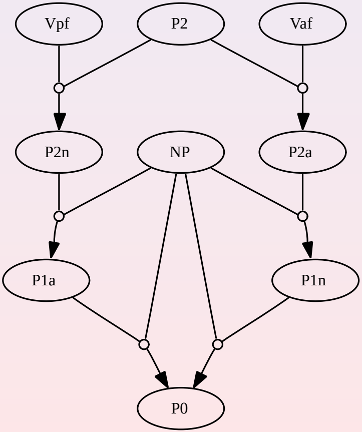
- In bal(Kor), 9 invariant sets ⬭ and 6 funcs ∘ (inter alia)



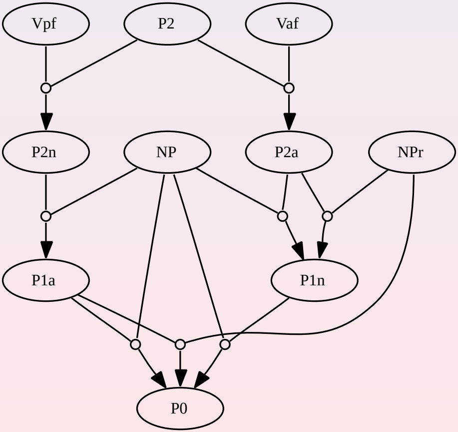
- In bal(Kor), NPr breaks the symmetry:



- In bal(Toba), 9 invariant sets ⬭ and 6 funcs ∘



- In bal(Toba), NPr breaks the symmetry:



"For algebras, there is only one reasonable way to define the concepts of subalgebra, homomorphism, and congruence relation. For partial algebras we will define three different types of subalgebra, three types of homomorphism, and two types of congruence relation." [2]

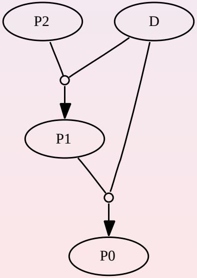
- $h : A \rightarrow B$ is a *strong homomorphism* from $(A, \langle f_1, \ldots, f_n \rangle)$ to $(B, \langle g_1, \ldots, g_n \rangle)$ iff for all $0 < i \leq n$ both
  - $\langle s_1, \ldots, s_m \rangle \in \text{dom}(f_i)$ iff $\langle h(s_1), \ldots, h(s_m) \rangle \in \text{dom}(g_i)$ and
  - $h(f_i(s_1, \ldots, s_m)) = g_i(h(s_1), \ldots, h(s_m))$.

- There is a *strong polynomial homomorphism* of $bal(A, \mathcal{F})$ into $(B, \mathcal{G})$ iff there are $f_1, f_2, \ldots, f_n \in bal(\mathcal{F})$ such that there is a strong homomorphism from $(A, \langle f_1, f_2, \ldots, f_n \rangle)$ to $(B, \mathcal{G})$.

- Let 'minimal predicative' $\mathcal{P} = \langle Lex, \langle m1, m2 \rangle \rangle$, with

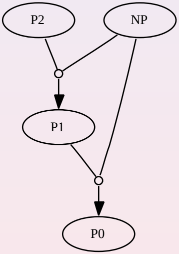
$$Lex = \{ \ \langle \mathrm{a}, \mathrm{D} \rangle, \langle \mathrm{b}, \mathrm{D} \rangle, \langle \mathrm{p}, \mathrm{P1} \rangle, \langle \mathrm{q}, \mathrm{P1} \rangle, \langle \mathrm{r}, \mathrm{P2} \rangle, \langle \mathrm{s}, \mathrm{P2} \rangle, \langle \mathrm{w}, \mathrm{W} \rangle \ \},$$

$$\begin{array}{ll} m1: & \langle \langle \mathrm{s}, \mathrm{D} \rangle, \langle \mathrm{t}, \mathrm{P1} \rangle \rangle \mapsto \langle \mathrm{st}, \mathrm{P0} \rangle \\ m2: & \langle \langle \mathrm{s}, \mathrm{D} \rangle, \langle \mathrm{t}, \mathrm{P2} \rangle \rangle \mapsto \langle \mathrm{st}, \mathrm{P1} \rangle \end{array}$$



- **(H)** $\exists$ strong polynomial homomorphism from G to $\mathcal{P}$, for G any balanced grammar for a human language.

