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On the presuppositional strength of interrogative clauses

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Abstract A central question in the study of presuppositions is how a presupposition trigger contributes to the meaning of a complex expression containing it. Two competing answers are found in the literature on quantificational expressions. According to the first, a quantificational expression presupposes that every member of its domain satisfies the presuppositions triggered in its scope, and according to the second, a quantificational expression presupposes that at least one member of its domain satisfies the presuppositions triggered in its scope. The former view implies that an interrogative clause, a kind of quantificational expression, presupposes all of its possible answers' presuppositions, whereas the latter view implies that an interrogative clause presupposes that the presuppositions of at least one of is possible answers are satisfied. This paper contributes to the debate by showing that 'alternative' interrogatives, formed with *or*, project presuppositions in the same, distinctive manner that other disjunctive constructions do: generally, universally. A theory that treats disjunctive words as restricted variables, bindable by various quantificational operators, is extended to account for the presuppositions of 'alternative' interrogatives, disjoined declaratives, and disjoined conditional antecedents in a uniform manner. The paper then explores some ways to reconcile the proposal with two special cases where interrogatives have been claimed to have weaker presuppositions: (1) constituent interrogatives in presupposition-weakening contexts, and (2) polar interrogatives containing bias-inducing scalar particles like *even*.

Keywords Disjunction · Presuppositions · Questions · Alternative questions

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1 Introduction

It is often claimed that interrogative clauses project their presuppositions universally (see, for example, Schlenker 2008 and Abrusán 2014). By this we mean that the presuppositions of all their possible replies need to be satisfied. Some motivation for this claim comes from *wh*-interrogative clauses such as the one in (1), which intuitively presupposes that Mary invited all ten relevant boys, a presupposition triggered by the emotive factive verb *regret* and derived by conjoining the presuppositions of all the possible replies.

(1) Who among those ten boys does Mary regret that she invited?
Possible replies: {Mary regrets that she invited Bill, Mary regrets that she invited Fred, …}

Yet there is no consensus in the literature concerning this. For example, it is argued in Guerzoni (2003, 2004) that the well-formedness of the polar interrogative in (2), which contains the idiomatic NPI *lift a finger*, is explained by the weaker requirement that at least one possible reply to an interrogative has to have satisfied presuppositions.

(2) Did John even lift a finger to help?
Possible replies: {John even did the bare minimum to help,
John didn't even do the bare minimum to help}

Unlike the polar interrogative (3), whose possible replies have the same presupposition (namely, that Mary invited Bill), the possible replies to (2) do not have the same presupposition.

(3) Does Mary regret inviting Bill?
Possible replies: {Mary regrets inviting Bill, Mary does not regret inviting Bill}

Guerzoni assumes (with Heim 1984, Horn 1989, and Lahiri 1998) that *lift a finger* picks out the low endpoint of a scale associated with *even*. *John even did the bare minimum to help* presupposes something false (namely, that doing the bare minimum is less likely than doing more than the bare minimum) and is therefore infelicitous. By contrast, *John didn't even do the bare minimum to help* has a satisfiable presupposition (namely, that not doing the bare minimum is less likely than not doing more than the bare minimum). This, according to Guerzoni, accounts for the observation that (2) is a well-formed but negatively-biased interrogative (Borkin 1971; Ladusaw 1979), unlike (3), which is non-biased. Crucially, if all possible replies were required to have satisfied presuppositions, (2) would simply be ill-formed.

We test these two competing hypotheses on the presuppositional strength of interrogative clauses against 'alternative' interrogative clauses, i.e., interrogatives such as (4a) when uttered with the intonation that brings about the meaning in (4b). As it turns out, (4a) has the universal presupposition in (4c).[1]

(4) a. Did John eat the cake or (did he eat) the candy?
    b. Which of {John ate the cake, John ate the candy} is true?
    c. Presupposes: There is cake and there is candy.

The interrogative in (4a) resembles (2) in that its possible replies—{John ate the cake, John ate the candy}—do not have the same presupposition (one presupposes that there is cake and the other that there is candy).

We argue, based on similar projection facts in other disjunctive constructions, that the universal presupposition of (4a) is imposed by the meaning of *or*. If interrogatives in general had only an existential presupposition, (4c) could be weakened to make (4a) felicitous as a biased interrogative in a context where there is no candy, on a par with (2). Yet such weakening is not possible. This suggests that the hypothesis that all interrogatives have an existential presupposition is incorrect.

We begin by spelling out our assumptions regarding *or* and then show how they account for the presuppositions of various disjunctive constructions, including 'alternative' interrogatives (Sect. 2). We then discuss some potential counterexamples and address the question of how presuppositions generally project from interrogative clauses (Sect. 3). Finally, we test the predictions of our proposal vis-à-vis the projection properties of focus-sensitive items such as *even* (Sect. 4).

2 Flexible presuppositional disjunction

Consider the unembedded disjunction in (5a), the disjunctive conditional in (5b), and the 'alternative' interrogative in (4a) repeated in (5c). They all intuitively presuppose the conjunction of the presuppositions of *John ate the cake* and *John ate the candy*, namely, that there is (a unique) cake and that there is candy.

(5) a. John ate the cake or (he ate) the candy.
    b. If John ate the cake or (he ate) the candy, then he is not hungry.
    c. Did John eat the cake or (did he eat) the candy?

However, as discussed in Karttunen (1973, 1974) and Karttunen and Peters (1979), a presupposition does not project globally from a disjunct when the negation of some other disjunct guarantees the satisfaction of that presupposition. For example, (6) does not presuppose that Jack has children because the falsity of *Jack has no children* guarantees the felicity of *Jack's children are away*.

[1] Following the practice in Guerzoni and Sharvit (2014), we use *'alternative'* to refer to the sort of interrogative in (4a) (with the meaning implied by (4b)). *Alternative*—without ''—has its standard use (as in *an alternative solution*).

(6) Either Jack has no children or his children are away.

We refer to this as the 'K–P effect'.[2] Notice that the counterpart of (5b) in (7) and the counterpart of (5c) in (8) also exhibit the K–P effect. Accordingly, (7) does not intuitively presuppose that Jack has children. In addition, while (8) intuitively presupposes that one of {Jack has no children, Jack's children are away} is true, it does not presuppose that Jack has children (as observed in Abenina-Adar and Sharvit 2018).

(7) If Jack is (either) childless or ashamed of his children, he won't admit that his house is always empty.
(8) Does Jack have no children or are his children away?

To provide a uniform account of these facts, we make two crucial assumptions about *or*: (a) that it has no quantificational force of its own, resembling an indefinite in the sense of Kamp (1981) and Heim (1982), and (b) that it has a conditional presupposition (adapted from Heim 1992), which accounts for its projection properties in all three constructions. We present the proposal in two steps. In Sect. 2.1 we ignore the presuppositions of *or* and focus on how its apparent force is determined, and in Sect. 2.2 we introduce a presuppositional version of *or*.

2.1 The flexible quantificational force of disjunction

In disjunctive constructions such as (9a), the disjunctive word *or* appears to have inherent existential force, as (9a) intuitively entails that one of {Mary is swimming, Mary is dancing} is true. This suggests that it has the LF in (9b) (where strikethrough indicates surface ellipsis). The connective $or^{\exists}$ has the existential semantics in (10), where w is a possible world. Accordingly, $or^{\exists}$ takes as arguments two semantic objects of type st (i.e., two propositions), and (9a) is true in w if and only if at least one of the arguments of $or^{\exists}$ is true in w, as shown in (9c).[3] (See Appendix 1 for a generalized counterpart of (10)–(10')—which does not restrict the arguments of $or^{\exists}$ to type st.[4])

[2] With Karttunen and Peters (1979), we treat *or* as symmetrical (for example, we take *Either Jack's children are away or he has no children* to be semantically equivalent to (6)). There is no consensus in the literature—or among speakers—regarding this. See Sharvit (2020) for an argument against asymmetric treatments of disjunction.

[3] Type t is the type of truth values (elements of {1, 0}) and type e is the type of individuals. For any types σ and ρ: (σ, ρ)—sometimes abbreviated to σρ—is the type of functions from entities of type σ to entities of type ρ, and (s, ρ)—sometimes abbreviated to sρ—is the type of functions from possible worlds to entities of type ρ.

[4] We assume that meaning composition proceeds generally along the lines of Heim and Kratzer (1998). See Appendix 1 for more detail.

(9) a. Mary is (either) swimming or dancing.

b. $^\wedge$[*Mary swimming*] $or^{\exists}$ $^\wedge$[~~*Mary*~~ *dancing*]

c. $[\![or^{\exists}]\!]^{w}([\![^\wedge[\textit{Mary dancing}]\,]\!])([\![^\wedge[\textit{Mary swimming}]\,]\!]) =$

$[\![or^{\exists}]\!]^{w}(\lambda w'.\ [\![\textit{dancing}]\!]^{w'}(\mathbf{m}))(\lambda w'.\ [\![\textit{swimming}]\!]^{w'}(\mathbf{m})) =$

1 iff $[\![\textit{dancing}]\!]^{w}(\mathbf{m}) = 1 \vee [\![\textit{swimming}]\!]^{w}(\mathbf{m}) = 1$

(10) For any p_1 and p_2 of type st:
$[\![or^{\exists}]\!]^{w}(p_1)(p_2)=1$ iff $p_1(w)=1 \vee p_2(w)=1$

However, a disjunctive word does not always have existential force. The syntactic environment in which the disjunction appears may provide non-existential force. One of the most striking examples of this is provided by conditional sentences, as first observed in Rooth and Partee (1982). Consider the conditional sentences in (11)–(12), whose antecedent clause is (9a). The conditional in (11) is unambiguous; the one in (12)—whose consequent clause contains an elided verb phrase (VP)—is ambiguous in a way that reflects how the elided VP is interpreted: it may be interpreted as identical to the VP in the antecedent (yielding the strict VP reading), or as co-varying with each alternative mentioned in the antecedent (yielding the sloppy VP reading).

(11) If Mary is (either) swimming or dancing, then Sue is smoking.
Reading: (‘Mary is swimming or dancing’ → ‘Sue is smoking’)

(12) If Mary is (either) swimming or dancing, then Sue is.
Strict VP reading:
(‘Mary is swimming or dancing’ → ‘Sue is swimming or dancing’)
Sloppy VP reading:
(‘Mary is swimming’ → ‘Sue is swimming’) ∧ (‘Mary is dancing’ → ‘Sue is dancing’)

The strict VP reading of (12) is expected, given (11) and given that VP-ellipsis is generally allowed (for example, *Sue is swimming or dancing* follows intuitively from *Mary is swimming or dancing; Sue also is*). The sloppy VP reading of (12) is not expected without additional assumptions.

Inspired by the theory of indefinites in Kamp (1981) and Heim (1982), Rooth and Partee account for the ambiguity of (12) by treating the disjunctive word as a pronoun whose index may be bound by various operators. Faithful to the spirit—though not the letter—of Rooth and Partee (1982), we may account for the ambiguity of (12) with: (i) the pronominal connective or_k defined in (13) (where k is a numerical index and g is a variable assignment), (ii) the existential “closer” $\exists$ defined in (14), and (iii) the property-forming *Op* defined in (15).

(13) For any $k \in \mathrm{Dom}(g)$ and any P_1 and P_2 of the same type:
$[\![or_k]\!]^g(P_1)(P_2)=1$ iff $g(k)=P_1 \vee g(k)=P_2$

(14) For any X of type (st, t):
$[\![\exists]\!]^w(X)=1$ iff $\{p|\ p(w)=X(p)=1\}\neq\varnothing$

(15) For any Z of type ((s, et), t), x of type e, and Q of type (s, et):
$[\![Op]\!]^w(Z)(x)(Q)=1$ iff $Q(w)(x)=Z(Q)=1$

The index of *or*$_k$—k—may be abstracted over at various syntactic levels. For example, (16a), where the index of *or* is abstracted over at the level of [^[*Mary swimming*] *or*$_2$ ^[~~*Mary*~~ *dancing*]], is interpreted as (the characteristic function of) the set of propositions $\{[\lambda w'.\ [\![dancing]\!]^{w'}(\mathbf{m})],\ [\lambda w'.\ [\![swimming]\!]^{w'}(\mathbf{m})]\}$. (17b), where the index of *or* is abstracted over at the level of [^*swimming or*$_2$ ^*dancing*], is interpreted as (the characteristic function of) the set of properties $\{[\lambda w'.\ [\![dancing]\!]^{w'}],\ [\lambda w'.\ [\![swimming]\!]^{w'}]\}$.

(16) a. [2 [^[*Mary swimming*] *or*$_2$ ^[~~*Mary*~~ *dancing*]]]
b. $\lambda p^{st}.\ [\![or_2]\!]^{[2\to p]}([\![\wedge[Mary\ dancing]\,]\!])([\![\wedge[Mary\ swimming]\,]\!]) =$
$\lambda p^{st}.\ p = [\![\wedge[Mary\ dancing]\,]\!] \vee p = [\![\wedge[Mary\ swimming]\,]\!]$

(17) a. [2 [^*swimming or*$_2$ ^*dancing*]]
b. $\lambda Q^{(s,\ et)}.\ [\![or_2]\!]^{[2\to Q]}([\![\wedge dancing]\!])([\![\wedge swimming]\!]) =$
$\lambda Q^{(s,\ et)}.\ Q = [\![\wedge dancing]\!] \vee Q = [\![\wedge swimming]\!]$

These LFs may be embedded in larger LFs. For example, (9a) has the LF (18a), which embeds the sub-LF (16a) and where $\exists$ serves as the "closer" of the disjunction.

(18) a. $\exists$ 2 [^[*Mary swimming*] *or*$_2$ ^[~~*Mary*~~ *dancing*]]
b. $[\![\exists]\!]^w(\lambda p^{st}.\ p = [\![\wedge[Mary\ swimming]\,]\!] \vee p = [\![\wedge[Mary\ dancing]\,]\!]) =$
1 iff $\{p|\ p(w) = 1 \wedge (p = [\![\wedge[Mary\ swimming]\,]\!] \vee p = [\![\wedge[Mary\ dancing]\,]\!])\} \neq \varnothing$
iff $[\![dancing]\!]^w(\mathbf{m}) = 1 \vee [\![swimming]\!]^w(\mathbf{m}) = 1$

Similarly, the strict VP reading of (12) has the LF in (20a), where *if*^{1} is the universal quantifier over worlds defined in (19) (Acc_w is the set of worlds accessible from w). Accordingly, the sub-LF [2 ^[t_3 *swimming*] *or*$_2$ ^[t_3 *dancing*]] (cf. (16a)) is $\exists$-"closed" and the index of t_3—the trace of *Mary/Sue* left behind by Quantifier Raising—is abstracted over above $\exists$ (see (20b)). The intensions of the antecedent sub-LF *Mary* [3 [$\exists$ 2 [^t_3 *swimming or*$_2$ ^t_3 *dancing*]]] and the consequent sub-LF *Sue* [3 [$\exists$ 2 [^[t_3 *swimming*] *or*$_2$ ^[t_3 *dancing*]]]] are the st-arguments of *if*^{1} (see (20c)).

(19) For any q and p of type st:
$[\![if^1]\!]^w(q)(p)=1$ iff $\{w'|\ w' \in Acc_w \wedge q(w')=1\}\subseteq\{w'|\ p(w')=1\}$.

