Class 16 (Week 8, T)
Learning Models III: numerical learning algorithms for constraint grammars

To do
- ☐ Read **Kirby (2013)** for Thursday (Nov. 19)
  - presenters, if you e-mail me your handout as a PDF by noon Thurs., I can print
- ☐ Prepare at least one **question or point for discussion** on the reading
- ☐ Computing **homework** using OTSoft is due Tuesday (Nov. 24). Turn in write-up on paper; I will enable file upload on CCLE for output files.

Overview: We know how a grammar works once we have a ranking or set of weights, but how does the learner arrive at that ranking/set of weights? Last week we looked at ranking (no numbers) and hidden structure. Now we look at learning numerical weights.

1. MaxEnt review

- The learner's job is to maximize
  - *log probability that grammar assigns to observed data – penalty for non-default weights*
  - $= \sum_{j=1}^{m} \ln(p(datum_j \mid grammar)) \; - \; \sum_{i=1}^{n} weightPenaltyForConstrain_i$
  - $= \sum_{j=1}^{m} \ln\left(\frac{e^{-sumOfWeightedConstrainViolations_j}}{total\_e^{-sumWghtedConstrViol\_ForAllCandsInTableau_j}} \right) - \sum_{i=1}^{n} \frac{(actualWeight_i - defaultWeight_i)^2}{2willingnessToChange_i^2}$
  - $\sum_{j=1}^{m} \ln\left(\frac{e^{-\sum_{i=1}^{n} w_i C_i(datum_j)}}{Z_j} \right) - \sum_{i=1}^{n} \frac{(w_i - \mu_i)^2}{2\sigma_i^2}$
  - $= \sum_{j=1}^{m} (-\sum_{i=1}^{n} w_i C_i(datum_j) - \ln(Z_j)) \; - \sum_{i=1}^{n} \frac{(w_i - \mu_i)^2}{2\sigma_i^2}$

  I hope I got all the negative signs right!

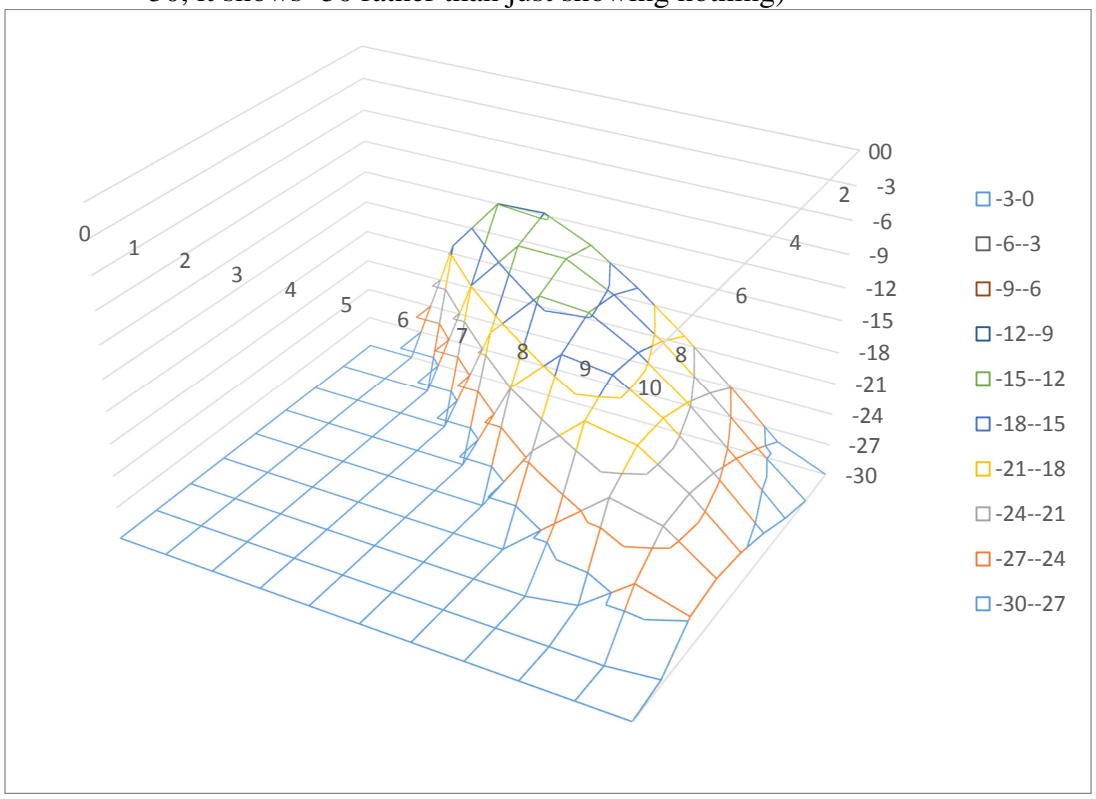
2. But how does the computer find the best weights?

(The good news: this part is really not relevant for us—the computer does it all.)

