Class 8: Optimality Theory, part I

0. Business
- Fill out the first two columns of this Google Doc:
  [https://docs.google.com/document/d/1jduAyW7xrPjK1cjLyFrz77Ygk-6bXSstd8BpXxTylwo/edit?usp=sharing](https://docs.google.com/document/d/1jduAyW7xrPjK1cjLyFrz77Ygk-6bXSstd8BpXxTylwo/edit?usp=sharing) (I will paste in chat)
- Kie: start recording

1. Recall the “conceptual crisis” (Prince & Smolensky 2004, p. 1)
   - On the one hand, we want constraints in our theory
   - On the other hand, we can’t decide exactly how they’re supposed to work.

2. Prince & Smolensky’s solution: Optimality Theory

<table>
<thead>
<tr>
<th>rule-based grammar with constraints</th>
<th>OT grammar</th>
</tr>
</thead>
<tbody>
<tr>
<td>start with UR/input (from mental lexicon, maybe after morphology)</td>
<td>apply all possible rules, producing a (large!) set of candidate outputs</td>
</tr>
<tr>
<td>apply rules in sequence—intermediate representation is known at all times</td>
<td>constraints pick the best candidate</td>
</tr>
<tr>
<td>constraints may block or trigger rules</td>
<td>candidate outputs are (potential) surface forms =&gt; full look-ahead to end of each possible derivation</td>
</tr>
<tr>
<td>look-ahead: nonexistent or sketchy</td>
<td>constraints interact through strict domination</td>
</tr>
<tr>
<td>interaction of constraints: nonexistent or sketchy</td>
<td>similarity to UR is enforced by faithfulness</td>
</tr>
<tr>
<td>similarity to UR results from not applying too many rules, not having too many constraints</td>
<td>constraints</td>
</tr>
<tr>
<td>end with SR/output (send it to the phonetic system)</td>
<td></td>
</tr>
</tbody>
</table>

3. Gen(): function that creates set of candidate outputs from input
   - One way to think of it:\(^1\) apply all possible rules to the input, any number of times (deletion, insertion, feature changing, maybe changing order).

   \[
   \text{Gen}(\text{/ab/}) = \{[ab], [a], [b], [ba], [], [ta], [at], [ae], \ldots\}
   \]

   ![Diagram of candidate outputs](image)

   ¿ Why is the resulting set of candidates infinite (assuming a finite alphabet of symbols)?

---

\(^1\) This is what P&S call ‘anharmonic serialism,’ but with a set of rules broad enough to get “all possible variants”.
4. Constraints

- In standard OT, a markedness constraint can be a function from a candidate output to a natural number (the number of violations). A lower number means greater harmony (goodness):
  \[ \text{NoCODA}([\text{bak}]) = 1 \quad \text{NoCODA}([\text{tik}.\text{pad}]) = 2 \]

- Similarly, a faithfulness constraint can be a function from input-output pair to natural number:
  \[ \text{DON'TDELETE}(/\text{bak}/, [\text{ba}]) = 1 \quad \text{DON'TDELETE}(/\text{bak}/, [\text{bak}]) = 0 \]

- More generally, a constraint \( C_i \) is a function that imposes a strict partial order \( \succ_i \) (“is more harmonic than with respect to \( C_i \)”) on a set of candidates...
  - Transitive: if \( a \succ_i b \) and \( b \succ_i c \), then \( a \succ_i c \).
  - Irreflexive: \( a \npreceq_i a \).
  - Asymmetric: if \( a \succ_i b \), then \( b \npreceq_i a \).

Show that asymmetry follows from the other two properties.

Show that irreflexivity follows from asymmetry.
...with these additional properties:
  - “Stratified”: if $a \nRightarrow b$ and $b \nRightarrow a$, then for any $x \Rightarrow a$, $x \Rightarrow b$ too; and for any $y$ such that $a \Rightarrow y$, $b \Rightarrow y$ too. (In other words, if $a \nRightarrow b$ and $b \nRightarrow a$, then $a$ and $b$ are of equivalent harmony.)
    (In Wilson 2001, the stratification requirement is relaxed.)
  - Bounded from above: There exists some $a$ such that there is no $x \Rightarrow a$. (I.e., even in an infinite set of candidates, one or more are the most harmonic; there’s not necessarily a set of least-harmonic candidate, though.)

**NoCoda:**

```
NOCODA:  ta.da  bo  ba.du.pi
        tak.do  i.tek  o.tek.lao
        tak.kat  bad.ku.pit  sik.lep.bu
        ...
```

Let’s verify that assigning a (non-unique) natural number (0, 1, 2, …) to each candidate meets all these ordering requirements.

Why are there no least-harmonic candidates for NOCODA?

Can you recall a case from P&S where numbers of violations weren’t used?