(20) a. LF of strict VP:

if^1 ^[*Mary* [3 [$\exists$2 [^[t_3 *swimming*] or_2 ^[t_3 *dancing*]]]]]

^[*Sue* [3 [$\exists$2 [^[t_3 ~~*swimming*~~] ~~or_2~~ ^[t_3 ~~*dancing*~~]]]]]

b. $[\![3\ \exists 2\ [\hat{}[t_3\ \textit{swimming}]\ or_2\ \hat{}[t_3\ \textit{dancing}]]\ ]\!]^{w'} =$

$\lambda x.\ [\![\textit{swimming}]\!]^{w'}(x) = 1 \vee [\![\textit{dancing}]\!]^{w'}(x) = 1$

c. $[\![if^1]\!]^w(\lambda w'.\ [\![\textit{swimming}]\!]^{w'}(\mathbf{m}) = 1 \vee [\![\textit{dancing}]\!]^{w'}(\mathbf{m}) = 1)(\lambda w'.\ [\![\textit{swimming}]\!]^{w'}(\mathbf{s}) = 1 \vee [\![\textit{dancing}]\!]^{w'}(\mathbf{s}) = 1) = 1$ iff for all $w' \in Acc_w$ such that $([\![\textit{swimming}]\!]^{w'}(\mathbf{m}) = 1 \vee [\![\textit{dancing}]\!]^{w'}(\mathbf{m}) = 1)$: $[\![\textit{swimming}]\!]^{w'}(\mathbf{s}) = 1 \vee [\![\textit{dancing}]\!]^{w'}(\mathbf{s}) = 1$

The sloppy VP reading of (12) has the LF in (22), where if^2 is the universal quantifier over world-property pairs defined in (21) and the sub-LF [2 [^*swimming* or_2 ^*dancing*]] (see (17a)) is one of the arguments of *Op*. The extension of the antecedent sub-LF [*Mary Op* [2 [^*swimming* or_2 ^*dancing*]]] is (the characteristic function of) a set of properties, as is the extension of the consequent sub-LF [6 [*Sue* $^{\vee}$*pro*$_6$]] (see (22b) and (22c)). Their intensions are the (s, ((s, et), t))-arguments of if^2 (see (22d)).

(21) For any $\mathscr{Q}$ and $\mathscr{P}$ of type (s, ((s, et), t)):
$[\![if^2]\!]^w(\mathscr{Q})(\mathscr{P})=1$ iff $\{(w', P) |\ w' \in Acc_w \wedge \mathscr{Q}(w')(P)=1\} \subseteq \{(w', P) |\ \mathscr{P}(w')(P)=1\}$.

(22) a. LF of sloppy VP:

if^2 ^[*Mary Op* [2 [^*swimming* or_2 ^*dancing*]]] ^[6 [*Sue* $^{\vee}$*pro*$_6$]]

b. $[\![\textit{Mary Op}\ [2\ [\hat{}\textit{swimming}\ or_2\ \hat{}\textit{dancing}]]]\!]^{w'} =$

$\lambda Q^{(s,et)}.\ Q(w')(\mathbf{m})=1 \wedge (Q= [\![\hat{}\textit{swimming}]\!] \vee Q= [\![\hat{}\textit{dancing}]\!])$

c. $[\![6\ [\textit{Sue}\ {}^{\vee}\textit{pro}_6]]\!]^{w'} =$

$\lambda Q^{(s,et)}.\ Q(w')(\mathbf{s})=1$

d. $[\![if^2]\!]^w(\lambda w'.\ \lambda Q^{(s,et)}.\ Q(w')(\mathbf{m})=1 \wedge (Q=[\![\hat{}\textit{swimming}]\!] \vee Q= [\![\hat{}\textit{dancing}]\!]))(\lambda w'.\ \lambda Q^{(s,et)}.\ Q(w')(\mathbf{s})=1) = 1$ iff for all (w', P) such that $w' \in Acc_w \wedge P(w')(\mathbf{m})=1 \wedge (P=[\![\hat{}\textit{swimming}]\!] \vee P= [\![\hat{}\textit{dancing}]\!])$: $P(w')(\mathbf{s})=1$

Notice that it is the lack of existential closure in (22a) that allows if^2 to universally quantify over the properties in the set $\{Q |\ Q(w')(\mathbf{m})=1 \wedge Q= [\![\hat{}\textit{swimming}]\!] \vee Q= [\![\hat{}\textit{dancing}]\!]\}$. This is not possible when if^2 embeds a sub-LF where or_2 is $\exists$-"closed", as in if^2 ^[*Mary* [3 [$\exists$ 2 [^[t_3 *swimming*] or_2 ^[t_3 *dancing*]]]]] ^[6 [*Sue* $^{\vee}$*pro*$_6$]], which is uninterpretable due to a type mismatch.[5]

[5] The mismatch could, in principle, be overcome by type-shifting. The derived reading (for all (w', Q) such that $w' \in Acc_w$ and Mary is swimming or dancing in w': Sue is Qing in w'), which is false unless there is no accessible world where Mary is swimming or dancing, would be ruled out by general pragmatic principles.

Given the existence of sloppy VP readings of disjunctive conditionals, we expect there to be other disjunctive constructions where the disjunction is not $\exists$-"closed". 'Alternative' interrogatives, to which we now turn, seem to be such constructions.

Following Hamblin (1973), we assume that the intension of an interrogative LF is a question—that is, a function that maps each world w to (the characteristic function of) a set of propositions (intuitively, the possible answers in w). For example, the intension of the LF of the constituent interrogative *Who danced?* is the question that maps every world w to the following function:

(23) $\lambda p^{st}.\ \{x|\ [\![person]\!]^{w}(x)=1 \wedge p= [\![{}^\wedge[t_4\ danced]]\!]^{[4\to x]}\} \neq \varnothing$

An 'alternative' interrogative such as (24a) contains *or*, and its canonical pronunciation places a high pitch accent on some position in each disjunct (in this case, on *swim*- and on *dan*-) and ends with a falling final boundary tone (in this case, on *-cing*).[6] The issuer of (24a) expects the reply to be among {Mary is swimming, Mary is dancing}. We take the LF of (24a) to be an "open" disjunction; specifically, it is the sister of $\exists$ in (18a). Disjunction-induced ellipsis-under-identity yields the reduced 'alternative' interrogative in (24b), which has the same LF as (24a). In every world w, the extension of that LF is the function in (24c) (=(16b)).[7]

(24) a. Is Mary swimming$_{H*}$ or is she dancing$_{H*L\text{-}L\%}$
LF: [2 [^[*Mary swimming*] *or*$_2$ ^[*Mary dancing*]]]
b. Is Mary swimming$_{H*}$ or dancing$_{H*L\text{-}L\%}$
LF: [2 [^[*Mary swimming*] *or*$_2$ ^[~~*Mary*~~ *dancing*]]]
c. $\lambda q^{st}.\ q = [\![{}^\wedge[Mary\ swimming]\ ]\!] \vee q = [\![{}^\wedge[Mary\ dancing]\ ]\!]$

Other theories of 'alternative' interrogatives treat them as "open" disjunctions; see Biezma and Rawlins (2015) for a useful survey. What is new about the current proposal is the explicit claim that an "open" disjunction—with no meaningful

[6] We limit our discussion to canonical pronunciations. Variations on the canonical pronunciation of 'alternative' interrogatives are attested, but all pronunciations share a falling final boundary tone (see Bartels 1999; Biezma and Rawlins 2012, 2015; Pruitt and Roelofsen 2013).

[7] It follows from this analysis, and from the assumption that *or* can take more than two disjuncts (see (13'), the generalized counterpart of (13), in Appendix 1), that *"*Is Mary singing swimming or dancing*"*, which can be pronounced with more than one 'alternative' prosodic pattern, supports more than one 'alternative' reading.

(i) a. Is Mary singing$_{H*}$ (or) swimming$_{H*}$ or dancing$_{H*L\text{-}L\%}$
{Mary is singing, Mary is swimming, Mary is dancing}
b. Is Mary singing or swimming$_{H*}$ or dancing$_{H*L\text{-}L\%}$
{Mary is singing or swimming, Mary is dancing}
c. Is Mary singing$_{H*}$ or swimming or dancing$_{H*L\text{-}L\%}$
{Mary is singing, Mary is swimming or dancing}

In (i.a) the three disjuncts form an "open" disjunction. In (i.b), the first two disjuncts are $\exists$-"closed"; the derived disjunct and the third form an "open" disjunction. In (i.c), the last two disjuncts are $\exists$-"closed"; the derived disjunct and the first form an "open" disjunction. Similarly, "If Mary is singing or dancing or swimming then Sue is" can be pronounced with more than one prosodic pattern, supporting more than one sloppy VP reading.

question word—is the only source of 'alternative' questions; no meaningful question word ever directly manipulates an "open" disjunction to yield a question meaning. This is supported by the fact, observed in Han and Romero (2004, fn. 14), that an overt *whether* appears twice in a non-reduced 'alternative' interrogative embedded under *wonder* (25b), as opposed to once in its reduced variant (25a).

(25) a. John wondered whether Mary is swimming$_{\mathrm{H*}}$ or dancing$_{\mathrm{H*L\text{-}L\%}}$.
b. John wondered whether Mary is swimming$_{\mathrm{H*}}$ or whether she is dancing$_{\mathrm{H*L\text{-}L\%}}$.

On the current proposal the question word *whether*, like subject-auxiliary inversion in (24), has no meaning and fulfills only a morpho-syntactic role (whatever this role might be). Accordingly, (25a) and (25b) have the same meaning (like (24a) and (24b)).

There are two conceivable versions of this hypothesis. The strong version says that the only question words in natural language are *wh*-words such as *who/which*, so even a polar interrogative such as (26a) and a disjunctive polar interrogative such as (27a)—whose canonical pronunciations have final rising intonation—are, underlyingly, special 'alternative' interrogatives whose possible answers are of the form {p, ¬p}.

(26) a. Is Mary dancing$_{\mathrm{L*H\text{-}H\%}}$
b. Possible answers: {Mary is dancing, Mary is not dancing}
c. LF: [2 [^[*Mary dancing*] *or*$_2$ ^[*not Mary dancing*]]]

(27) a. Is Mary dancing or swimming$_{\mathrm{L*H\text{-}H\%}}$
b. Possible answers: {Mary is dancing or swimming, Mary is neither dancing nor swimming}
c. LF: [3 [^[*∃* 2 [^[*Mary dancing*] *or*$_2$ ^[*Mary swimming*]]] *or*$_3$ ^[*not ∃* 2 [^[*Mary dancing*] *or*$_2$ ^[*Mary swimming*]]]]]

A weaker version of this hypothesis still says that "open" disjunctions are the only source for 'alternative' questions, but does not treat polar interrogatives such as (26a) and (27a) as 'alternative' interrogatives. Instead, a designated question operator may yield a polar question from *Mary dancing* and from *Mary dancing or swimming*. The current proposal is compatible with both the weak and strong versions.[8]

The following concern might arise regarding any version of the current proposal. We might expect a "higher" *∃* to derive the 'Someone danced'-meaning in (28) for

[8] See Guerzoni and Sharvit (2014) for an argument for the view that polar interrogatives are underlyingly 'alternative' interrogatives. Guerzoni and Sharvit's theory of 'alternative' interrogatives is inspired by Larson (1985) and Han and Romero (2004); it assumes a designated 'alternative' question-forming word (*whether*) and takes questions to be functions from worlds to sets of true propositions (as in Karttunen 1977). In such a framework, the proposition-level disjunctive word and existential closure are as in (i)–(ii) (cf. Rooth and Partee 1982; see Appendix 2).

(i) $[\![or^{\spadesuit}_{k}]\!]^{w,g}(p_1)(p_2) = 1$ iff $g(k)(w)=1 \wedge (g(k) = p_1 \vee g(k) = p_2)$
(ii) $[\![\exists^{\spadesuit}]\!](X)=1$ iff $\{Z \mid X(Z)=1\} \neq \emptyset$

Who danced (cf. (18)), in addition to the meaning in (23), yet *Who danced* lacks the meaning in (28).

(28) $[\![\exists]\!]^{w}(\lambda p^{st}.\ \{x|\ [\![person]\!]^{w}(x)=1 \wedge p=[\![{}^{\wedge}[t_4\ danced]]\!]^{[4\to x]}\}\neq\varnothing)$
$= 1$ iff $\{x|\ [\![person]\!]^{w}(x)=\ [\![danced]\!]^{w}(x)=1\}\neq\varnothing$

While there is no simple solution to this puzzle, it arises in other frameworks too. Yet it is well known that many languages do use the same word to express *who*-interrogatives and their corresponding existential declaratives (see Kratzer and Shimoyama 2002 and references therein).

2.2 Presupposition projection in disjunctions

Recall that presuppositions project from various disjunctive constructions, subject to the K–P effect. The relevant examples are repeated below.

(29) a. John ate the cake or (he ate) the candy.
b. Did John eat the cake$_{H*}$ or (did he eat) the candy$_{H*L\text{-}L\%}$
c. If John ate the cake or (he ate) the candy, then he is not hungry.

(30) a. Either Jack has no children or his children are away.
b. Does Jack have$_{H*}$ no children or are his children away$_{H*L\text{-}L\%}$
c. If Jack is (either) childless or ashamed of his children, he won't admit that his house is always empty.

These projection facts are accounted for by the assumption that *or*, *∃*, and *if* are presuppositional.

Let us start with *if*. We assume that if^1 has the presuppositional meaning in (31), adapted from Heim (1992). This meaning is inspired by Stalnaker (1975) and Karttunen (1973, 1974). Accordingly, 'if q then p' asserts that the closest-to-w q-worlds are p-worlds, presupposes that the presuppositions of q are satisfied in w, and may "pass up" some presuppositions of p not entailed by q. The former presupposition is expressed by (31.i); the latter is expressed by (31.ii). The second presupposition and the assertion refer to the 'similarity function' SIM: SIM(w)(∅) is undefined, and for any set of worlds X≠∅, SIM(w)(X)={w'| w' ∈ X, and w' resembles w no less than any w''≠w' such that w'' ∈ X}.[9]

(31) For any q and p of type st, $[\![if^1]\!]^{w}(q)(p) \in \{1, 0\}$ iff:
(i) $q(w) \in \{1, 0\}$, and
(ii) $SIM(w)(\{w'|\ q(w')=1\})\subseteq\{w'|\ p(w') \in \{1, 0\}\}$.
If $[\![if^1]\!]^{w}(q)(p) \in \{1, 0\}$, $[\![if^1]\!]^{w}(q)(p)=1$ iff $SIM(w)(\{w'|\ q(w')=1\})$ $\subseteq\{w'|\ p(w')=1\}$.

[9] The Limit Assumption (Lewis 1973) guarantees that there is such a set. The context parameter that fixes the similarity relation is implicit. For the generalized counterpart of (31), which subsumes a presuppositional version of if^2, see (31') in Appendix 1.

The presupposition in (31.i) accounts for the fact that the antecedent has satisfied presuppositions. For example, *If John's mother is at home, he is happy* presupposes that John has a mother. The presupposition in (31.ii) accounts for the context-dependency of presupposition filtering (see Karttunen 1973, 1974; Karttunen and Peters 1979; Heim 1983; Geurts 1996, 1999). Consider *If John is a scuba diver, he will bring his wetsuit*. In a world where scuba divers are obliged to have wetsuits, the sentence may be felicitous even if John does not have a wetsuit as long as the closest worlds where John is a scuba diver are worlds where he has a wetsuit. In a world where no relevant laws prevent John from being a scuba diver who fails to own a wetsuit, *If John is a scuba diver, he will bring his wetsuit* presupposes that John has a wetsuit (or else there would be some closest worlds where he is a scuba diver and fails to own a wetsuit).