- Some form of *gradient ascent/descent*
- Simple Excel example: two constraints (I'll show this on the screen)

	sigmas:	2	2
	mus:	0	0
/rad/	observed prob.	Ident(voice)	*FinalVoicedObstruent
[rad]	2	0	1
[rat]	98	1	0

- A plot of the objective function, for various values of the two weights:
  - (Excel quirk: because I limited the z-axis to -30 through 0, for all areas that fall below -30, it shows -30 rather than just showing nothing)



  - The learner doesn't have access to this full landscape
  - But, it can start somewhere, and figure out the local **gradient** (slope)
    - How? A whole bunch of linear algebra.
    - Some methods move in just one dimension at a time (i.e., change one weight), while others can move in any direction.
    - See Shewchuk (1994) for a tutorial on one method (still difficult for me, despite title)
  - Then, the learner just has to take a small step uphill, check the gradient again, and repeat.
  - Terminology: people talk about *gradient descent* when they want to minimize something, like error, and *gradient ascent* when they want to maximize something, like probability.
- The important thing is that in MaxEnt, we are guaranteed that the surface is **convex**
  - As long as we keep going uphill, we'll get to the peak (global optimum).
  - We don't have to worry about getting stuck at a local optimum.

3. Noisy Harmonic Grammar review

- Draw a Harmonic Grammar tableau for the schematic example above. Pick weights that make [rat] the winner.

- How do you make Harmonic Grammar "noisy"?

4. A way to find HG weights: the Gradual Learning Algorithm

- The Gradual Learning Algorithm is *error-driven*.
  - What makes a learning algorithm error-driven?

- How it works for Noisy Harmonic Grammar
  - Hear an utterance
    - Will usually be [rat], but sometimes will be [rad]
  - Use the current grammar to choose a candidate (flip coin to break tie)
    - With starting weights as shown, what will happen?

weight	0	0
/rad/	IDENT(voice)	*FINALVOICEDOBSTRUENT
[rad] (2%)		*
[rat] (98%)	*	

  - If the chosen candidate doesn't match what you heard...
    - slightly increase the weight of any constraints that prefer the winner
    - slightly decrease the weight of any constraints that prefer the loser
    - Compare and contrast with Error-Driven Constraint Demotion

  - Repeat.

- There's no point when the learner knows it's done
  - If adult data is variable, then the grammar can never completely stop making errors
  - A typical strategy is to keep lowering the increment/decrement value until it's zero.
  - Another possibility is just to stop running the learner after a certain number of iterations.
    - You could also do both—lower the increment/decrement, but stop before it hits zero.

5. Stochastic OT (Boersma 1998, Boersma & Hayes 2001 for a more tutorial presentation)

- We're now in a good position to go over another model of variation in constraint grammar, which you read about in Jarosz (submitted)
- It works a lot like Noisy HG, in that...
  - Each constraint has a number
    - called "ranking value" rather than "weight"
  - Each time you want to talk, you add a little noise, drawn from a normal distribution

ranking value	37.0	40.0
noise on this occasion	0.7	-1.2
ranking value + noise ("selection point")	37.7	38.8
/rad/	IDENT(voice)	*FINALVOICEDOBSTRUENT
[rad]		*
[rat]	*	

- The difference is that instead of adding up harmony to find the best candidate...
  - You just rank the constraints according to their selection points
  - And apply strict-ranking OT in the usual way
- Also, it's fine for ranking values to be negative, since the absolute numbers don't matter, only the relative ranking that results.

- As with Noisy HG, the easiest way to get candidate probabilities from ranking values is to simulate them (e.g., run the grammar 10,000 times)
  - But you can also do numerical integration
- As with Noisy HG, the most popular learner is the Gradual Learning Algorithm

6. Problems with the Gradual Learning Algorithm

- Unlike MaxEnt, it's not guaranteed to work.
- The GLA can fail to converge, e.g., demoting one constraint ever more
- If the GLA does converge, we have no guarantee that the ranking values it finds are the "best"
  - e.g., these are the ranking values that produce the lowest error rate, or that maximize the probability of the observed data
- If you want to know more: Boersma & Pater 2013; Pater 2008; Magri 2012

7. Partial Ordering OT

- A Stochastic OT grammar defines a probability distribution over strict-ranking OT grammars
  - E.g., 50% A>>B, 50% B>>A
  - or 10% A>>B, 90% B>> A
    - How would the difference between these two grammars be represented?

- Not just any distribution, though—it has to be capturable with ranking values
  - impossible: 80% A>>B>>C, 20% C>>A>>B, 0% for the rest
    - Why impossible?

- Anttila (1997) proposes an even more restricted theory of probability distributions over strict rankings
  - Define a **strict partial order** over constraints
    - Anyone who's taken Math Ling: remind us what the crucial properties of a strict partial order are.

    - Draw any strict partial order of 5 constraints, A, B, C, D, E

  - Enumerate all the total orderings consistent with that partial order
  - Each of those total orderings is equally probable
    - I.e., if there are *n* such total orderings, then each one's probability is _____.