---

2 I don’t know if there’s a real math term for “stratified”. Samek-Lodovici & Prince 1999 use this term, following Tesar 1995, who uses it to describe partial orderings of constraints rather than of candidates.
5. Eval()

- **Eval()** is a function
  - arguments of the function: input, set of output candidates, ordered list (Con) of constraints
  - output of function: subset of the candidates that is optimal

- Typically we use it this way:
  - Eval(input, Gen(/input/), Con) = {[output]}

- But Eval() also can work on a smaller set of candidates:
  - Eval(/bak/, {[bak],[ba]}, <NOCODA, DON’TDELETE>) = {[ba]}

- And, the output set can have a tie:
  - Eval(/bak/, {[bak],[ba], [bo]}, <NOCODA, DON’TDELETE>) = {[ba], [bo]}

- **Eval()** takes the orderings imposed by the various constraints and assembles them into one giant ordering
  - Giant ordering has the same properties: transitive, irreflexive, asymmetric, stratified, bounded above.

- We can think of many ways this could be done…**strict ranking** is the mechanism used in standard OT for adjudicating harmony disagreements among constraints.

---

3 In the original P&S manuscript, the output candidate always contains all the information about the input, so we don’t need to include the input as an argument to Eval().
6. Alphabetization as strict ranking

- Constraints impose partly conflicting orderings on words (I know the last column isn’t fully visible—wouldn’t fit):

We reconcile the orderings by adding only pairwise orderings that don’t contradict what we have so far:
7. **How about finding just the first word?**
   - find the members that have the earliest first letter—and discard the rest
   - from the new, smaller set, pick the members that have the earliest second letter, etc.
   - Once a word is ruled out, it can’t redeem itself by, e.g., having lots of *as* later on.

  Can we imagine some other ways that constraints could conceivably interact?

8. **Eval() works the same way**
   - To find just the winners, if you have *n* constraints…
     - Find the candidates that tie for being ‘best’ on the top-ranked constraint *C*₁; discard the rest.
     - Of the remaining candidates, find those the next constraint, *C*₂, deems best; discard the rest.
     - Repeat for *C*₃,…,*C*ₙ.
     - Whatever candidates are still left at the end are tied for being the winner (if you have enough constraints, there is normally just one winner).

**Q:** How can that be computable? Wouldn’t you have to go through an infinite list of candidates just to do the first step?

**A:** For that reason, most computational implementations of OT (Albro 2005, Eisner 1997, Ellison 1994, Riggle 2004) represent the candidate set as a regular expression, which is a finite way to represent a certain class of infinite sets. For example, \(ab^*a\) is the set \{\(aa\), \(aba\), \(abba\), \(abbba\), …\}. These expressions can then be manipulated algorithmically, either in a fairly literal translation of the above (as in Eisner 1997) or by other means.

   - More declaratively, a candidate \(a\) is optimal iff, for any \(b\) and \(C_j\) such that \(b \succ_j a\), there exists some \(C_i\) such that \(i < j\) (i.e., \(C_i\) is higher ranked than \(C_j\)) and \(a \succ_i b\).
   - In words, for \(a\) to be optimal, any candidate that does better than \(a\) on some constraint must do worse than \(a\) on another, higher-ranked constraint.

9. **Two types of constraint**
   - In pre-OT approaches to constraints, constraints were all *markedness* constraints: they penalized certain surface structures, such as CCC clusters.
   - So, on first hearing about OT, many people’s second reaction (after worrying about infinity) was to wonder why, if it’s all about constraints, every word isn’t maximally unmarked.
In rule theories, what prevents every word from coming out [baba] (or whatever the least marked word is)?

How do P&S prevent every word from coming out [baba]?

- **Markedness** constraints look at the surface representation.
  - The simplest ones can be defined by the structural description that they ban: *[+voice]#, *[C]_o_.
  - Typical markedness constraints reflect articulatory ease, or perceptual clarity, rhythmic organization, or other “natural” drives.⁴
    - You can (and should!) give a constraint a helpful mnemonic name, like NOCODA for *[C]_o_, as long as you precisely define the constraint somewhere.
      - In other words, the constraint name isn’t the definition
    - A good constraint definition should make it clear not just what is banned, but **how the number of violations is assessed**.

What are some different ways that NOCODA might count violations?

- **Faithfulness** constraints look at the relationship between the underlying and surface representations (the standard ones require similarity but we can imagine other possibilities).
  - P&S’s PARSE (≈ don’t delete) and FILL (≈ don’t insert), were quickly superseded by McCarthy & Prince’s correspondence constraints (the theory behind which we’ll see another time), so let’s start using the newer names now:
    - MAX-X: don’t delete X (e.g., MAX-C, MAX-V)
    - DEP-X: don’t insert X (e.g., DEP-C, DEP-V)
    - IDENT-F: don’t change a segment’s value for the feature F

⁴ Or maybe they are just arbitrary and learned by speakers in response to whatever cards history has dealt them. Or, maybe both natural and unnatural constraints are possible, but learners treat them differently. See Moreton 2008.
People often have a hard time at first with IDENT-F.

- The most common confusion is thinking it means “don’t delete a segment that is +F”.
- The next most common mistake is thinking it means “don’t alter a segment that is +F (e.g., by changing its values for some other feature G)”.