We propose that the proposition-level variant of or_k has the presuppositional meaning in (32), where the second presupposition is a conditional presupposition which, like (31.ii), is stated in terms of SIM. It amounts to the following: "If p_1 is infelicitous in w then its felicity is guaranteed in the non-p_2 worlds closest to w, and if p_2 is infelicitous in w then its felicity is guaranteed in the non-p_1 worlds closest to w".[10]

(32) For any p_1 and p_2 of type st, $[\![or_k]\!]^{w,g}(p_1)(p_2) \in \{1, 0\}$ iff:

a. $g(k)(w) \in \{1, 0\}$, and

b. $(p_1(w) \in \{1, 0\} \lor SIM(w)(\{w' | p_2(w')=0\}) \subseteq \{w' | p_1(w') \in \{1, 0\}\}) \land$ $(p_2(w) \in \{1, 0\} \lor SIM(w)(\{w' | p_1(w')=0\}) \subseteq \{w' | p_2(w') \in \{1, 0\}\})$

If $[\![or_k]\!]^{w,g}(p_1)(p_2) \in \{1, 0\}$, $[\![or_k]\!]^{w,g}(p_1)(p_2)=1$ iff $g(k)=p_1 \lor g(k)=p_2$.

Accordingly, an 'alternative' interrogative—namely, an "open" disjunction—has a universal presupposition modulo the K–P effect. Let us assume that a question Q can be issued in an utterance context c only if the set of possible answers to Q is non-empty in c. Since we assume that Q is a Hamblin question intension, this amounts to the requirement that for all w in the common ground of c (the set of worlds compatible with the shared beliefs of the discourse participants of c), $\{p | Q(w)(p)=1\} \neq \varnothing$. Suppose all the worlds in the common ground are like our world, in the sense that they are more or less "normal" (for example, no relevant law or principle in w derives the existence of candy from John not eating the cake, or the existence of a unique cake from John not eating the candy, and Jack's children, if he has any, can in principle be away). It follows that (i) (33a) presupposes that there is both cake and candy, and (ii) (34a) does not presuppose that Jack has children.

[10] For the presuppositional property-level disjunctive connective, as well as a generalized counterpart, see Appendix 2.

(33) a. Did John eat the cake$_{H*}$ or the candy$_{H*L-L\%}$

b. [2 [^[*John ate the cake*] *or*$_2$ ^[~~*John ate*~~ *the candy*]]]

(34) a. Does Jack have$_{H*}$ no children or are his children away$_{H*L-L\%}$

b. [2 [^[*Jack has no children*] *or*$_2$ ^[*Jack's children are away*]]]

By the meaning of *or*$_k$, and given the nature of the worlds in the common ground, *John ate the cake* and *John ate the candy* must have satisfied presuppositions. Accordingly, (35) holds of any w in the common ground.

(35) a. $[\![(33b)]\!]^w = \lambda p: w \in Dom(p) \wedge$
$w \in Dom([\![\wedge[\textit{John ate the cake}]]\!]) \wedge$
$w \in Dom([\![\wedge[\textit{John ate the candy}]]\!])$.
$p = [\![\wedge[\textit{John ate the cake}]]\!] \vee p = [\![\wedge[\textit{John ate the candy}]]\!]$

b. $\{p | [\![(33b)]\!]^w(p) = 1\} \neq \emptyset$ iff
$\{p | p(w) \in \{1, 0\}\} \supseteq \{[\![\wedge[\textit{John ate the cake}]]\!], [\![\wedge[\textit{John ate the candy}]]\!]\}$

On the other hand, the closest worlds to w where *Jack has no children* is false are worlds where the presuppositions of *Jack's children are away* are true. Accordingly, (36) holds of any w in the common ground.

(36) a. $[\![(34b)]\!]^w = \lambda p: w \in Dom(p) \wedge$
$w \in Dom([\![\wedge[\textit{Jack has no children}]]\!])$.
$p = [\![\wedge[\textit{Jack's children are away}]]\!] \vee p = [\![\wedge[\textit{Jack has no children}]]\!]$

b. $\{p | [\![(34b)]\!]^w(p) = 1\} \neq \emptyset$ iff $\{p | p(w) \in \{1, 0\}\} \supseteq \{[\![\wedge[\textit{Jack has no children}]]\!]\}$

'Alternative' interrogatives under *wonder* project their presuppositions locally, subject to the K–P effect. Thus, if the worlds compatible with Bill's beliefs resemble our world in the relevant sense, (37a) presupposes—by the meaning of *wonder*—that Bill believes that there was cake and that there was candy, and (37b) does not presuppose that Bill believes that Jack has children.

(37) a. Bill wonders whether John ate the cake$_{H*}$ or the candy$_{H*L-L\%}$.

b. Bill wonders whether Jack has$_{H*}$ no children or whether his children are away$_{H*L-L\%}$.

Recall that in our system, the sister of *∃* is an interrogative LF (see (18)). Assuming that the presuppositional meaning of *∃* is as in (38), this amounts to requiring that the interrogatives embedded in (39b) and (40b) have a non-empty set of possible answers. Assuming that a common ground can be updated with a proposition q only if for every w in the common ground, q(w) ∈ {1, 0}, (39a) presupposes that there is cake and candy, but (40a) does not presuppose that Jack has children. By the meaning of *believe*, those presuppositions are relativized to the worlds compatible with Bill's beliefs in (41a,b).[11,12]

(38) For any X of type (st, t): $[\![\exists]\!]^w(X) \in \{1, 0\}$ iff $\{p|X(p)=1\} \neq \emptyset$.
If $[\![\exists]\!]^w(X) \in \{1, 0\}$, $[\![\exists]\!]^w(X)=1$ iff $\{p|\, p(w)=X(p)=1\} \neq \emptyset$.

(39) a. John ate the cake or the candy.
b. $\exists$ 2 [^[*John ate the cake*] *or*$_2$ ^[~~*John ate*~~ *the candy*]]
c. $[\![(39b)]\!]^w \in \{1, 0\}$ iff $\{p|\, p(w) \in \{1, 0\}\} \supseteq \{[\![$^[*John ate the cake*] $]\!]$, $[\![$^[*John ate the candy*] $]\!]\}$.

(40) a. (Either) Jack has no children or his children are away.
b. $\exists$ 2 [^[*Jack has no children*] *or*$_2$ ^[*Jack's children are away*]]
c. $[\![(40b)]\!]^w \in \{1, 0\}$ iff $\{p|\, p(w) \in \{1, 0\}\} \supseteq \{[\![$^[*Jack has no children*]$]\!]\}$

(41) a. Bill believes that John ate the cake or the candy.
b. Bill believes that either Jack has no children or his children are away.

It is instructive to note that an alternative meaning of *or*, according to which it has a SIM-less conditional presupposition $((p_1(w)=1 \vee p_2(w) \in \{1, 0\}) \wedge (p_2(w)=1 \vee p_1(w) \in \{1, 0\}))$ (cf. Karttunen and Peters (1979)), fails to account for the context-dependency of filtering. For example, it incorrectly predicts (33a) and (39a) to merely presuppose that there is cake or candy, even in a context where not eating the cake has no effect on the existence of candy (and not eating candy has no effect on the existence of a cake). Similarly, it incorrectly predicts that *Either Jack has no children or his children are with his assistant*—and its corresponding 'alternative' interrogative—need not presuppose that Jack has an assistant even in a context where having children has no effect on having an assistant (whereas by (32), whether the presupposition that Jack has an assistant is filtered out depends on whether Jack can, in principle, be a parent and fail to have an assistant). In addition, the alternative SIM-less conditional presupposition $((p_1(w) \in \{1, 0\} \vee \{w'|\, p_2(w')=0)\} \subseteq Dom(p_1)) \wedge (p_2(w) \in \{1, 0\} \vee \{w'|\, p_1(w')=0)\} \subseteq Dom(p_2)))$ incorrectly predicts that *Either Jack has no children or his children are with his assistant*—and its corresponding 'alternative' interrogative—presuppose that Jack has

[11] If the LF of the polar *Did John eat the cake or the candy*$_{L*H\text{-}H\%}$ is (i) (see Sect. 2.1), the fact that it presupposes that there is cake and candy is also accounted for by the presuppositions of *or* and $\exists$.

(i) [6 [^$\exists$ 5 [^[*John ate the cake*] *or*$_5$ ^[~~*John ate*~~ *the candy*]]] ~~*or*~~$_6$
^[~~*not*~~ [^$\exists$ 5 [^[~~*John ate the cake*~~] ~~*or*~~$_5$ ^[~~*John ate the candy*~~]]]]]

[12] Note that *(Either) 3 equals 3 or Jack's children are away* is correctly predicted to be infelicitous unless Jack has children because SIM(w)($\emptyset$) is undefined. Yet the conditional presupposition in (32) does not suffice to account for (i)–(ii). Explaining (i)–(ii), along with their corresponding 'alternative' interrogatives, conditionals, and related facts observed in Hurford (1974) requires reference to pragmatic constraints.

(i) #Either Jack has children or his children are away.
(ii) Jack has no daughters. Either he (also) has no sons, or his sons/#children are away.

children (and enforces the presupposition that Jack has an assistant regardless of the context).

The K–P effect of disjunctive conditionals is predicted by the meaning of if^1 in (31) and the meaning of *or* in (32). The former projects the presuppositions of the disjunctive antecedent, and so (29c) presupposes that there is cake and candy and (30c) does not presuppose that Jack has children. (See Appendix 2 for an account of K–P effects in strict and sloppy readings of disjunctive conditionals with VP-ellipsis in the consequent.)

To sum up so far, the conditional presupposition of or_k in (32), together with the assumption that 'alternative' interrogatives are "open" disjunctions, explains why 'alternative' interrogatives universally project the presuppositions of their possible answers (modulo the K–P effect), as do other disjunctive constructions.

Let us briefly consider the following contrasting hypothesis regarding 'alternative' interrogatives: they are formed by a question operator which applies to (the intension of) an "open" disjunction. What would such an operator encode? It stands to reason that it would impose a presupposition that accounts for the fact that the interrogative in (42a) intuitively presupposes that Mary is either swimming or dancing, but not both. The intuition consists in the fact that R1 is a reply that the issuer of the interrogative expects to receive, but R2 and R3 are not (see, for example, Karttunen and Peters 1976; Bartels 1999; Biezma and Rawlins 2012). R2 implies that $\{p|\ p(w)= [\![2\ [{}^\wedge[\textit{Mary swimming}]\ or_2\ {}^\wedge[\textit{Mary dancing}]]]\!](p)=1\}=\varnothing$ for any relevant w, and R3 implies that $\{p|\ p(w)= [\![2\ [{}^\wedge[\textit{Mary swimming}]\ or_2\ {}^\wedge[\textit{Mary dancing}]]]\!](p)=1\}=\{\ [\![{}^\wedge[\textit{Mary swimming}]]\!],\ [\![{}^\wedge[\textit{Mary dancing}]]\!]\ \}$. (42b) shows that R2 and R3 are blocked when relativized to the subject of *wonder*.

(42) a. Int: Is Mary swimming$_{H*}$ or (is she) dancing$_{H*L\text{-}L\%}$
R1: She is swimming.
R2: ?She is neither swimming nor dancing.
R3: ?She is doing both – swimming and dancing.
b. #John thinks that it's possible that Mary is neither swimming nor dancing/ both swimming and dancing, and he is wondering whether she is swimming$_{H*}$ or dancing$_{H*L\text{-}L\%}$

Suppose the facts in (42) are accounted for by the 'alternative'-question forming operator *Alt*, which imposes the presupposition that there is one possible true answer that entails all other possible true answers:

(43) $$[\![Alt]\!]^w=\lambda Q^{(s,(st,t))}: \{p|\ p(w)=Q(w)(p)=1 \wedge \{q|\ q(w)=Q(w)(q)=1\}\subseteq\{q|\ p=>q\}\}\neq\varnothing.\ Q(w)$$

Alt has the virtue of "passing up" the presuppositions of *or*. However, given (25), and since all disjunctive constructions—and not just 'alternative' interrogatives—project the presuppositions of all their disjuncts (modulo K–P), it seems more explanatory to treat 'alternative' interrogatives as open disjunctions and account for the facts in (42) by appealing to more general principles. Given this, we adopt the felicity condition in (44a) (where cg_c is the common ground of c), and the presupposition of *wonder* in (44b) (where $BEL_{w,x}$ is the set of worlds compatible with what x believes in w). Both refer to the answerhood operator Ans in (44c), which is independently motivated by *wh*-interrogatives (see Dayal 1996) and "passes up" the presuppositions of *or*.

(44) For any question Q:

a. $\{c|\ c$ is an utterance context and Q is issuable in the world of $c\} \subseteq \{c|\ cg_c \subseteq \mathrm{Dom}(\mathrm{Ans}(Q))\}$.

b. For any individual x, $\{w|\ [\![wonder]\!]^w(Q)(x) \in \{1, 0\}\} \subseteq \{w|\ BEL_{w,x} \subseteq \mathrm{Dom}(\mathrm{Ans}(Q))\}$.

c. $\mathrm{Ans}(Q) =$
$\lambda w\colon \{p|\ p(w)=Q(w)(p)=1 \wedge \{q|\ q(w)=Q(w)(q)=1\} \subseteq \{q|\ p=>q\}\} \neq \varnothing.$
the p such that $p(w)=Q(w)(p)=1 \wedge \{q|\ q(w)=Q(w)(q)=1\} \subseteq \{q|\ p=>q\}$

The facts in (42) are thus accounted for, together with the universal projection of presuppositions (modulo K–P) in all disjunctive constructions.[13]

To sum up, while the current proposal does not explain why natural language disjunction has the conditional presupposition in (32) (any more than the theory in Heim (1983, 1992) explains why connectives have the presuppositions that they are claimed to have), it does explain why all disjunctive constructions—including 'alternative' interrogatives—have universal presuppositions (modulo K–P). In Sect. 3 we discuss some potential counterexamples and address the issue of how presuppositions generally project from interrogative clauses.

3 Potential counterexamples

Some disjunctive clauses do not behave as expected given the proposal in Sect. 2. We divide these examples into the following groups: Group I consists of disjunctions with special presupposition triggers, Group II consists of disjunctive interrogatives with a special intonation pattern, and Group III consists of interrogatives whose behavior can only be understood within a general theory of the presuppositions of interrogatives.

3.1 Group I

There are acceptable disjunctions—interrogative as well as non-interrogative—where the conditional presupposition of *or* in (32) appears to be bluntly violated. Their disjuncts contain verbs such as *stop* and definite descriptions such as *the king*.

[13] According to Biezma and Rawlins (2012), the disjuncts in an 'alternative' interrogative are determined by the alternatives set up by the Question-under-Discussion (QUD). In the Ans-based system, the QUD requirement can be made part of (44a).

For example, (45) below, taken from Hausser (1976), is expected—given the conditional presupposition of *or*—to be infelicitous due to the contradiction between fermenting in the past and not fermenting in the past; and (46), from Beaver (2001), is expected to be infelicitous due to the (pragmatic) oddity of having both a king and a president. Yet they can both be felicitous. The corresponding disjunctive conditionals and 'alternative' interrogatives have the same intuitive global presuppositions as (45) and (46), respectively.

(45) The liquid of this tank has either stopped fermenting or it has not yet begun to ferment.
First disjunct presupposes: The liquid was fermenting, in the past.
Second disjunct presupposes: The liquid was not fermenting, in the past.
In addition, the closest worlds where the liquid has begun to ferment are worlds where the liquid was not fermenting, and the closest worlds where the liquid has not stopped fermenting are worlds where the liquid was fermenting.
Intuitive global presupposition: There is liquid.