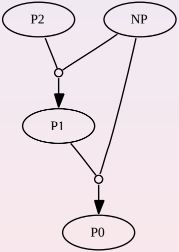
- **Thm** $\exists$ strong polynomial homomorphism from bal(Kor) to $\mathcal{P}$.



$$m2 = \mathsf{PA}_{\mathsf{KPnP2}}(\mathsf{CM}_{\mathsf{KnNP}}(\langle\text{-nom}, \mathsf{Kn}\rangle, \epsilon^2_1), \epsilon^2_2) : NP \times P2 \rightarrow P1a$$

$$m1 = \mathsf{PA}_{\mathsf{KPaP1a}}(\mathsf{CM}_{\mathsf{KaNP}}(\langle\text{-acc}, \mathsf{Ka}\rangle, \epsilon^2_1), \epsilon^2_2) : NP \times P1a \rightarrow P0$$

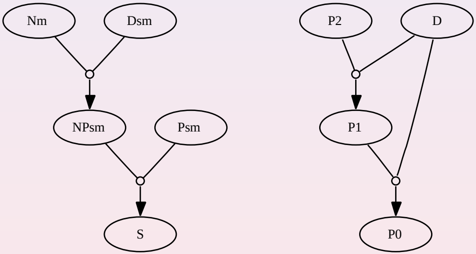
Thm $\exists$ strong polynomial homomorphism from bal(Toba) to $\mathcal{P}$.



$$m2 = \mathsf{PA}_{\mathsf{P2sfNP}}(\mathsf{VM}_{\mathsf{VafP2}}(\langle \mathsf{mang\text{-}}, \mathsf{Vaf}\rangle, \epsilon_2^2), \epsilon_1^2) : NP \times P2 \rightarrow P1sf$$

$$m1 = \mathsf{PA}_{\mathsf{P1sfNP}}(\epsilon_2^2, \epsilon_1^2) : NP \times P1sf \rightarrow P0$$

- **Thm** $\nexists$ strong polynomial homomorphism from $\mathcal{D}$ to $\mathcal{P}$.



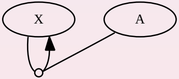
If h maps all Dsm and Psm into elements of D, we violate the "strong homomorphism" requirements...

- Define a 'minimal modifier' structure $\mathcal{M} = (Lex, \langle m \rangle)$

$$\Sigma = \{a,b,p,q,w\}, \text{ and Cat} = \{A,X,W\},$$

$$Lex = \{\ \langle a, A\rangle, \langle b, A\rangle, \langle p, X\rangle, \langle q, X\rangle, \langle w, W\rangle\ \},$$

$$m : \langle\langle s, A\rangle, \langle t, X\rangle\rangle \mapsto \langle st, X\rangle$$



- **(H)** $\exists$ strong polynomial homomorphism from G to $\mathcal{M}$, for G any balanced grammar for a human language

- (M) We can wash out artefacts of description by expanding to clones of balanced grammars
- (H) For balanced human grammars G, $\exists$ strong polynomial homomorphisms from G to $\mathcal{P}$ and to $\mathcal{M}$
- (F0) (H) has content before knowing all possible kinds (all 'parameters') of predication, modification systems
- (F1) In lexicalized MGs, TAGs, TLGs..., (H) a lexical hypothesis. There, lexicons contain not only "what makes English English" but also "what makes a MCS/P set a human language." (cf mt, alg)
- (N) More syn + syn/sem universals...

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