10. Exposition: the tableau

- Someday, we’ll all check our analyses with software that evaluates the infinite candidate set.\(^5\)
  - In the meantime, we illustrate an analysis with a tableau\(^6\) showing a finite subset of candidates that have been chosen to demonstrate aspects of the constraint ranking.
  - (The danger here is obvious—what if you didn’t think of some important candidate?)

- This tableau shows a ranking argument:
  - NOCODA prefers \(a\) (the winner), whereas DEP-V prefers \(b\).
  - If that’s the only difference between the candidates—no other constraint not known to be ranked below DEP-V prefers \(a\) over \(b\)—then NOCODA must outrank (\(\gg\)) DEP-V.

<table>
<thead>
<tr>
<th></th>
<th>NOCODA</th>
<th>DEP-V</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varphi a)</td>
<td>[a.tɔ.ka]</td>
<td>*</td>
</tr>
<tr>
<td>(b)</td>
<td>[at.ka]</td>
<td>*!</td>
</tr>
</tbody>
</table>

Parts of the tableau:

- input
- output candidates (not all structure shown)
- constraints (highest-ranked on left)
- asterisks
- exclamation marks
- shading
- pointing finger (you can use an arrow)

These three don’t add any new information, but are there for the convenience of the reader.

11. How do I know which candidates and constraints to include in my tableaux?

This procedure works reasonably well:

- Start with the winning candidate and the fully faithful candidate.
- If the winning candidate ≠ the fully faithful candidate…
  - Add the markedness constraint(s) that rule out the fully faithful candidate.
  - Add the faithfulness constraints that the winning candidate violates.
  - Think of other ways to satisfy the markedness constraints that rule out the fully faithful candidate. Add those candidates, and the faithfulness and markedness constraints that rule them out. How far to take this step is a matter of judgment.

---

\(^5\) See Jason Riggle for some software along these lines: http://hum.uchicago.edu/~jriggle/ riggleDiss.html

\(^6\) French for ‘table’. The singular tableau is pronounced [tabló] in French; a typical English adaptation is [tʰæblóʊ]. The plural tableaux is also pronounced [tabló] in French, [tʰæblóʊ] or [tʰæblóʊz] in English.
• If the winning candidate = the fully faithful candidate, then you are probably including this example only to show how faithfulness prevents satisfaction of a markedness constraint that, in other cases, causes deviation from the underlying form.
  ■ Add that markedness constraint.
  ■ Add one or more candidates that satisfy that markedness constraint.
  ■ Add the faithfulness constraints that rule out those candidates.

❓ Let’s try it for /atka/ → [atøka]: do it on paper and then hold it up for the rest of us to see

❓ One of the candidates below is unnecessary in arguing for the constraint ranking. Why?

<table>
<thead>
<tr>
<th>/at+ka/</th>
<th>*CC</th>
<th>DEP-V</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>[atøka]</td>
<td>*</td>
</tr>
<tr>
<td>b</td>
<td>[atka]</td>
<td>*!</td>
</tr>
<tr>
<td>c</td>
<td>[atökøa]</td>
<td>**!</td>
</tr>
</tbody>
</table>

• A candidate is harmonically bounded if it could not win under any constraint ranking.

❓ Here’s a subtler case of harmonic bounding—explain:

<table>
<thead>
<tr>
<th>/at+kap+so/</th>
<th>*CC</th>
<th>DEP-V</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>atkapso</td>
<td><em>!</em></td>
</tr>
<tr>
<td>b</td>
<td>atkaposo</td>
<td>*!</td>
</tr>
<tr>
<td>c</td>
<td>atøkapøso</td>
<td>**</td>
</tr>
</tbody>
</table>
12. **Comparative tableaux, if time**

- An innovation of Alan Prince. They convey the same information, but in a different form.

<table>
<thead>
<tr>
<th></th>
<th>/at+ka/ → [atəka]</th>
<th>*CC</th>
<th>DEP-V</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>[atəka] vs. [atka]</td>
<td>W</td>
<td>L</td>
</tr>
<tr>
<td>b</td>
<td>[atəka] vs. [atəkəa]</td>
<td>W</td>
<td></td>
</tr>
</tbody>
</table>

Each line compares the winner to one losing candidate, and shows whether each constraint prefers the winner (W) or the loser (L).

- Comparative tableaux are nice because you can easily see if your ranking is correct: the first non-blank cell in each row must say W.

❓ We also see easily why [atəkəa] is irrelevant to the ranking—explain.

❓ Draw a comparative tableau for /at+kap+so/ too. Then try to make one where b wins.

13. **Wrap-up**

- **Next week:** Hands-on practice with OT; correspondence theory; targets vs. processes
- **Kie:** stop the recording
- Fill out the third columns of the Google Doc: https://docs.google.com/document/d/1jduAyW7xrwpJkJ1cjLyFrz77YgkJ6bXstd8BpXxTylwo/edit?usp=sharing (I will paste in chat)
References
Tesar, Bruce. 1995. Computational Optimality Theory.. University of Colorado.