(46) Either the King of Buganda is now opening parliament, or the President of Buganda is.
First disjunct presupposes: Buganda has a (unique) king.
Second disjunct presupposes: Buganda has a (unique) president.
In addition, the closest worlds where the President of Buganda is not opening parliament are worlds where Buganda has a president, and the closest worlds where the King of Buganda is not opening parliament are worlds where Buganda has a king.
Intuitive global presupposition: Buganda has a king or a president.

(47) a. If the liquid of this tank has stopped fermenting or has not yet begun to ferment, then we should use another tank.
b. If the King of Buganda is opening parliament or the President of Buganda is, then the other speeches will be delivered the following day.

(48) a. Has the liquid of this tank stopped$_{H*}$ fermenting or has it not yet begun$_{H*L\text{-}L\%}$ to ferment
b. Is the King$_{H*}$ of Buganda opening parliament or is the President$_{H*L\text{-}L\%}$ of Buganda

Upon more careful reflection, judgments regarding (45)–(48) are not inconsistent with the conditional presupposition of *or*, because the presupposition triggers in (45)–(48) are special (see also Zehr et al. 2017). The presupposition triggers in (45) are "soft" presupposition triggers in the sense of Abusch (2002, 2010); the definite noun phrases in (46) are 'role'-definites, which typically "pick out" an individual who holds a position held—due to social convention—by at most one individual at any given time. "Soft" presuppositions are easily cancellable, as shown in (49): *stop* is a "soft" presupposition trigger, but the emotive factive verb *regret* is a "hard" presupposition trigger; the latter projects its presuppositions from under negation/disjunction. The same contrast arises with conditionals and 'alternative' interrogatives, as shown in (50) and (51).

(49) a. I don't think John stopped smoking; in fact, he never smoked.
b. #I don't think Bill regrets going to grad school; in fact, he never went to grad school.
c. #Either Bill regrets going to grad school or he regrets turning down a job on Wall Street; I can't remember if he went to grad school or turned down a job on Wall Street.

(50) a. If John stopped smoking or doing drugs, we can hire him. I can't remember if he used to smoke or do drugs.
b. If John regrets inviting Bill or Fred, we should cancel the meeting. #I can't remember which of these guys he invited.

(51) a. Did John stop smoking$_{H*}$ or doing drugs$_{H*L\text{-}L\%}$
I can't remember if he smoked or did drugs.
b. Does John regret inviting Bill$_{H*}$ or inviting Fred$_{H*L\text{-}L\%}$
#I can't remember which of these guys he invited.

As for 'role'-definites, their existence presuppositions need not project from the predicate position of a negated copular sentence, as shown in (52) (see Halliday 1967, Fodor 1970, Higgins 1973, and others), but they do project from the non-predicate position. Noun phrases that are typically not 'role'-definites project their existence presupposition from the predicate position.

(52) a. I don't think John Smith is the President. In fact, we don't have a president.
b. I don't think the President is John Smith. #In fact, we don't have a president.
c. I don't think Peter Baldwin is the president whose daughter died yesterday. #In fact, no president's daughter died yesterday.

While many definite noun phrases can—in the right context—acquire the status of a 'role'-definite, this is clearly not automatic, as confirmed by (53), where the definite noun phrases are typically not 'role'-definites and project their presuppositions (cf. (46)), and by the contrasts in (54). Similar contrasts arise with corresponding conditionals and 'alternative' interrogatives.

(53) Either the cab-driver from last night or the beggar from last night greeted me; #I can't remember if there was a cab-driver or a beggar.

(54) a. My best friend is neither the King of Buganda nor the President of Buganda;
– I can't remember if Buganda has a king or a president.
– Buganda is in a state of chaos right now and has no leader.
b. #Neither the King of Buganda nor the President of Buganda is my best friend.

c. Neither the president whose daughter died nor the one whose wife left him is my best friend. #I can't remember whether some president's daughter died or some president's wife left him.
d. My best friend is neither the president whose daughter died nor the one whose wife left him. #I can't remember whether some president's daughter died or some president's wife left him.

(55) a. If the President or the Prime Minister greeted you, then your visit is probably over. I can't remember if this country has a president or a prime minister.
b. If the student you met yesterday greeted you or the student you met this morning greeted you, then you are done. #I can't remember if you met a student yesterday or this morning.

(56) a. Did the President$_{\text{H}*}$ greet you or the Prime Minister$_{\text{H}*\text{L-L}\%}$
I can't remember if this country has a president or a prime minister.
b. Did the student you met yesterday$_{\text{H}*}$ greet you or the student you met this morning$_{\text{H}*\text{L-L}\%}$
#I can't remember if you met a student yesterday or this morning.

Let us use the term "soft triggers" as a cover term for the presupposition triggers in (45)–(46). Accounting for soft triggers is a complicated matter, and different explanations might be needed for different kinds of soft triggers. Regardless, (45)–(56) strongly suggest that the theory of soft triggers is independent of the theory of disjunction. Assuming cancellation in disjunctions is possible in principle, the context described in (57) may favor the inference in (58) over (59), for (46).

(57) The law in Buganda: there is at most one leader.
(58) Buganda has a king or the closest worlds where Buganda has no president who is opening parliament are worlds where it has a king.
(59) Buganda has a king.

We therefore maintain that *or* has the proposed conditional presupposition, which governs the projection of "hard" presuppositions.

3.2 Group II

A non-trivial empirical challenge for the proposal in Sect. 2 is posed by disjunctive interrogatives such as (60a) and (61a), where each disjunct is pronounced with a rising intonation, unlike the disjunctive interrogatives (60b) and (61b), which have the canonical 'alternative' intonation.

(60) a. Is Mary learning French$_{\text{L}*\text{H-H}\%}$ or (is she learning) Italian$_{\text{L}*\text{H-H}\%}$
b. Is Mary learning French$_{\text{H}*}$ or (is she learning) Italian$_{\text{H}*\text{L-L}\%}$

(61) a. Is Mary at her sister's$_{\text{L*H-H\%}}$ or at her mother's$_{\text{L*H-H\%}}$
b. Is Mary at her sister's$_{\text{H*}}$ or at her mother's$_{\text{H*L-L\%}}$

It is claimed in Hoeks and Roelofsen (2019) (cf. Hoeks 2018) that *She is learning neither* is an expected reply to (60a) (though, as we saw, it is not an expected reply to (60b)). On the other hand, *She is not learning French* is not an expected reply to (60a). These facts, together with the fact that (61a) appears to presuppose that Mary has a sister or a mother (unlike (61b) which, in a world with "normal" laws, presupposes that she has both a sister and a mother), challenge both the claim that all interrogatives of the form 'p or q' are simply "open" disjunctions with no question word, and the claim that all interrogatives of the form 'p or q' project their presuppositions universally (modulo K–P).

Within the framework of Inquisitive Semantics, (60a) is analyzed in Hoeks and Roelofsen (2019) as an interrogative generated by a designated question word, which guarantees that the possible answers are {Mary is learning French, Mary is learning Italian, Mary is learning neither French neither French nor Italian}. This analysis indeed predicts *Mary is not learning French* to be an unacceptable reply to (60a), but does not account for the fact that (61a) and (61b) do not have the same presuppositions.

If our hypothesis that all interrogatives of the form 'p or q' are "open" disjunctions with no question word is to be maintained, (60a) and (61a) can only be analyzed as a disjunction of two polar speech/question acts, contra the claim in Krifka (2001) that speech/question acts cannot be disjoined. This is what we propose for (60a) and (61a), and it seems to be supported by the fact that the rising intonation on each disjunct resembles that of an independent polar interrogative (cf. (26a)).

Speech act modifiers can override the prohibition against disjoining speech acts (if indeed there is one; see Hirsch (2017), Hoeks and Roelofsen (2019) and works cited there for some relevant). This is shown, for example, by the acceptability of *Where did you go? Or rather, Who did you see?* (from Szabolcsi 1995). In (62a), the prohibition is overridden by the presence of *alternatively,* which introduces the question *Is Mary learning Italian*$_{\text{L*H-H\%}}$ as an alternative to the question *Is Mary learning French*$_{\text{L*H-H\%}}$, indicating the asker's willingness to prioritize one of the questions and be satisfied with receiving an answer to just one of them. In (62b)—which bears 'alternative' prosody—*alternatively* modifies a proposition-level disjunct (Italian is an alternative to French).

(62) a. Is Mary learning French$_{\text{L*H-H\%}}$ or, alternatively, Italian$_{\text{L*H-H\%}}$
b. Is Mary learning French$_{\text{H*}}$ or alternatively Italian$_{\text{H*L-L\%}}$

We suggest that (60a) and (61a) contain a covert speech act modifier akin to *if not.* Notice the oddity of the *if not*-variant of (64) versus the acceptability of its *if not*-less variant and the acceptability of both variants of (63).

(63) Is Jack childless$_{L^*H\text{-}H\%}$ or(, if not,) are his children away for the summer$_{L^*H\text{-}H\%}$

(64) Are Jack's children away for the summer$_{L^*H\text{-}H\%}$ or(, #if not,) is he childless$_{L^*H\text{-}H\%}$

We take this to indicate that the speech act modifier *if not* is asymmetric; it introduces the possibility that the first "inverted" disjunct—*Jack is childless* in (63), *Jack's children are away* in (64)—is false (since speech acts are discourse-anchored, it comes as no surprise that some speech act modifiers are asymmetric, even if *or* itself is not). We suggest that (60a) and (61a) and the *if not*-less variants of (63) and (64) include, underlyingly, a default, asymmetric speech act modifier that introduces the possibility that the answer to the first interrogative is known (in which case the question is infelicitous) or the possibility that the interrogative is infelicitous or irrelevant for other reasons (e.g., unsatisfied presuppositions of the possible answers). Other speech act modifiers that filter out presuppositions are *alternatively, more relevantly*, *better yet,* and the like.

Accordingly, (60a) and (61a) may be paraphrased as follows: "the first or second question is issued (and it might turn out mid-utterance that the second is more relevant than the first)." This explains the lack of universal projection in (61a). It also explains a judgment reported to us by a *Natural Language Semantics* reviewer, according to which *She is not learning French* is a felicitous answer to (60a) when the context makes it clear that it is the best answer available. Crucially, the prediction regarding 'alternative' interrogatives remains intact: no interrogative with 'alternative' prosody can fail to project the presuppositions of its disjuncts, modulo the K–P effect.

3.3 Group III

This section deals with how the projection of presuppositions from 'alternative' interrogatives relates to the more general question of how presuppositions project from all interrogatives. As it turns out, depending on the context, some interrogatives sometimes project the presuppositions of their possible answers in a non-universal manner.

As mentioned in Sect. 1, it has been claimed that interrogatives in general project the ("hard") presuppositions of all their answers. Thus, the *wh*-interrogative in (65) from Schwarz and Simonenko (2017) presupposes, out of the blue, that all our colleagues have Australian relatives.

(65) Which of our colleagues brought their Australian relatives?

However, (66), also from Schwarz and Simonenko (2017), illustrates a context where the interrogative in (65) is felicitous despite the fact that some possible answers have presuppositions that are not entailed by the common ground.

(66) (i) A: Some of our colleagues brought their Australian relatives.
B: Which of our colleagues brought their Australian relatives?
(ii) Context: A and B agree on who their colleagues are and for each colleague x, B lacks an opinion about whether x has Australian relatives.

Similarly, (67) is felicitous in the context described in (68).

(67) Which of these players does Fred know scored?
(68) (i) A: Crazy Fred is turning into a real problem. Whenever he finds out that one of our players scored a goal, he sends that player a threat.
B: We must protect our players! Which of them does Fred know scored?
(ii) Context: It is common knowledge between A and B that the players are $r_1 \ldots r_n$ ($n > 3$). For each of $r_1 \ldots r_3$, it is common knowledge that they scored. For each of $r_4 \ldots r_n$, it is common knowledge that they did not score.

There are two ways to go from here: abandon the hypothesis that presuppositions of interrogatives project universally, or treat the counterexamples above as special cases, felicitous only in special circumstances. With Schwarz and Simonenko, we opt for the latter, as it seems to be the case that non-universal projection is indeed possible only in special circumstances.

Schwarz and Simonenko assume that *wh*-interrogatives do not have semantic presuppositions of their own—only their possible answers do. Specifically, (65) and (67) have the Hamblin-extensions in (69) and (70), respectively, which are total functions.

(69) $\lambda p.\ \{x|\ [\![\textit{colleague of ours}]\!]^{w}(x)=1 \wedge p=\ [\![{}^{\wedge}[t_4\ \textit{brought}\ t_4\textit{'s Australian relatives}]]\!]^{[4\rightarrow x]}\} \neq \varnothing$
(70) $\lambda p.\ \{x|\ [\![\textit{one of these players}]\!]^{w}(x)=1 \wedge p=\ [\![{}^{\wedge}[\textit{Fred knows}\ {}^{\wedge}[t_4\ \textit{scored}]]]\!]^{[4\rightarrow x]}\} \neq \varnothing$

Schwarz and Simonenko propose that the intuitive presuppositions of *wh*-interrogatives are obtained from a set of pragmatic principles, which we formulate as in (71) (Q is a *wh*-question, c is a context, and cg_c is the common ground of c). When all three principles in (71) are met, universal projection is guaranteed (that is to say, (71a,b,c) entail (72); see Schwarz and Simonenko for the formal proof).

(71) a. $\forall p[cg_c \subseteq \{w|\ Q(w)(p)=1\} \rightarrow (cg_c \subseteq Dom(p) \vee cg_c \cap Dom(p)=\varnothing)]$
b. $\forall p[cg_c \subseteq \{w|\ Q(w)(p)=1\} \rightarrow cg_c \cap Dom(p) \neq \varnothing]$
c. $\forall w,w'[(w \neq w' \wedge w,w' \in cg_c) \rightarrow \{p|\ Q(w)(p)=1\}=\{p|\ Q(w')(p)=1\}]$
(72) $cg_c \subseteq \{w|\ \forall p[p \in Q(w) \rightarrow w \in Dom(p)]\}$

The scenario described in (66) is one where (65) violates (71a), and the scenario described in (68) is one where (67) violates (71b) (see Schwarz and Simonenko for examples of contexts where (71c) is legitimately violated). As Schwarz and Simonenko note, more needs to be said about when these principles can be violated without penalty, but the proposal seems promising as the expectation is still that, in the default case, presuppositions will project universally. As we now show, however, some non-*wh* interrogatives necessarily violate one of the principles in (71), resulting in some unwelcome predictions.

In Sect. 1 we mentioned the view expressed in Guerzoni (2003, 2004) according to which interrogatives are only required to project their presuppositions existentially. Schwarz and Simonenko cite Guerzoni's work as one of the arguments for the need to incorporate pragmatic weakening into the theory of questions. The phenomenon that drives Guerzoni's theory is illustrated by the polar interrogative in (73), which contains *even*. Suppose that the common ground provides that John is least likely to eat the cake (among the available food options). In such a context, a discourse participant who utters (73) need not have any expectations about whether John ate the cake.

(73) Did John even eat the cake$_{\text{L*H-H\%}}$

But (73) sometimes has a different meaning. Suppose the common ground instead provides that the food options are the cake and the candy, that John typically loves the cake, and that he showed up without an appetite. In such a context, a discourse participant who utters (73) expects the reply to be *No*. Let us call the interpretation in the former kind of context "unbiased" and the interpretation in the latter kind of context "negatively-biased".