- As far as I know no learning algorithm has been proposed for this theory.
  - But you can sort of fake it with Stochastic OT and the Gradual Learning Algorithm:
    - Set the learning increment/decrement to be huge for the whole learning period, e.g., 20
    - The result is that the constraints effectively sort themselves into strata
    - This can't capture a non-stratified partial order, though

8. Grammar models roundup

	model name	*typical learning algorithms*	*easiest way to get candidate probabilities*
probability distribution over strict rankings	Classic OT (trivially)	Error-Driven Constraint Demotion Recursive Constraint Demotion	inspect tableau
	Stochastic OT	Gradual Learning Algorithm	simulate
	Partial Ordering OT	?	enumerate linear rankings
	Pairwise Ranking Grammar	Expectation Maximization	simulate
constraint weighting	Harmonic Grammar	Gradual Learning Algorithm	simulate
	MaxEnt	Gradient descent/ascent	calculate directly from grammar

9. Learner roundup

- Some of our algorithms are guaranteed to work (at least on their own terms), others not

- We've seen that some of our learners were **batch** and others **on-line**—discuss the difference, pros and cons

10. Back to the question of smoothing/regularization (and other priors)

- Recall: the purpose of smoothing is _____

- Smoothing will always result in a _____ fit to the training data.
- So how do we *know* whether we're over- or under-fitting?
- Ideally, we would be able to test different grammars, say against wug-test data.
  - The one that best predicts the wug test data (after training on existing data) is the one that was the right fit.
- This is similar to the machine-learning perspective:
  - The right fit is the one that does the best job of predicting new data
- But what if we don't have new data?

11. Cross-validation

- Randomly divide your data into two sets.
- One set is the training data
  - Fit various models to it

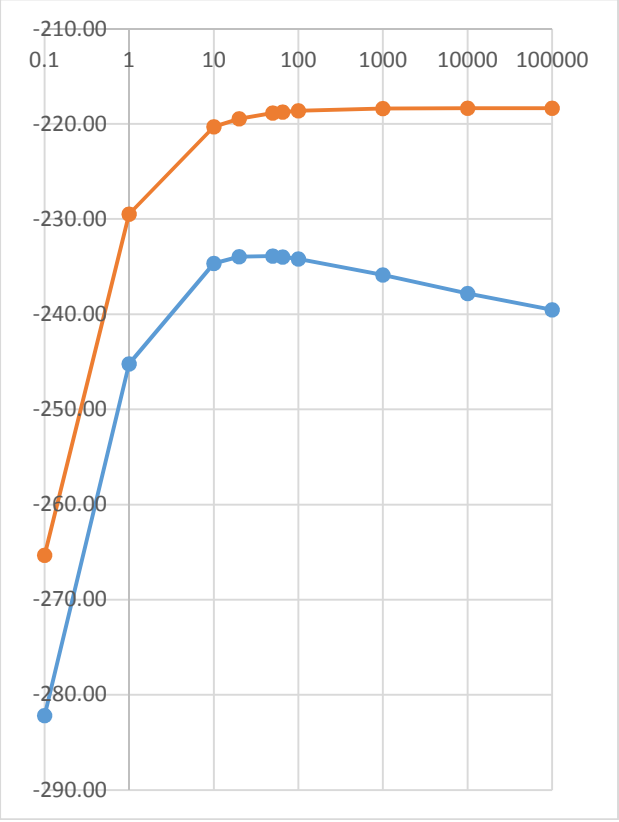
- The other[1] is the cross-validation set
  - See how well each model does at predicting these "new" data

(Another common approach: divide the data in tenths, and do the whole procedure 10 times, each time using a different tenth as the cross-validation set.)

I tried it on Shona

- Randomly selected 20% of tokens to hold back[2]
- Fitted a series of a series of MaxEnt grammars to the other 80% of the data
  - All have the same constraints
  - For simplicity of cross-validation, just two kinds
    - DEP-*X* (5 of them, one for each vowel): always giant σ^2 of 100,000
    - DEP-*X*/C__T (5), DEP-*X*/NearestVIs*Y* (25), DEP-*X*/PrecedingCIs*Y* (20): σ varies
- Test each grammar on the 20%
- Check the fit (log likelihood—the first term of the objective function)
  - log likelihood of test data under the grammar
  - log likelihood of training data under the grammar (divided by 4 for easier graphical comparison)
  - Discuss the results:

σ^2 for complex constraints	log likelihood of test data	log likelihood of training data	(log likelihood of training data)/4
100000	-239.54	**-873.41**	-218.35
10000	-237.81	-873.43	-218.36
1000	-235.88	-873.57	-218.39
100	-234.19	-874.55	-218.64
65	-233.98	-875.05	-218.76
50	**-233.89**	-875.46	-218.86
20	-233.94	-877.80	-219.45
10	-234.66	-881.21	-220.30
1	-245.23	-917.93	-229.48
0.1	-282.18	-1061.32	-265.33



[1] Usually there's yet a third set, the testing data, used for getting an estimate of model performance on new data, once the cross-validation set has been used to tweak the parameters. You can think about the following: why can't you just use the cross-validation set for this purpose?

[2] Used the rbinom() function in R.

12. Coming up

- Phonologization: How do learners end up turning phonetics into phonology?

References

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