Following Karttunen and Peters (1979) and Wilkinson (1996), Guerzoni adopts a scope theory of *even*. She assumes that *even* is a focus-sensitive operator that introduces the presupposition that the focus-alternatives to its prejacent are more likely than the prejacent itself.[14] The focus-associate of *even* in an unembedded declarative clause is canonically pronounced with prominence, as in (74a), where the focus-induced prominence is indicated by capital letters.[15]

(74) a. John even ate the CAKE.
b. Focus alternatives: {John ate the cake, John ate the candy, John ate the pizza, …}

Let us, then, adopt the meaning of *even* in (75), according to which *even* takes a domain restrictor as one of its arguments, and assume that the LF of (74a) is (76b), where the domain restrictor of *even* is provided by a pronoun. The value of the restrictor of *even* is determined by the method in Rooth (1992): the restrictor is constrained—via '$\sim$' in (77)—to be a contextually relevant subset of the focus value of *even*'s prejacent (by convention, if X_k is a free pronoun, $[\![X_k]\!]^g \equiv$Xk). In a context where the food options include the cake, and John is allergic to the cake

[14] Greenberg (2018) argues that *even*'s scalar presupposition involves comparison along a contextually-supplied dimension (not necessarily likelihood). Most of our examples involve alternatives that are on contextual rather than logical scales, so the predictions could be reproduced with her lexical entry as well.

[15] In (73), *cake* need not be pronounced with more prominence than in the *even*-less *Did John eat the cake*$_{\text{L*H-H\%}}$. We take this to imply that in (73) the canonical polar pronunciation is sufficient to signal the prominence of the focus-associate of *even*.

(and therefore not likely to eat it), C1$\subseteq${$\underline{[\![{}^\wedge[\textit{John ate the cake}]]\!]}$, $[\![{}^\wedge[\textit{John ate the candy}]]\!]$, …} and (74a) is a felicitous declarative sentence (F-marking does not affect ordinary semantic values and, by convention, underlined items must be in C1).

(75) Where C is a set of propositions, $[\![\textit{even}]\!]^{w}(C)(p) \in \{1, 0\}$ only if:
(i) $C \supset \{p\}$, and
(ii) for all $q \in C$: $q(w) \in \{1, 0\}$ and if $q \neq p$, p is less likely than q in w.
If $[\![\textit{even}]\!]^{w}(C)(p) \in \{1, 0\}$, $[\![\textit{even}]\!]^{w}(C)(p)=1$ iff $p(w)=1$.

(76) a. John even ate the CAKE. (74a)
b. LF: *even*-C_1 [$^\wedge$[*John ate the cake*$_F$] $\sim C_1$]
c. Presupposition: John eating the cake is least likely among the C1-alternatives.

(77) $[\![{}^\wedge\alpha \sim C_k]\!]^g$ is defined only if $Ck \subseteq \{Y|$ there is a β such that: (i) $Y= [\![{}^\wedge\beta]\!]^g$, and (ii) $\beta=\alpha$, or β is just like α except that at least one F-marked node γ in α is replaced
in β with some $\delta \neq \gamma$ such that δ is a type-identical alternative of $\gamma\}$. If defined, $[\![{}^\wedge\alpha \sim C_k]\!]^g = [\![{}^\wedge\alpha]\!]^g$.

The ambiguity of (73) is accounted for as follows. Like Schwarz and Simonenko, Guerzoni (2003, 2004) takes question extensions to be total functions from propositions to truth values. A polar interrogative, according to Guerzoni, does not have a silent *or* in its LF. Rather, the polar meaning is obtained with a designated question-forming word, and the position of *not* in the "negative" possible answer corresponds to the position of the trace of the question-forming word in the interrogative's LF. Thus, the unbiased meaning of (73) is obtained when, in effect, *not* scopes above *even* in the "negative" possible answer, as in (78b). Its negatively-biased meaning is obtained when, in effect, *even* scopes above *not* in the "negative" possible answer, as in (78c). Suppose the food options are the cake and the candy, so C1$\subseteq${$\underline{[\![{}^\wedge[\textit{John ate the cake}]]\!]}$, $\underline{[\![{}^\wedge[\textit{John ate the candy}]]\!]}$, …}, and C2$\subseteq${$\underline{[\![{}^\wedge[\textit{not John ate the cake}]]\!]}$, $\underline{[\![{}^\wedge[\textit{not John ate the candy}]]\!]}$, …}. The unbiased meaning in (78b) may come about when John is allergic to the cake and not likely to eat it. The negatively-biased meaning in (78c) may come about when John loves the cake but showed up without an appetite. In the latter case, only the "negative" answer has a satisfied presupposition in the common ground.[16]

[16] A similar explanation would apply to the negative bias of polar interrogatives with minimizing NPIs such as *lift a finger* (as in *Did John (even) lift a finger?*; see (2) in Sect. 1).

(78) a. Did John even eat the cake$_{\text{L*H-H\%}}$ (73)

b. Unbiased meaning (*not* scopes above *even* in the "negative" answer):
$\lambda p.\, p = [\![{}^\wedge[\textit{even-}C_1\ [{}^\wedge[\textit{John ate the cake}_F] \sim C_1]]]\!]^g \vee$
$p = [\![{}^\wedge[\textit{not even-}C_1\ [{}^\wedge[\textit{John ate the cake}_F] \sim C_1]]]\!]^g$
Presupposition of both possible answers: same as (76c).

c. Negatively-biased meaning (*even* scopes above *not* in the "negative" answer):
$\lambda p.\, p = [\![{}^\wedge[\textit{even-}C_1\ [{}^\wedge[\textit{John ate the cake}_F] \sim C_1]]]\!]^g \vee$
$p = [\![{}^\wedge[\textit{even-}C_2\ [{}^\wedge[\textit{not John ate the cake}_F] \sim C_2]]]\!]^g$
Presupposition of "positive" answer: same as (76c).
Presupposition of "negative" answer: John not eating the cake is the least likely among the C2-alternatives.

The LF where *even* scopes above *not* in the "negative" answer violates (71b).

Now, according to the theory outlined in Sect. 2, 'alternative' interrogatives have definedness conditions of their own (see (32)). This makes (71a,b) non-violable by definition, when Q is an 'alternative' question, and (71c) violable in principle. Let us test the predictions with respect to the 'alternative' interrogative in (79).

(79) Does Trump regret collaborating with foreign leaders$_{\text{H*}}$ or does he regret hiring a spy$_{\text{H*L-L\%}}$

Crucially, (79) is infelicitous in the context in (80).

(80) A: Trump collaborated with foreign leaders, but he didn't hire a spy.
B: OK. #The question on my mind is, Does he regret collaborating with foreign leaders$_{\text{H*}}$ or does he regret hiring a spy$_{\text{H*L-L\%}}$

That (79) does not have an unbiased reading when the common ground entails that Trump did not hire a spy follows straightforwardly from the conditional presupposition of *or* (as an unbiased reading cannot have the same singleton possible answer throughout the common ground). However, we expect there to be contexts where (79) has a biased reading with *Trump collaborated with foreign leaders* as the only possible answer throughout the common ground. But (79) does not have the same kind of biased reading that (78a) has. It can be biased and felicitous in principle (for example, when the common ground entails that Trump collaborated with foreign leaders and hired a spy, and he never regrets hiring anyone), but it cannot be used when one of its disjuncts is undefined throughout the common ground to seek confirmation for the bias (and it is similarly constrained under *wonder*). By contrast, (78a) may be used to seek confirmation for the negative bias (and may be negatively-biased under *wonder*).

One might try to defend the theory by explaining the facts regarding (79) as follows. By the meaning of *or* in (32), if the common ground does not entail that Trump hired a spy, (79)'s felicity depends on the existence of a law or principle that prohibits, in principle, Trump's not regretting collaborating with foreign leaders and not having hired a spy, and if the common ground does not entail that Trump collaborated with foreign leaders, (79)'s felicity depends on the existence of a law or principle that prohibits, in principle, Trump's not regretting hiring a spy and his not having collaborated with foreign leaders (at least in some worlds of the common ground). The existence of such laws/principles is not easy to establish without compelling linguistic or non-linguistic evidence (this is probably why (79) usually presupposes that Trump collaborated with foreign leaders and hired a spy). And yet, as pointed out to us by a *Natural Language Semantics* reviewer, (79) is felicitous in a quiz show context like (81), whose common ground contains worlds where Trump hired a spy and did not collaborate with foreign leaders and worlds where Trump collaborated with foreign leaders and did not hire a spy (in accordance with the violability of (71c)).

(81) Now, for $1000: does Trump regret collaborating with foreign leaders$_{\text{H*}}$ or does he regret hiring a spy$_{\text{H*L-L\%}}$

It is not entirely clear why, in the absence of compelling evidence for the existence of the required laws, (79) is felicitous in the quiz show context. Suppose the reason is that the issuer of the question, in such a context, is not agnostic about it. If this is the case, then any context where the issuer is not agnostic about the question should be equally tolerant. Given this, (79) is still expected to have a biased reading when one of its disjuncts is undefined throughout the common ground, on a par with (78a).

It is worth noting that if we treated 'alternative' interrogatives as total functions (and assumed that *or* lacks the conditional presupposition in (32)), the set of possible answers to (79) would always be {Trump regrets collaborating with foreign leaders, Trump regrets hiring a spy}. The felicity of (79) in the quiz show context in (81) would simply be the result of violating (71a) (on a par with (65) in the context described in (66)). But (79) would also be predicted—incorrectly—to be felicitous in contexts where one of its possible answers is undefined throughout the common ground, violating (71b) (on a par with (78a) when its meaning is the one in (78c), and with (67) in the context described in (68)).

The lesson we take from this is that pragmatic weakening in the style of Schwarz and Simonenko is not restrictive enough; it must be paired with a general theory of biased interrogatives that accounts for the contrast between (78a) and (79). As an alternative to Guerzoni's analysis of polar interrogatives with *even*, let us briefly consider the hypothesis that the negative bias of polar interrogatives with *even* is not the result of weakening the universal presupposition of interrogatives. Instead, in the biased LF of (73), *even* scopes above a (silent) interrogative speech act operator—*Ask*—as in (82a), which applies to a question Q and yields 'true' in w if and only if the speaker issues Q in w.

(82) a. *even*-C_3 [^[*Ask* ^[… ^[*John ate the cake*$_{\text{F}}$]]] $\sim C_3$]
b. C3 ⊆ {<u>The speaker issues {John ate the cake, John didn't eat the cake}</u>,
The speaker issues {John ate the candy, John didn't eat the candy}, …}

The pre-*Ask* position is non-monotonic with respect to the focus associate; as such, it is an appropriate landing site for *even* (see Crnič 2014), from which *even* triggers the presupposition that the speaker is less likely to issue the question denoted by *Did John eat the cake*$_{\text{L*H-H\%}}$ than any of the other C3-alternatives.

This idea follows other works that exploit the assumption that interrogatives are always syntactically embedded, an assumption that provides a wide-scope landing site to surface embedded operators. In Krifka (2001) this assumption is used to account for pair-list readings of *who*-interrogatives with quantifiers (see (83)), and in Sauerland and Yatsushiro (2017) the assumption is used to account for *remind-me* readings of interrogatives with *again* (see (84)).[17]

(83) a. Who does every man love?
 b. Intuitive meaning: "For each man x: tell us which y is such that x loves y."

(84) a. What was your name again?
 b. Intuitive meaning: "Tell us again the answer to: What is your name?"

The LF in (82) is also inspired by the idea in Iatridou and Tatevosov (2016) that question-focusing *even* conveys that the current question is less likely to be asked than its alternatives.

Importantly, while the acceptability of (73) is accounted for with *even*-over-*Ask*, its negative bias is not accounted for in any obvious way. In the absence of a theory of asking that relates likelihoods of issuing questions to specific epistemic states of the issuers of those questions, the *even*-over-*Ask* proposal does not—as of yet—account for the negative bias of (73).

To sum up, interrogatives generally have a universal presupposition, though the source of this presupposition may not be the same in *wh*- and 'alternative' interrogatives (in the latter, the source is the meaning of *or*). Bias in interrogatives remains a puzzle.

4 Reduced disjunctions and *even*

We have shown that the bias effect of *even* in polar interrogatives does not undermine the claim that 'alternative' interrogatives (and interrogatives in general) have a universal presupposition. However, our argument remains incomplete without testing the following prediction: in the absence of presupposition-cancelling disjuncts (the K–P effect), presuppositions introduced by *even* project universally from 'alternative' interrogatives (as in *Did John even eat the cake*$_{\text{*H}}$ *or did he even eat the candy*$_{\text{H*L-L\%}}$). We show that such interrogatives do indeed project the presuppositions of *even* universally, as predicted, but the conditional presupposition of *or* does not suffice to account for all the facts, as many 'alternative' interrogatives are reduced disjunctions with ellipsis of *even*. Therefore, the current proposal is supplemented with a constraint on anaphora resolution in ellipsis constructions.

Consider the interrogatives in (85). (85a) is a polar interrogative enriched with an occurrence of *even* following the subject; (85b) is an 'alternative' interrogative

[17] In Krifka (2001), the speech act operator does not yield a truth value. In Sauerland and Yatsushiro (2017), the speech act operator is decomposed into several components. *Ask* is compatible with all other aspects of these proposals.

enriched with an occurrence of *even* following the subject. As observed in Abenina-Adar and Sharvit (2018), the latter is ill-formed.

(85) a. Did John even eat the cake or the candy$_{\text{L*H-H\%}}$
b. *Did John even eat the cake$_{\text{H*}}$ or the candy$_{\text{H*L-L\%}}$

Crucially, (85b) does not have an object-focus *even* reading. By this we mean that it does not have a reading according to which the expected reply is among {John even ate the CAKE, John even ate the CANDY}, nor does it have a reading according to which the expected reply is among {John even ate the CAKE, John ate the candy}. To see that these readings are not ruled out in principle, consider the 'alternative' interrogatives in (86). (86a) is a non-reduced counterpart of (85b), and (86b) is a counterpart of (85b)—reduced like (85b)—but with an overt occurrence of *even* in each disjunct. (86a,b) have the indicated object-focus readings.

(86) a. Did John even eat the cake$_{\text{H*}}$ or did he eat the candy$_{\text{H*L-L\%}}$
Expected reply is among {John even ate the CAKE, John ate the candy}
b. Did John even eat the cake$_{\text{H*}}$ or even (eat) the candy$_{\text{H*L-L\%}}$
Expected reply is among {John even ate the CAKE, John even ate the CANDY}

The findings in (87) illustrate the pertinent judgments regarding (85) and (86). They are based on our own judgments and judgments we elicited in an informal setting, from colleagues who are native speakers of English and from non-specialist native speakers.

(87) (i) (85a) is acceptable in Scenario 2 in (88).
(ii) (85b) is not acceptable in any scenario in (88).
(iii) (86a) is acceptable in Scenarios 1 and 3 in (88).
(iv) For many speakers, (86b) is acceptable in Scenario 3 in (88) (it may be used to ask which of the two unexpected things happened).

(88) *Scenario 1.* At Sam's birthday party, there are only two food options: cake and candy. John is least likely to eat the cake (because he is allergic to flour).
Scenario 2. At Sam's birthday party, there are three food options: cake, candy, pizza. John is not allowed to eat sweets; he is more likely to eat the pizza than the cake or the candy.
Scenario 3. At Sam's birthday party, there are more than two food options. Among the flour-based food options, John is least likely to eat the cake (because it is too sweet compared to, say, the pita bread), and among the flour-less desserts, he is least likely to eat the candy (because his dentist told him to avoid sticky food).

Crucially, no scenario makes (85b) acceptable. One might suspect that the reason is that (85b) violates some interrogative-specific constraint, but this does not seem to be the case, as suggested by the fact that the declarative counterparts of (85a), (86a), and (86b) have the same presuppositions.

(89) a. John even ate the CAKE or the CANDY. (cf. (85a))
b. John even ate the CAKE or ate the candy. (cf. (86a))
c. John even ate the CAKE or even ate the CANDY. (cf. (86b))

What is least likely in (89a) is that John ate one of {the cake, the candy}; in (89b), on the other hand, John eating the cake is least likely among {John ate the cake, John ate the candy}; and in (89c) John eating the cake is least likely relative to one set of alternatives and John eating the candy is least likely among another set of alternatives. Given this, we do not blame the absence of an object-focus reading of (85b) on some interrogative-specific constraint.[18,19] The absence of an object-focus reading of (85b), rather, is the interaction between the meaning of *or* and a constraint which we call DU.

(90) *Domain Uniformity (DU):* A disjunction is well-formed only if every *elided* quantifier that it contains has the same domain restrictor as its antecedent.

Here is how DU and the meaning of *or* work together. Merely for simplicity, let us assume that underlyingly, a polar interrogative is a special 'alternative' interrogative, containing a silent *or not* followed by a silent copy of the clause that precedes *or not* (see (26c) and (27c) in Sect. 2.1). Accordingly, the declarative disjunction in (89a) and the polar interrogative in (85a), whose LFs are (91a) and (91b) respectively, are potentially acceptable because DU and the conditional presupposition of *or* can both be respected when *even* scopes above ∃.

(91) a. *even*-C_2 [^∃ 5 [^[*John ate the cake*$_F$] *or*$_5$ ^[~~*John*~~ ~~*ate*~~ *the candy*$_F$]] $\sim C_2$] (89a)
b. [6 [^[*even*-C_2 [^∃ 5 [^[*John ate the cake*$_F$] *or*$_5$ ^[~~*John*~~ ~~*ate*~~ *the candy*$_F$]] $\sim C_2$]] ~~*or*~~$_6$
^[~~*not*~~ ~~*even*~~-C_2 [^∃ 5 [^[~~*John ate the cake*~~$_F$] ~~*or*~~$_5$ ^[~~*John ate the candy*~~$_F$]] $\sim C_2$]]]] (85a)
c. C2 ⊆ {⟦^∃ 5 [^[*John ate the cake*] *or*$_5$ ^[*John ate the candy*]] ⟧,
⟦^∃ 5 [^[*John ate the pizza*] *or*$_5$ ^[*John ate the pizza*]] ⟧, …}
Even-induced presupposition projected by *or* and ∃, satisfiable:
John eating one of {the cake, the candy} is least likely among the C2-alternatives.

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[18] This includes treating *even* as an intervener in the sense of Beck (1996, 2006) and Beck and Kim (2006), or as a negative polarity item (NPI), or part of an NPI, along the lines of Lee and Horn (1994) and Crnič (2014), reducing the unacceptability of (85b) to the exclusion (observed in Higginbotham 1993) of NPIs from 'alternative' interrogatives. Importantly, declarative clauses such as those in (89) are neither intervention environments nor NPI environments.

[19] *Even* may associate with an item outside the surface disjunction, given the right context.

A: John took syntax and phonology. He is a very good student, but like everyone in his class he failed SOMEthing.

B: Really? Did even HE fail syntax$_{H*}$ or phonology$_{H*L\text{-}L\%}$

On the other hand, the LFs in (92a,b) are ill-formed, since the disjuncts have an underlying *even* whose restrictor, by DU, leads to contradictory presuppositions (alternative LFs with "functional" *even*-restrictors are considered and discarded in Appendix 3).

(92) a. * *∃* [5 [^[*even*-C_3 [^[*John ate the cake*$_F$] ~C_3]] *or*$_5$ (89a)
^[~~*even*~~-C_3 [^[~~*John*~~ ~~*ate*~~ *the candy*$_F$] ~C_3]]]]

b. * [5 [^[*even*-C_3 [^[*John ate the cake*$_F$] ~C_3]] *or*$_5$ (85b)
^[~~*even*~~-C_3 [^[~~*John*~~ ~~*ate*~~ *the candy*$_F$] ~C_3]]]]

c. C3 ⊆ {⟦^[*John ate the cake*] ⟧, ⟦^[*John ate the candy*] ⟧, …}
Even-induced presuppositions projected by *or* and *∃*, not satisfiable:
For any w in the common ground:
⟦^[*John ate the cake*] ⟧ is least likely in w among the C3-alternatives, or
SIM(w)({w'| ⟦*even*-C_3 [^[*John ate the candy*$_F$] ~C_3] ⟧$^{w',g}$ = 0} ⊆ {w'| ⟦^[*John ate the cake*] ⟧ is least likely in w' among the C3-alternatives}
and
⟦^[*John ate the candy*] ⟧ is least likely in w among the C3-alternatives, or
SIM(w)({w'| ⟦*even*-C_3 [^[*John ate the cake*$_F$] ~C_3] ⟧$^{w',g}$ = 0}) ⊆ {w'| ⟦^[*John ate the candy*] ⟧ is least likely in w' among the C3-alternatives}

This means that (89a) has only the reading corresponding to (91a), and that the 'alternative' interrogative in (85b)—whose prosody forces the illicit (92b)—is simply unacceptable. Without DU, (92a,b) would escape the unsatisfiable presuppositions in (92c), because *even* could have different restrictors in the two disjuncts. Without the conditional presupposition of *or*, (92a,b) would also escape those unsatisfiable presuppositions.

It is worth noting that many works (for example, Karttunen and Peters 1979) attribute to *even* an additive presupposition. Accordingly, *John even ate the CAKE* presupposes that John ate at least one relevant thing other than the cake (and the cake is the least likely thing for him to eat). Indeed, if we phrase the additive presupposition of *even* as a requirement that at least one alternative to the prejacent —one that is not entailed by it—be true, this would, together with DU and an adjusted Ans, explain the unacceptability of (85b) even if *or* has no conditional presupposition. However, the projection facts regarding 'even'-less disjunctive constructions, interrogative and non-interrogative, would be unexplained if we adopted this view. We conclude that while the facts discussed here are compatible with additivity as part of the meaning of *even*, additivity itself cannot replace the conditional presupposition of *or*, nor is it needed to account for (85).

Some additional consequences are worth noting. Not every focus-sensitive item causes a fatal presupposition. For example, if we replace *even* in (85b) with *only*, which lacks a likelihood presupposition, the result is acceptable.

(93) a. Did John only eat the cake$_{\mathrm{H*}}$ or the candy$_{\mathrm{H*L\text{-}L\%}}$

LF: [5 [^[*only*-C_1 [^[*John ate the cake*$_{\mathrm{F}}$] ~C_1]] *or*$_5$

^[~~*only*~~-C_1 [^[~~*John*~~ ~~*ate*~~ *the candy*$_{\mathrm{F}}$] ~C_1]]]]

b. *Only*-induced presuppositions (see Horn 1996) projected by *or*, satisfiable: {John ate something}

Reduced disjunctions obey a parallelism constraint that rules (94) out: if (94) were a possible LF of (85b), no contradiction would arise.

(94) [5 [^[*even*-C_1 [^[*John ate the cake*$_{\mathrm{F}}$] ~C_1]] *or*$_5$ ^[~~*John*~~ ~~*ate*~~ *the candy*]]]

We do not formulate the parallelism constraint, but independent evidence for it is provided by the fact that the acceptable reduced (93a) (with *only*) and the acceptable reduced (95) (with *sometimes*) do not have the meanings implied by (96a) and (96b), respectively.

(95) Does John sometimes watch the news$_{\mathrm{H*}}$ or Jeopardy$_{\mathrm{H*L\text{-}L\%}}$

[5 [^[*sometimes John watches the news*] *or*$_5$ ^[~~*sometimes*~~ ~~*John*~~ ~~*watches*~~ *Jeopardy*]]]

(96) a. [5 [^[*only*-C_1 [^[*John ate the cake*$_{\mathrm{F}}$] ~C_1]] *or*$_5$ ^[~~*John*~~ ~~*ate*~~ *the candy*]]]

b. [5 [^[*sometimes John watches the news*] *or*$_5$ ^[~~*John*~~ ~~*watches*~~ *Jeopardy*]]]

Non-reduced disjunctions are exempt from parallelism; in addition, a non-elided *even* is exempt from DU. As a result, (i) a disjunction of two polar speech acts allows *even* in one of them or both of them (see (97a), (98a)); (ii) a non-reduced 'alternative' interrogative can have an occurrence of *even* in the first disjunct with or without an occurrence of *even* in the second (see (97b), (98b)); and (iii) a disjunction with *even* in one disjunct and *only* in the other can be felicitous (see (99)).

(97) a. Did John even eat the cake$_{\mathrm{L*H\text{-}H\%}}$ or, alternatively, did he eat the candy$_{\mathrm{L*H\text{-}H\%}}$

b. Did John even eat the cake$_{\mathrm{H*}}$ or did he eat the candy$_{\mathrm{H*L\text{-}L\%}}$

c. … ^[*even*-C_1 [^[*John ate the cake*$_{\mathrm{F}}$] ~ C_1]] … ^[*John ate the candy*] …

(98) a. Did John even eat the cake$_{\mathrm{L*H\text{-}H\%}}$ or, alternatively, did he even eat the candy$_{\mathrm{L*H\text{-}H\%}}$

b. Did John even eat the cake$_{\mathrm{H*}}$ or did he even eat the candy$_{\mathrm{H*L\text{-}L\%}}$

c. … ^[*even*-C_1 [^[*John ate the cake*$_{\mathrm{F}}$] ~ C_1]] … ^[*even*-C_2 [^[*John ate the candy*$_{\mathrm{F}}$] ~ C_2]] …

(99) Did John even eat the cake$_{\mathrm{H*}}$ or only the candy$_{\mathrm{H*L\text{-}L\%}}$

Finally, a sloppy VP reading of (100a) (whose *even*-less counterpart is *If Mary is dancing or swimming, then Sue is*, which has the LF in (22)) with a narrow scope *even* is unavailable due to the conflicting presuppositions that project from the antecedent. A sloppy VP reading with a wide scope *even* is available in principle (cf. Guerzoni 2004).

(100) a. If Mary is even dancing or swimming, then Sue is.

b. Narrow-scope-*even* sloppy VP reading, unavailable:

(i) *if*^{2} ^[*Mary* ... *even* ... *swimming*$_{\mathrm{F}}$ ... ~~*even*~~ ... *dancing*$_{\mathrm{F}}$ …] ^[6 [*Sue* $^{\vee}$*pro*$_6$]]

(ii) The set of alternatives is a subset of

{⟦^[*Mary swimming*] ⟧, ⟦^[*Mary dancing*] ⟧, …}

(iii) *Even*-induced presupposition projected by *or* and *if*^{2}, not satisfiable:

'Mary dances' is least likely among the relevant alternatives and 'Mary swims' is least likely among the relevant alternatives.

c. Wide-scope-*even* sloppy VP reading, available:

(i) *even* ... [^[*if*^{2} ^[*Mary* ... *swimming*$_{\mathrm{F}}$ ... *dancing*$_{\mathrm{F}}$] ^[6 [*Sue* $^{\vee}$*pro*$_6$]] …]

(ii) The set of alternatives is a subset of

{⟦^*if*^{2} ^[*Mary* ... *swimming* ... *dancing*] ^[6 [*Sue* $^{\vee}$*pro*$_6$]] ⟧,

⟦^*if*^{2} ^[*Mary* ... *running* ... *hiking*] ^[6 [*Sue* $^{\vee}$*pro*$_6$]] ⟧, …}

(iii) *Even*-induced presupposition, satisfiable:

(['Mary dances' → 'Sue dances'] ∧ ['Mary swims' → 'Sue swims'])

is least likely among the relevant alternatives.

Some discussion regarding the status of DU is in order. It is commonly assumed (Westerståhl 1985, von Fintel 1994, and others) that the domains of determiners and quantifiers are contextually restricted. Moreover, it has been observed that domain restrictors may vary intra-sententially. Indeed, the following examples, where C1 and C2 represent distinct domain restrictors, are coherent.

(101) a. Everyone-C1 is asleep and being monitored by a-C2 research assistant. (Soames 1986)

b. The-C1 pig is grunting, but the-C2 pig with floppy ears is not grunting. (Lewis 1973)

Similarly, the unembedded declarative disjunction in (102a) and the 'alternative' interrogative in (102b) may be felicitous when the two occurrences of *the* have different domain restrictors.

(102) a. John ate the-C1 cake or the-C2 carrot cake.

b. Did John eat the-C1 cake$_{\mathrm{H*}}$ or the-C2 carrot cake$_{\mathrm{H*L\text{-}L\%}}$

In contexts where there is only a carrot cake, (102a) and (102b) are definitely odd, probably because one of the disjuncts is superfluous. In contexts where there are two salient cakes—salient relative to different domain restrictors—they may be felicitous, as expected.

As it turns out, ellipsis affects a determiner's freedom to "choose" a restrictor independently of its antecedent's restrictor. Thus, (103a,b) cannot be understood as implying what (104a,b) imply when the two occurrences of *the guard* have different domain restrictors. Presumably, DU prohibits pairing the elided occurrence of *the guard* with a different restrictor.

(103) a. The guard is asleep or watching TV.

b. Is the guard asleep$_{H*}$ or watching TV$_{H*L\text{-}L\%}$

(104) a. *∃ 5 [^[the-*C_1 *guard asleep] or*$_5$ ^[~~*the*~~-$C_{1/*2}$ ~~*guard*~~ *watching TV*]]

b. [5 [^[*the-*C_1 *guard asleep*] *or*$_5$ ^[~~*the*~~-$C_{1/*2}$ ~~*guard*~~ *watching TV*]]]

While alternative LFs with across-the-board movement and only one occurrence of *the guard* are available for these examples (see (105)), potentially providing an alternative explanation for the absence of multiple domain restrictors, it is less obvious that anything other than DU rules out multiple domain restrictors when *the guard* in (103a,b) is replaced by *everyone* (as scoping *everyone* above *or* results in a different meaning).

(105) a. *the-*C_1 *guard* [3 [*∃* 2 [^[t_3 *asleep*] *or*$_2$ ^[t_3 *watching TV*]]]]

b. *the-*C_1 *guard* [3 [*ASK* ^[2 [^[t_3 *asleep*] *or*$_2$ ^[t_3 *watching TV*]]]]]

In addition, in some cases there are no alternative across-the-board LFs: the unacceptability of *Did John even eat the cake*$_{H*}$ *or the candy*$_{H*L\text{-}L\%}$, and the fact that the potentially acceptable *Did John even eat the cake*$_{H*}$ *or even eat the candy*$_{H*L\text{-}L\%}$ lacks a reading that implies that asking *Did John eat the cake*$_{H*}$ *or eat the candy*$_{H*L\text{-}L\%}$ is less likely than asking the focus-alternatives (e.g., *Did John eat the chips*$_{H*}$ *or eat the pizza*$_{H*L\text{-}L\%}$; cf. Iatridou and Tatevosov 2016, fn. 39), show that *even* cannot scope across-the-board above *or* in ‘alternative’ interrogatives.

It stands to reason that DU follows from a more general ellipsis constraint: an elided quantifier (or determiner) and its restrictor always form a single unit, “searching for” a single antecedent (composed of a quantifier and a restrictor). Some independent, though indirect, evidence for this is provided by (106), where the relevant quantifier is focus-sensitive *only* and the relevant connective is *and*.

(106) a. The provost reprimanded only his ASSISTANT and only the DEAN. Other people who got reprimands are adjunct professors.

b. The provost reprimanded only his ASSISTANT and the DEAN. #Other people who got reprimands are adjunct professors.

c. In his office, the provost reprimanded only his ASSISTANT and the DEAN. Other people who got reprimands are outside his office.

Non-elided quantifiers can have different restrictors; hence the acceptability of the first sentence in (106a), as reflected by its LF in (107). No contradiction arises when each occurrence of *only* has a different restrictor.

(107) ^[*only-*C_1 [^[*the provost reprimanded* [*his assistant*]$_F$] ~C_1]] *and* ^[*only-*$C_{2/*1}$ [^[~~*the provost reprimanded*~~ [*the dean*]$_F$] ~$C_{2/*1}$]]

The oddity of the continuation in (106b) versus the felicity of the continuations in (106a,c) suggests the following. For independent reasons (which we do not fully understand), unless an alternative domain is invoked by the use of an overt item (e.g., a second overt *only* in (106a), *in his own office* in (106c)), the second sentence cannot be evaluated relative to a different domain restrictor; therefore, a contradiction between the first and second sentences arises in (106b) when the focused unit is *his assistant and the dean* is focused; see (108), the LF of the first sentence under this focusing.

(108) $only\text{-}C_1$ [^[*the provost reprimanded* [*his assistant and the dean*]$_F$] $\sim C_1$]

Given this, if elided quantifiers were not required to be fully anaphoric, the focused unit in (106b) could be *the dean*. Since (106b) is infelicitous, we conclude that (109) —the LF of its first sentence under this alternative focusing—is ill-formed; either due to a violation of the requirement that elided quantifiers be fully anaphoric, or due to a contradiction that arises when the elided quantifiers are in fact fully anaphoric.

(109) *^[$only\text{-}C_1$ [^[*the provost reprimanded* [*his assistant*]$_F$] $\sim C_1$]] *and* ^[~~*only*~~-$C_{2/1}$ [^[~~*the provost reprimanded*~~ [*the dean*]$_F$] $\sim C_{2/1}$]]

A parallel case, with *even* instead of *only*, is provided in (110).

(110) We are shocked. Administrators never get reprimanded, certainly not senior ones.
- a. The provost reprimanded even her ASSISTANT and even the ASSISTANT DEAN. Other people who got reprimands are even more senior.
- b. The provost reprimanded even her ASSISTANT and the ASSISTANT DEAN. #Other people who got reprimands are even more senior.
- c. In her own immediate environment, the provost reprimanded even her ASSISTANT and the ASSISTANT DEAN. Other people who got reprimands are outside her immediate environment.

In sum, together with the conditional presupposition of *or*, DU accounts for the projection properties of *even* in disjunctive clauses, including disjunctive interrogatives.

5 Conclusion

We have argued that the fact that disjunctions project all the presuppositions of all their disjuncts, modulo the K–P effect, follows from the conditional presupposition of *or*. This explains why 'alternative' interrogatives have universal presuppositions modulo the K–P effect, just like other disjunctive constructions. While the universality of the presuppositions of *wh*-interrogatives may result from the pragmatic principles in Schwarz and Simonenko (2017), the question of why the presupposition trigger *even* brings about a negative bias in polar interrogatives remains open.

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Appendix 1: Some rules and conventions

Generally following Heim and Kratzer (1998), we assume that for any possible world w and variable assignment g, (i)–(v), (10'), (13'), and (31') hold.

(i) If γ is a branching node whose daughters are α and β and $[\![\alpha]\!]^{w,g} \in$ Dom($[\![\beta]\!]^{w,g}$), then:
$[\![\gamma]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\alpha]\!]^{w,g})$

(ii) If $\gamma = [^\wedge \beta]$, then: $[\![\gamma]\!]^{w,g}=[\lambda w': \beta \in \mathrm{Dom}([\![\,]\!]^{w',g}).\ [\![\beta]\!]^{w',g}]$
If $\gamma = [^\vee \beta]$, then: $[\![\gamma]\!]^{w,g}=[\![\beta]\!]^{w,g}(w)$

(iii) If $\gamma = [k\ \beta]$ and k is a numerical index, then:
$[\![\gamma]\!]^{w,g} = [\lambda x: \beta \in \mathrm{Dom}([\![\,]\!]^{w,g[k\rightarrow x]}).\ [\![\beta]\!]^{w,g[k\rightarrow x]}]$,
where $\mathrm{Dom}(g_{[k\rightarrow x]})=(\mathrm{Dom}(g) \cup \{k\})$, $g_{[k\rightarrow x]}(k)=x$, and for all $m \in \mathrm{Dom}(g_{[k\rightarrow x]})$ such that $m\neq k$: $g_{[k\rightarrow x]}(m)=g(m)$

(iv) If γ is a pronoun or a trace and k is a numerical index, then:
$[\![\gamma_k]\!]^{w,g}$ is defined only if g(k) is defined. If defined, $[\![\gamma_k]\!]^{w,g} = g(k)$.

(v) a. $[\![\gamma]\!]^{g}$ is defined iff for all w' and w'', $[\![\gamma]\!]^{w',g}$ and $[\![\gamma]\!]^{w'',g}$ are defined and $[\![\gamma]\!]^{w',g} = [\![\gamma]\!]^{w'',g}$;
if $[\![\gamma]\!]^{g}$ is defined, then for all w': $[\![\gamma]\!]^{g} = [\![\gamma]\!]^{w',g}$.

b. $[\![\gamma]\!]^{w}$ is defined iff for all g' and g'', $[\![\gamma]\!]^{w,g'}$ and $[\![\gamma]\!]^{w,g''}$ are defined and $[\![\gamma]\!]^{w,g'} = [\![\gamma]\!]^{w,g''}$;
if $[\![\gamma]\!]^{w}$ is defined, then for all g': $[\![\gamma]\!]^{w} = [\![\gamma]\!]^{w,g'}$.

c. $[\![\gamma]\!]$ is defined iff for all w', w'', g' and g'', $[\![\gamma]\!]^{w',g'}$ and $[\![\gamma]\!]^{w'',g''}$ are defined and $[\![\gamma]\!]^{w',g'}= [\![\gamma]\!]^{w'',g''}$;
if $[\![\gamma]\!]$ is defined, then for all w' and g': $[\![\gamma]\!] = [\![\gamma]\!]^{w',g'}$.

(10') For any $n \geq 2$, sequence S, and any $P_1, P_2, \ldots,$ and P_n
such that $P_1(w)(S)$ is of type t and $P_2, \ldots,$ and P_n are of the same type as P_1:
$[\![or^{\exists}]\!]^w(P_1)\ldots(P_n)(S)=1$ iff $\{Q|\ Q(w)(S)=1 \wedge (Q=P_1 \vee \ldots \vee Q=P_n)\} \neq \varnothing$
(cf. (10), Sect. 2)

(13') For any $k \in Dom(g)$, any $n \geq 2$, and any $P_1, P_2, \ldots, P_n$ of the same type:
$[\![or_k]\!]^g(P_1)\ldots(P_n)=1$ iff $g(k)=P_1 \vee \ldots \vee g(k)=P_n$
(cf. (13), Sect. 2)

(31') For any $\mathscr{Q}$ and $\mathscr{P}$, $[\![if]\!]^w(\mathscr{Q})(\mathscr{P}) \in \{1, 0\}$ only if:
(i) $\{S|\ S$ is a sequence and $\mathscr{Q}(w)(S) \in \{1, 0\}\} \neq \varnothing$, and
(ii) for all S such that S is a sequence and $\mathscr{Q}(w)(S) \in \{1, 0\}$:
$SIM(w)(\{w'|\ \mathscr{Q}(w')(S)=1\}) \subseteq \{w'|\ \mathscr{P}(w')(S) \in \{1, 0\}\}$.
If $[\![if]\!]^w(\mathscr{Q})(\mathscr{P}) \in \{1, 0\}$, $[\![if]\!]^w(\mathscr{Q})(\mathscr{P})=1$ iff:
for all S such that S is a sequence and $\mathscr{Q}(w)(S) \in \{1, 0\}$:
$SIM(w)(\{w'|\ \mathscr{Q}(w')(S)=1\}) \subseteq \{w'|\ \mathscr{P}(w')(S)=1\}$.
(cf. (31), Sect. 2)

We use the following conventions:

(I) [λx: A. B] is shorthand for "the smallest function *f* such that *f* maps every x such that A to B" or "the smallest function *f* such that *f* maps every x such that A to 1 if B and to 0 otherwise" (whichever makes sense).
(II) For any Z whose type ends in t, $m \geq 0$, and m-long sequence S:
if $m=0$: $Z(S) \equiv Z$;
if $m > 0$ and $S=(X_1, \ldots, X_m)$: $Z(S) \equiv Z(X_1)\ldots(X_m)$.

Appendix 2: Presuppositions of disjunctive conditionals

Disjunctive conditionals, including those with ellipsis in the consequent, are subject to the K–P effect. Thus, in utterance contexts with more or less "normal" laws, both readings of (112a) intuitively presuppose that Mary has a son and a daughter and Sue has a son and a daughter, neither reading of (112b) presupposes that either woman has children, and the sloppy reading of (112b) presupposes that if Mary is a parent, Sue is a parent (we set aside the presuppositions of strict pronoun readings, where the consequent implies that Sue stands in some relation to Mary's children).

(112) a. If Mary avoids her son or her daughter, then Sue does.
Strict VP reading:
'Mary avoids Mary's son or daughter' $\rightarrow$ 'Sue avoids Sue's son or daughter'
Sloppy VP reading:
('Mary avoids Mary's son' $\rightarrow$ 'Sue avoids Sue's son') $\wedge$
('Mary avoids Mary's daughter' $\rightarrow$ 'Sue avoids Sue's daughter')

b. If Mary is (either) childless or abusive with her children, then Sue is.
Strict VP reading:
'Mary is childless or abusive with Mary's children' $\rightarrow$
'Sue is childless or abusive with Sue's children'
Sloppy VP reading:
('Mary is childless' $\rightarrow$ 'Sue is childless') $\wedge$
('Mary is abusive with Mary's children' $\rightarrow$ 'Sue is abusive with Sue's children')

This is predicted if the LF of the strict VP reading contains presuppositional if^1 (see (31) in Sect. 2), proposition-level presuppositional or_k and presuppositional $\exists$ (see (32) and (38)), and the LF of the sloppy VP reading contains presuppositional if^2 in (113) below and property-level presuppositional OR_k in (114) below. The predictions are as in (115)–(116).

(113) $[\![if^2]\!]^{w}(\mathscr{Q})(\mathscr{P}) \in \{1, 0\}$ only if:
(i) $\{P|\ \mathscr{Q}(w)(P) \in \{1, 0\}\} \neq \varnothing$, and
(ii) for all P such that $\mathscr{Q}(w)(P) \in \{1, 0\}$:
$SIM(w)(\{w'|\ \mathscr{Q}(w')(P)=1\}) \subseteq \{w'|\ \mathscr{P}(w')(P) \in \{1, 0\}\}$.

If $[\![if^2]\!]^{w}(\mathscr{Q})(\mathscr{P}) \in \{1, 0\}$, $[\![if^2]\!]^{w}(\mathscr{Q})(\mathscr{P})=1$ iff:
for all P such that $\mathscr{Q}(w)(P) \in \{1, 0\}$:
$SIM(w)(\{w'|\ \mathscr{Q}(w')(P)=1\}) \subseteq \{w'|\ \mathscr{P}(w')(P)=1\}$.

(114) For any x of type e, and P_1 and P_2 of type (s, et), $[\![OR_k]\!]^{w,g}(P_1)(P_2)(x) \in \{1, 0\}$ iff:
a. $g(k)(w)(x) \in \{1, 0\}$,
b. $g(k)=P_1 \vee g(k)=P_2$, and
c. $(P_1(w)(x) \in \{1, 0\} \vee SIM(w)(\{w'|\ P_2(w')(x)=0\}) \subseteq \{w'|\ P_1(w')(x) \in \{1, 0\}\}) \wedge$
$(P_2(w)(x) \in \{1, 0\} \vee SIM(w)(\{w'|\ P_1(w')(x)=0\}) \subseteq \{w'|\ P_2(w')(x) \in \{1, 0\}\})$

If $[\![OR_k]\!]^{w,g}(P_1)(P_2)(x) \in \{1, 0\}$, $[\![OR_k]\!]^{w,g}(P_1)(P_2)(x)=1$ iff $g(k)(w)(x)=1$.

(115) a. If Mary avoids her son or her daughter, then Sue does. (112a)

b. (i) LF of strict VP:

$\textit{if}^1$ $\hat{}$[$\textit{Mary}$ [1 [$\exists$2 [$\hat{}$[t_1 $\textit{avoids her}_1$ $\textit{son}$] $\textit{or}_2$ $\hat{}$[t_1 $\cancel{\textit{avoids}}$ $\textit{her}_1$ $\textit{daughter}$]]]]]

$\hat{}$[$\textit{Sue}$ [1 [$\exists$ 2 [$\hat{}$[t_1 $\cancel{\textit{avoids}}$ $\cancel{\textit{her}}_1$ $\cancel{\textit{son}}$] $\cancel{\textit{or}}_2$ $\hat{}$[t_1 $\cancel{\textit{avoids}}$ $\cancel{\textit{her}}_1$ $\cancel{\textit{daughter}}$]]]]]

(ii) LF of strict VP presupposes:

$\mathbf{m} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{son}]\!]^{w,[1\to\mathbf{m}]}) \wedge \mathbf{m} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w,[1\to\mathbf{m}]}) \wedge$

$\mathrm{SIM}(w)(\{w'|\ [\![\textit{avoids her}_1\ \textit{son}]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1 \vee$

$[\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1\}) \subseteq$

$\{w'|\ \mathbf{s} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{son}]\!]^{w',[1\to\mathbf{s}]}) \wedge \mathbf{s} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w',[1\to\mathbf{s}]})\}$

(Presupposition: Mary and Sue each have a son and a daughter in w.)

(iii) LF of strict VP asserts:

$\mathrm{SIM}(w)(\{w'|\ [\![\textit{avoids her}_1\ \textit{son}]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1 \vee$

$[\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1\}) \subseteq$

$\{w'|\ [\![\textit{avoids her}_1\ \textit{son}]\!]^{w',[1\to\mathbf{s}]}(\mathbf{s}) = 1 \vee [\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w',[1\to\mathbf{s}]}(\mathbf{s}) = 1\}$

c. (i) LF of sloppy VP:

$\textit{if}^2$ $\hat{}$[2 [$\textit{Mary}$ $\hat{}$[1 t_1 $\textit{avoids her}_1$ $\textit{son}$] $\textit{OR}_2$ $\hat{}$[1 t_1 $\cancel{\textit{avoids}}$ $\textit{her}_1$ $\textit{daughter}$]]]

$\hat{}$[6 [$\textit{Sue}$ ${}^{\vee}\textit{pro}_6$]]

(ii) $[\![$2 [$\textit{Mary}$ $\hat{}$[1 t_1 $\textit{avoids her}_1$ $\textit{son}$] $\textit{OR}_2$ $\hat{}$[1 t_1 $\cancel{\textit{avoids}}$ $\textit{her}_1$ $\textit{daughter}$]] $]\!]^{w,g}$ =

λQ: $Q(w)(\mathbf{m}) \in \{1, 0\} \wedge \mathbf{m} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{son}]\!]^{w,[1\to\mathbf{m}]}) \wedge \mathbf{m} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w,[1\to\mathbf{m}]}) \wedge (Q = [\![\hat{}[1\ t_1\ \textit{avoids her}_1\ \textit{son}]\ ]\!] \vee Q = [\![\hat{}[1\ t_1\ \textit{avoids her}_1\ \textit{daughter}]\ ]\!])$. $Q(w)(\mathbf{m}) = 1$

(iii) LF of sloppy VP presupposes:

$\mathbf{m} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{son}]\!]^{w,[1\to\mathbf{m}]}) \wedge \mathbf{m} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w,[1\to\mathbf{m}]}) \wedge$

$\mathrm{SIM}(w)(\{w'|\ [\![\textit{avoids her}_1\ \textit{son}]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1\}) \subseteq$

$\{w'|\ \mathbf{s} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{son}]\!]^{w',[1\to\mathbf{s}]})\} \wedge$

$\mathrm{SIM}(w)(\{w'|\ [\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1\}) \subseteq$

$\{w'|\ \mathbf{s} \in \mathrm{Dom}([\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w',[1\to\mathbf{s}]})\}$

(Presupposition: Mary and Sue each have a son and a daughter in w.)

(iv) LF of sloppy VP asserts:

$\mathrm{SIM}(w)(\{w'|\ [\![\textit{avoids her}_1\ \textit{son}]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1\}) \subseteq$

$\{w'|\ [\![\textit{avoids her}_1\ \textit{son}]\!]^{w',[1\to\mathbf{s}]}(\mathbf{s}) = 1\} \wedge$

$\mathrm{SIM}(w)(\{w'|\ [\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1\}) \subseteq$

$\{w'|\ [\![\textit{avoids her}_1\ \textit{daughter}]\!]^{w',[1\to\mathbf{s}]}(\mathbf{s}) = 1\}$

(116) a. If Mary is (either) childless or abusive with her children, then Sue is. (112b)

b. (i) LF of strict VP:

if^1 ^[*Mary* [1 [$\exists$ 2 [^[t_1 *childless*] or_2 ^[t_1 *abusive with her*$_1$ *children*]]]]]

^[*Sue* [1 [$\exists$ 2 [^[t_1 ~~*childless*~~] ~~*or*~~$_2$ ^[t_1 ~~*abusive with her*~~$_1$ ~~*children*~~]]]]]

(ii) LF of strict VP presupposes:

$\mathbf{m} \in \mathrm{Dom}([\![childless]\!]^{w}) \wedge \mathbf{s} \in \mathrm{Dom}([\![childless]\!]^{w})$

(No presupposition that either Mary or Sue has children.)

(iii) LF of strict VP asserts:

$\mathrm{SIM}(w)(\{w' \mid [\![childless]\!]^{w'}(\mathbf{m}) = 1 \vee [\![abusive\ with\ her_1\ children]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1\}) \subseteq$

$\{w' \mid [\![childless]\!]^{w'}(\mathbf{s}) = 1 \vee [\![abusive\ with\ her_1\ children]\!]^{w',[1\to\mathbf{s}]}(\mathbf{s}) = 1\}$

c. (i) LF of sloppy VP:

if^2 ^[2 [*Mary* ^*childless* OR_2 ^[1 t_1 *abusive with her*$_1$ *children*]]] ^[6 [*Sue* $^{\vee}pro_6$]]

(ii) $[\![$2 [*Mary* ^*childless* OR_2 ^[1 t_1 *abusive with her*$_1$ *children*]] $]\!]^{w,g}$ =

$\lambda Q{:}\ Q(w)(\mathbf{m}) \in \{1, 0\} \wedge \mathbf{m} \in \mathrm{Dom}([\![childless]\!]^{w}) \wedge$

$(Q = [\![{}^{\wedge}childless]\!] \vee Q = [\![{}^{\wedge}[1\ t_1\ abusive\ with\ her_1\ children]\ ]\!]).\ Q(w)(\mathbf{m}) = 1$

(iii) LF of sloppy VP presupposes:

$\mathbf{m} \in \mathrm{Dom}([\![childless]\!]^{w}) \wedge$

$\mathrm{SIM}(w)(\{w' \mid [\![childless]\!]^{w'}(\mathbf{m}) = 1\}) \subseteq \{w' \mid \mathbf{s} \in \mathrm{Dom}([\![childless]\!]^{w'})\} \wedge$

(if $\mathbf{m} \in \mathrm{Dom}([\![abusive\ with\ her_1\ children]\!]^{w,[1\to\mathbf{m}]})$, then

$\mathrm{SIM}(w)(\{w' \mid [\![abusive\ with\ her_1\ children]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1\}) \subseteq$

$\{w' \mid \mathbf{s} \in \mathrm{Dom}([\![abusive\ with\ her_1\ children]\!]^{w',[1\to\mathbf{s}]})\})$

(Presupposition: If Mary is a parent, Sue is a parent.)

(iv) LF of sloppy VP asserts:

$(\mathrm{SIM}(w)(\{w' \mid [\![childless]\!]^{w'}(\mathbf{m}) = 1\}) \subseteq \{w' \mid [\![childless]\!]^{w'}(\mathbf{s}) = 1\} \wedge$

(if $\mathbf{m} \in \mathrm{Dom}([\ [\![abusive\ with\ her_1\ children]\!]^{w,[1\to\mathbf{m}]})$, then

$\mathrm{SIM}(w)(\{w' \mid [\![abusive\ with\ her_1\ children]\!]^{w',[1\to\mathbf{m}]}(\mathbf{m}) = 1\}) \subseteq$

$\{w' \mid [\![abusive\ with\ her_1\ children]\!]^{w',[1\to\mathbf{s}]}(\mathbf{s}) = 1)$

This analysis comes at a cost. While *if*^{1} and *if*^{2} can be merged into one item (see their generalized counterpart (31') in Appendix 1), proposition-level presuppositional or_k and property-level presuppositional OR_k cannot be merged into one item: both or_k and OR_k have a (generalizable) conditional presupposition, but or_k asserts '$g(k)=p_1 \vee g(k)=p_2$', whereas OR_k presupposes '$g(k)=P_1 \vee g(k)=P_2$' and asserts '$g(k)(w)(x)=1$'. It is worth noting that the generalized presuppositional disjunctive word in (117), with the necessary adjustment of existential closure in (118), covers both property-level and proposition-level disjunction and makes the same predictions as or_{k} and OR_{k} (see Sharvit 2020, for a similar proposal).

(117) For any n≥2, P_1, P_2, …, and P_n, sequence S, and numerical index k:
$[\![or^{\spadesuit}{}_{k}]\!]^{w,g}(P_1)\ldots(P_n)(S) \in \{1, 0\}$ iff:
a. $g(k)(w)(S) \in \{1, 0\}$,
b. $g(k)=P_1 \vee \ldots \vee g(k)=P_n$, and
c. for all $P \in \{P_1, \ldots, P_n\}$: $P(w)(S) \in \{1, 0\} \vee$
$\{D| D\subseteq\{P_1, \ldots, P_n\} \wedge \{w'| D\subseteq\{Q| Q(w')(S) \in \{1, 0\}\} \not\subseteq \{w'| P(w')(S) \in \{1, 0\}\} \wedge$
$SIM(w)(\{w'| D\subseteq\{Q| Q(w')(S)=0\}\})\subseteq\{w'| P(w')(S) \in \{1, 0\}\}\}\neq\varnothing$.
If $[\![or^{\spadesuit}{}_{k}]\!]^{w,g}(P_1)\ldots(P_n)(S) \in \{1, 0\}$, $[\![or^{\spadesuit}{}_{k}]\!]^{w,g}(P_1)\ldots(P_n)(S)=1$ iff $g(k)(w)(S)=1$.

(118) $[\![\exists^{\spadesuit}]\!](X) \in \{1, 0\}$ iff $Dom(X)\neq\varnothing$.
If $[\![\exists^{\spadesuit}]\!](X) \in \{1, 0\}$, $[\![\exists^{\spadesuit}]\!](X)=1$ iff $\{Z \in Dom(X)| X(Z)=1\}\neq\varnothing$.

Adopting (117) requires adopting the view that the extension of an interrogative is a set of true possible answers as in Karttunen (1977), and not a set of merely possible answers as in Hamblin (1973). Presupposition projection in polar and ‘alternative’ interrogatives are still accounted for under (117), as questions are still required to have satisfied presuppositions in order to be issued. However, the semantic presuppositions of *wh*-interrogatives and their pragmatics (see Sect. 3.3) would need to be reworked if Karttunen (1977) is adopted.

Appendix 3: Functional restrictors

It has been proposed (see, for example, von Fintel 1994) that quantificational restrictors can be “functional”. For example, in (119), where the subject of the prejacent of *even* varies across possible answers, the restrictor of *even* is the complex f_2-pro_4. *Who* binds its trace, t_4, and the co-indexed pro_4, which is the argument of the free pronoun f_2, whose value is determined by the context.

(119) a. Who even ate the CAKE?

b. $[\![$*who-C*$_1]\!]^g(\lambda x \lambda p.\ p = [\![$^[*even-f*$_2$*-pro*$_4$ [^[t_4 ate the cake$_F$] $\sim$ *f*$_2$*-pro*$_4$]]$]\!]^{g[4\rightarrow x]})$

Which x ∈ C1 is such that 'x ate the cake' is least likely in f2(x)?

c. C1 ⊇ {**j**, **b**, **k**}

f2(**j**) ⊆ {$\underline{[\![}$^[*John* *ate the cake*]$\underline{]\!]}$, $[\![$^[*John ate the candy*]$]\!]$, $[\![$^[*John ate the chips*]$]\!]$, …}

f2(b) ⊆ {$\underline{[\![}$ ^[*Bill* *ate the cake*]$\underline{]\!]}$, $[\![$^[*Bill ate the candy*]$]\!]$, $[\![$^[*Bill ate the chips*]$]\!]$, …}

f2(**k**) ⊆ {$\underline{[\![}$^[*Kat* *ate the cake*]$\underline{]\!]}$, $[\![$^[*Kat ate the candy*]$]\!]$, $[\![$^[*Kat ate the chips*]$]\!]$, …}

This raises the possibility that the 'alternative' interrogative in (120a) below (= (85b) in Sect. 4) could, in principle, have a satisfiable presupposition: f_3 in (120b) has a different silent pronominal argument in each of the disjuncts: the first is anaphoric to *the cake* and the second to *the candy*.

(120) a. *Did John even eat the cake$_{H*}$ or the candy$_{H*L\text{-}L\%}$

b. [5 [^[*even-f*$_3$*-pro*$_1$ [^[*John ate the cake*$_F$] ~*f*$_3$*-pro*$_1$]] *or*$_5$ ^[~~*even*~~-*f*$_3$*-pro*$_2$ [^[~~*John*~~ ~~*ate*~~ *the candy*$_F$] ~*f*$_3$*-pro*$_2$]]]]

c. f3($[\![$*pro*$_1]\!]^g$) (= f3($[\![$^[*the cake*] $]\!]$)) ⊆ {$[\![$^[*John ate the cake*] $]\!]$, …}

f3($[\![$*pro*$_2]\!]^g$) (= f3($[\![$^[*the candy*] $]\!]$)) ⊆ {$[\![$^[*John ate the candy*] $]\!]$, …}

We claim that (120b) is still excluded by DU, since the functional restrictors *f*$_3$*-pro*$_1$ and *f*$_3$*-pro*$_2$ are not identical.

Yet DU alone does not suffice to rule out (120a) once we acknowledge the availability of functional restrictors, because functional restrictors are not generally banned from constructions that involve ellipsis. For example, thanks to the assumption that Q(uantifier) R(aising) is available, the elided quantifier in (121) is licit.

(121) a. John met every student, and Bill did too. John's students have an exam tomorrow, but Bill's students don't – their exam is next week.

b. ^[*John* [1 *every student f*$_3$*-pro*$_1$ [2 t_1 *met* t_2]]] *and* ^[*Bill* [1 ~~*every*~~ ~~*student*~~ *f*$_3$*-pro*$_1$ [2 t_1 ~~*met*~~ t_2]]]

Nevertheless, the QR option is not available for (120a), for independent reasons. In other words, an independent constraint bans (122), where the focus-associates of *even* are traces bound from above *even* by *the cake* and *the candy*. That constraint does not ban either (119b) or (121b), which do not contain illicit traces.

(122) *[5 [^[*the cake* 1 [*even*-f_3-*pro*$_1$ [^[*John ate* [t_1]$_F$] ~f_3-*pro*$_1$]]] *or*$_5$
^[*the candy* 1 [*even*-f_3-*pro*$_1$ [^[*John ate* [t_1]$_F$] ~f_3-*pro*$_1$]]]]]

That such a constraint is indeed at play is corroborated by the fact that a QR-ed phrase cannot bind a trace that is the focus-associate of the focus-sensitive superlative operator *est*, despite the fact that a *wh*-phrase can bind such a trace. While (123a,b) are both acceptable (Szabolcsi 1986; Heim 1999), the reading of (124b) where *every student* binds a trace that is the focus-associate of *est* is not available.

(123) a. Who gave the most books to Joe? (cf. *ANN gave the most books to Joe*)
⟦*who*-C_1⟧^{g}(λzλp. p=
⟦^[*est*-f_3-*pro*$_1$ [^[2 [t_1]$_F$ *gave* d_2-*many books to Joe*] ~f_3-*pro*$_1$]]]⟧$^{g[1\rightarrow z]}$)
Which z ∈ C1 is such that ∀x≠z[x ∈ f3(z)]: z gave Joe more books than x gave Joe?

b. Who did Ann give the most books to? (cf. *Ann gave the most books to JOE*)
⟦*who*-C_1⟧^{g}(λzλp. p=
⟦^[*est*-f_3-*pro*$_1$ [^[2 *Ann gave* d_2-*many books to* [t_1]$_F$] ~f_3-*pro*$_1$]]]⟧$^{g[1\rightarrow z]}$)
Which z ∈ C1 is such that ∀x≠z[x ∈ f3(z)]: Ann gave z more books than Ann gave x?

(124) a. JANE gave the most books to every student.
every student [1 *est*-f_3-*pro*$_1$ [^[2 *Jane*$_F$ *gave* d_2-*many books to* t_1] ~f_3-*pro*$_1$]]
For every student x and every z in f3(x) such that z≠Jane:
Jane gave x more books than z gave x.

b. Jane gave the most books to EVERY STUDENT/every student.
**every student* [1 *est*-f_3-*pro*$_1$ [^[2 *Jane gave* d_2-*many books to* [t_1]$_F$] ~f_3-*pro*$_1$]]
For every student x and every z in f3(x) such that z≠x:
Jane gave x more books than Jane gave z.

(125) Where C is a set of degree-properties, $[\![est]\!]^{w}(C)(P) \in \{1, 0\}$ only if $P \in C$ and $\{d \mid P(w)(d)=1\} \neq \varnothing$.
If $[\![est]\!]^{w}(C)(P) \in \{1, 0\}$, $[\![est]\!]^{w}(C)(P)=1$ iff $\{d \mid \{Q \in C \mid Q(w)(d)=1\} =\{P\}\} \neq \varnothing$.
(Howard 2014; cf. Heim 1999)

Finally, it is worth noting that (100a) in Sect. 4—*If Mary is even dancing or swimming, then Sue is*—has a narrow-scope-*even* strict VP reading. Its DU-compliant LF in (126) below contains functional restrictors but does not contain illicit focused traces.

(126) *If*[1] ^[*Mary* 1 *even*-f_2-*pro*$_1$ [^$\exists$ 5 [^[t_1 *dance*$_F$] *or*$_5$ ^[t_1 *swim*$_F$]] ~f_2-*pro*$_1$]]
^[*Sue* 1 ~~*even*~~-f_2-*pro*$_1$ [^$\exists$ 5 [^[t_1 ~~*dance*~~$_F$] ~~*or*~~$_5$ [~~t_1~~ ~~*swim*~~$_F$]] ~f_2-*pro*$_1$]]

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